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DELAY DIFFERENTIAL EQUATIONS MODEL FOR CHILD BIRTH SPACING

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Abstract

Birth interval is a major determinant of the rate of fertility. Event histories such as birth, pregnancy and marriage have been used by social scientists to study fertility behavour of women. Birth history analysis undoubtedly provides useful information regarding reproduction and family formation. Fertility depends not only on the decision of couples but also on many social-economic, demographic, health related, education as well as traditional related and emotional factors. The factors affecting fertility may have varying effect on child spacing. Thus birth intervals experienced by women may reveal insight about their reproduction patterns. In this paper a delay differential equation model was formulated to determine the factors, effects and significance of birth interval to aid family formation. Stability theorem was used to carry out the stability analysis of the formulated model. In the analysis the model show instability in the system when delay is present. We used forth-order Runge-kutta method to solve the formulated model. Data was collected from health and medicals centre's in Taraba State to verify our model equations. The result show that when delay such as advance in women age at marriage and traditional and cultural practice that discourage marriages are introduced to the model it affects child birth spacing adversely.

Keywords**:** Child Birth spacing, Fertility, women education and age at marriage, traditional norrns and cultural practices

1.0 Introduction

Delay Differential Equation (DDEs) is a differential equation in which the derivatives of the unknown function at a certain time is given in terms of the values of the function at previous times. DDEs are also known as Time Delay System, Systems with after effect or dead-time, hereditary system, Equation with deviation argument or Differentia Difference Equations.

Birth interval is a major determinant of the rate of fertility. Event histories such as birth, pregnancy and marriage have been used by social scientists to study fertility behavior of women. Birth history analysis undoubtedly provides useful information regarding reproduction and family formation. Fertility depends not only on the decision of couples but also on many socio-economic, demographic, health-related as well as tradition-related and emotional factors. The factors affecting fertility may have varying effects on child spacing. Thus birth intervals experienced by women may reveals insights about their reproduction patterns. Furthermore a detailed analysis of the sequence of steps in the childbearing procedure could provide a more complete picture of the dynamics of fertility transitions [1].

The level of fruitfulness in a group of women depends mainly on four midway variables; the proportion that is married, post partum infecundability, contraception and induced abortion [2]. In other words, differences in exposure to the danger of pregnancy and differences in the length of time between births, when women are exposed may add to differentials in childbearing levels [3]. Whatever the cause, the length of birth intervals may vary from one population of women to another.

Social scientists suppose that differences in birth interval lengths are explained by varying breast feeding patterns, contraceptive use, rate of intercourse, incidence of abortion and fecundity [3]. Differences in other factors such as women's roles and status and the value of children may also influence the birth intervals. There is no doubt that the socioeconomic, demographic, health and cultural background of a country, consequently that of women, affects the above factors.

Women's education and age at marriage are the most widely analysed determinants of birth intervals. The former is found to have a considerable effect on birth intervals ([1,3]). However, in a study done in a village of Kerala State in India, did not find any important effects in terms of the education women on the birth intervals [5]. Nevertheless, male education and profession were found to be significant determinants of fruitfulness in Indonesia and Philippians [3].

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A couple's decision on timing of the first baby or the second or the third may depends on traditional norms and cultural practices as well. Analysis of birth intervals suggested that religion is also an important factor in the fertility behavior of women [5].

Wife of Lagos State Governor, Mrs. Abimboola Fashola has urged women to adopt proper child birth spacing to ensure survival and successful future of a child. Fashola made the call in Lagos at the 12th Babies and Moms EXPO on Wednesday organized by Afribaby Initiative and permitted "Great Mothers, Happy Babies". She said the inadequate child spacing could affect not only the health of children and mothers but also a child's future successes. According to her, two and a half of three years between births is usually best for the wellbeing of the mother and her children. Studies have shown that if couples space birth two to three years, death rate of children under the age of five could definitely drop. Planning enough time in between pregnancies increases the chances of healthy mother and existing children.

"It also allows parents to devote more time to each child in the early years, easing pressures on the family's finances and giving parents more time for other activities than child bearing alone". She said [12].

For the effectiveness of birth spacing, factors such as women's education and age at marriage should be given much consideration. It is the most widely analysed determinant of birth intervals. It has considerable effect on birth intervals ([1,4]).

Across the developed world couples are postponing parenthood. This review assesses the consequences of delayed family formation from a demographic and medical perspective. One main focus is on the quantitative importance of pregnancy postponement [11]. Rescheduling of parenthood is linked to a higher rate of involuntarily childlessness and smaller families than desired due to increased childlessness and fetal death with higher female and male age. For women, the increased risk of prolonged TTP, (time to pregnancy), infertility, spontaneous abortion, ectopic pregnancies and trisomy 21 starts at around 30 years of age with a more pronounced effects ?35 years whereas the increasing risk of premature births and still births starts at around 35 years with a more pronounced affect >40 years. Over the past forty years almost all developed countries have witnessed an increase in the age at entry into parenthood, albeit from different starting points. The delay in entry into motherhood is often referred to as fruitfulness delay [6]. The rearrangement of childbearing to later ages has vital implications for example in terms of health outcomes, increased infecundity and smaller completed family sizes. There remain a strong relationship between age at entry into parenthood and family size ([7-9]). Whilst it is difficult to statistically identify a causal relationship between them, there are a number of reasons why it can be difficult for women starting their childbearing in their mid to later thirties to fully recover births which were delayed from earlier ages. These include declining fruitfulness with age, competing commitments, and lack of a suitable partner with whom they want to have children [10].

In a number of European countries, the mean age of women at childbearing has surpassed the age of 30. However, such a late pattern of childbearing was not exceptional in the past, when families were larger and women often continued bearing children until the end of their reproductive age. The most radical transformation is related to the start of family building—the age when women give birth to their first child. In the countries of Western, Northern, and Southern Europe, first-time mothers are on average 26 to 29 years old, up from 23 to 25 years at the start of the 1970s. Postponement of first births, which in some countries has continued uninterrupted for more than three decades, has become one of the most prominent features of fertility patterns in developed societies.

These research explore several questions regarding fertility behavior of women and as well mortality rate using "Delay Differential Equation Model" a case study of Taraba State in Nigeria seek to know the factors, effects and significance of birth interval and will assist in the reduction of mortality rate. Despite the present concerns of health practitioners, parents, non-governmental organization and the government on reducing child mortality and ensuring child survival in Nigeria, the situation is rapidly increasing in both urban and rural areas. Taraba State is not exempted from these child mortality and survival problems. For this cause, we formulate a Delay Differential equation Model to predict child survival. Fourth order Runge-kuttanumerical method of solving Differential equation was use to solve the model.

2.0 METHODS

In this section, we present a model for a delay differential equation model for birth space, assumptions and method of analysis of the model.

The researchers begins the model formulation by first, presenting the assumptions, variables and parameters of the model.

Assumption of the model

The model equations base on the basic assumption that; (1) mortality rate (Q) increases due to inadequate birth spacing (δ). (2) The survival (β) of children under the age of five depends on time delay (τ) between one child birth to the next child (I). (3) Mortality (α) decreases due to proper "birth spacing (ε)". (4) Traditional norms and cultural practice (T) affect birth spacing at the rate $(\bar{x})$ and grows at the rate (μ). It drops at the rate (z). (5) The former can be regulated by women's education (θ) and age at marriage (t). When women are educated, the risk of improper birth space is minimized. (6) Women's education grows at the rate (w) and affect birth space at the rate (7) The rate at which improper birth spacing drops (γ) depends on a change in traditional norms and women education. (8) Religion (R) as well is a factor not to overlook when dealing with birth spacing. (9) The stability (S) of their laws affects birth space at the rate (σ) i.e. $dR/dt = 0(steadystate)$.

10. We assumed that any delay in child birth cause delay in subsequence birth.

To reduce mortality (α) an effort (ϵ) has to be applied and death rate (k) drops due to applied effort. The effort here means measures emanating from the government, health practitioners, parents and non-governmental organizations to ensure that the high rate of child mortality in Nigeria is reduced. Effort such as the use of contraceptives(η), induced abortion (I_a) to mention few are to be properly managed for maximal benefit.

Definition of variables and parameters of the model

ξ = Birth space
γ = Improper birth spacing
I = Interval between one child birth to the next
η = The use of contraceptives
τ = Delay term
T = Traditional norms and cultural practices
π = Rate at which traditional norms and cultural practices affect birth spacing
θ_t = Women's education and age at marriage
ω = Women's education grow rate
z = Rate at which traditional norms and cultural practices drops
E = Effort to reduce or minimize mortality
ρ = Rate at which mortality is minimized due to applied effort (E)
t = Delay term coefficient

Formulation of Model Equations

The Formulation of our model equations is hinge on the basic factors that affect birth spacing. These factors are as follows;

(i) Women education at marriage
(ii) Women age at marriage
(iii) Traditional norms and cultural practices

The model equations are given below
The transcendental equation of the delayed differential equation, at steady state determined for $\tau = o$ of the model is

$$\xi(t,\tau) = I_1(t) + I_2(t)e^{-t\tau} = 0 \qquad (1)$$
$$\xi(t,\tau) = I(t)e^{-t\tau} \qquad (2)$$

Where τ is the length of the discrete time delay and I is the interval between one child birth to the next in time t.
When one introduces a time delay into a system of differential equations, it is often of interest to determine whether or not birfurcations occur for various lengths of the delay. In particular, a stable steady state can become unstable if, by increasing the length of the time delay, the eigenvalues of the system go from having negative real parts to having positive real parts, and this occurs only if they traverse the imaginary axis. Many authors have utilized certain methods for determining if and when a birfurcation occurs about a steady state.
From equation (1), we can assume that the steady state about which we have linearised is stable in the absence of the delay. For $\tau = 0$ all of the polynomial have negative real part. As τ varies, these roots change. We are interested in a case where $\tau \neq 0$.
Hence; at $\tau \neq 0$
The systems of delay ordinary differential equations are;

$$\frac{dT}{dt} = T(z-\pi)e^{-t\tau} \qquad (3)$$
$$\frac{d\theta}{dt} = \theta(\omega-\eta)e^{-t\tau} \qquad (4)$$
$$\frac{d\gamma}{dt} = \gamma[(\omega+z-\sigma)+\rho^E]e^{-t\tau} \qquad (5)$$

Equation (3),(4) and (5) are the model equations which will be used to determine the interval of birth spacing between one child birth to the next by applying the fourth-order Runge-Kutta method of solving ordinary differential equation which is below

$$k_iT_n = hf[T_0(z-\pi)e^{-t\tau}]$$
$$k_i\theta_n = hf[\theta_0(\omega-\eta)e^{-t\tau}]$$
$$k_i\gamma_n = hf[\gamma_0(\omega+z-\sigma)+\rho^E]e^{-t\tau} \qquad (6)$$
$$k_2T_n = hf\left[T_0+\frac{1}{2}\kappa_1T, \gamma_0+\frac{1}{2}\kappa_1\gamma\right]$$
$$k_2\theta_n = hf\left[\theta_0+\frac{1}{2}\kappa_1\theta, \gamma_0+\frac{1}{2}\kappa_1\gamma\right]$$
$$k_2\gamma_n = hf\left[T_0+\frac{1}{2}\kappa_1T, \gamma_0+\frac{1}{2}\kappa_1\gamma, \theta_0+\frac{1}{2}\kappa_1\theta\right] \qquad (7)$$
$$k_3T_n = hf\left[T_0+\frac{1}{2}\kappa_2T, \gamma_0+\frac{1}{2}\kappa_2\gamma\right]$$
$$k_3\theta_n = hf\left[\theta_0+\frac{1}{2}\kappa_2\theta, \gamma_0+\frac{1}{2}\kappa_i\gamma\right]$$
$$k_3\gamma_n = hf\left[T_0+\frac{1}{2}\kappa_2T, \gamma_0+\frac{1}{2}\kappa_2\gamma, \theta_0+\frac{1}{2}\kappa_2\theta\right] \qquad (8)$$

$k_4T_n = hf[T_0 + \kappa_3 T, \gamma_0 + \kappa_3\gamma]$
$k_4\theta_n = hf[\theta_0 + \kappa_3\theta, \gamma_0 + \kappa_3\theta]$
$k_4\gamma_n = hf[T_0 + \kappa_3 T, \gamma_0 + \kappa_3\gamma, \theta_0 + \kappa_3\theta]$ (9)

RESULTS

In this chapter, the researcher will be considering the model in details by carrying out existence and stability of birth spacing free equilibrium state, implementation of the model equations using fourth-order Runge-kutta scheme of ordinary differential equation on the model and plotting a graph to show the relationship between the major variables.

Existence and Stability of Birth Spacing Free Equilibrium State of the Model.

The researcher investigates for the existence and stability of the birth spacing free equilibrium state of the model.

Existence of Birth Spacing Free Equilibrium State of the Model

At steady state (equilibrium),

$$\frac{dT}{dt} = \frac{d\theta_t}{dt} = \frac{d\gamma}{dt} = 0$$

Therefore, equating the left hand side (LHS) of the model equations; (3)-(5) gives;

$0 = T(Z - \pi)e^{-t\tau}$ (10)
$0 = \theta(\omega - \eta)e^{-t\tau}$ (11)
$0 = \gamma[(\omega + z - \sigma) + \rho^E]e^{-t\tau}$ (12)

Assume that $(T_0, \theta_0, \gamma_0, 0)$ is an equilibrium state of the model, then;

$T_0(Z - \pi)e^{-t\tau} = 0$ Implies ,$T_0 = 0$ (14)
$\theta_0(\omega - \eta)e^{-t\tau} = 0$ $\theta_0 = 0$ (15)
$\gamma_0[(\omega + z - \sigma) + \rho^E]e^{-t\tau} = 0$ $\gamma_0 = 0$ (16)

Therefore, the birth spacing free equilibrium of the model is
$G_0 = (0,0,0,0)$.

Stability Analysis of Delay Differential Equation

The researcher examine the local stability of DDE-Free equilibrium $C_0 = (0,0,0,0)$
Using the method of linearised stability, let

$f_1 = T(z - \pi)e^{-t\tau}$ (17)
$f_2 = \theta(\omega - \eta)e^{-t\tau}$ (18)
$f_3 = [\gamma(\omega + z - \delta) + \rho^e]e^{-t\tau}$ (19)

Then;

$$\frac{df_1}{dT} = (z - \pi)e^{-t\tau}, \quad \frac{df_1}{d\theta_t} = 0, \quad \frac{df_1}{d\gamma} = 0$$
$$\frac{df_2}{d\theta_t} = (\omega - \eta)e^{-t\tau}, \quad \frac{df_2}{dt} = 0, \quad \frac{df_2}{d\gamma} = 0$$
$$\frac{df_3}{d\gamma} = (\omega + z - \delta)e^{-t\tau}, \quad \frac{df_3}{dt} = 0, \quad \frac{df_3}{d\theta_t} = 0$$

Thus the Jacobian matrix associated with equation (3)-(5) at equilibrium states is given as;

$$X = \begin{pmatrix} ze^{-t\tau} - \pi e^{-t\tau} & 0 & 0 \\ 0 & \omega e^{-t\tau} - \eta e^{-t\tau} & \omega e^{-t\tau} - \mathrm{z}e^{-t\tau} - \delta^{-t\tau} \\ 0 & 0 & 0 \end{pmatrix}$$

The eigenvalues are calculated from the characteristic equation $\det(\lambda - \lambda\mathrm{I}) = 0$, where I is a$(3 \times 3)$ identity matrix.

$$/A - \lambda\mathrm{I}/ = \begin{pmatrix} ze^{-t\tau} - \pi e^{-t\tau} - \lambda & 0 & 0 \\ 0 & \omega e^{-t\tau} - \eta e^{-t\tau} - \lambda & 0 \\ 0 & 0 & \omega e^{-t\tau} + \mathrm{z}e^{-t\tau} - \delta e^{-t\tau} - \lambda \end{pmatrix}$$

$$(ze^{-t\tau} - \pi e^{-t\tau} - \lambda)[(we^{-t\tau} - \eta e^{-t\tau} - \lambda)(we^{-t\tau} - ze^{-t\tau} - \delta e^{-t\tau} - \lambda] = 0$$
$$(ze^{-t\tau} - \pi e^{-t\tau} - \lambda)[\omega^2 e^{-2t\tau} + \omega e^{-2t\tau} - \omega\delta e^{-2t\tau} - \omega e^{-t\tau}\lambda - \eta\omega e^{-2t\tau} - \eta z e^{-t\tau} + \eta\delta e^{-2t\tau} + \eta e^{-t\tau}\lambda - \omega e^{-t\tau}\lambda - ze^{-t\tau}\lambda + \delta e^{-t\tau}\lambda + \lambda^2] = 0$$

$(\mathrm{Z}\mathrm{e}^{-t\tau} - \pi\mathrm{e}^{-t\tau} - \lambda[\lambda^2 + (\eta\mathrm{e}^{-t\tau} - 2\omega\mathrm{e}^{-t\tau} - z\mathrm{e}^{-t\tau})\lambda + (\omega^2\mathrm{e}^{-t\tau} + \omega z\mathrm{e}^{-t\tau} - \omega\delta\mathrm{e}^{-t\tau} - \eta z\mathrm{e}^{-t\tau} + \eta\delta\mathrm{e}^{-t\tau})$ (20)

Thus, the linearization of the model at birth spacing free-equilibriumc_{30} gives the above characterization equation.
Also from the equation, it can be seen that

$Z\mathrm{e}^{-t\tau} - \pi\mathrm{e}^{-t\tau} - \lambda = 0$
$\lambda_1 = z\mathrm{e}^{-t\tau} - \pi\mathrm{e}^{-t\tau}$

Which is the first positive eigeenvalue. To obtain other values, we consider the equation.

$\lambda^2 + (\eta\mathrm{e}^{-t\tau} - 2\omega\mathrm{e}^{-t\tau} - z\mathrm{e}^{-t\tau} + \delta\mathrm{e}^{-t\tau})\lambda +$
$(\omega^2\mathrm{e}^{-2t\tau} - \omega\delta\mathrm{e}^{-2t\tau} - \eta\omega\mathrm{e}^{-2t\tau} - \eta z\mathrm{e}^{-2t\tau} + \eta\delta\mathrm{e}^{-2t\tau}) = 0$ (21)

It can be seen that equation (21) is quadratic equation of the form

$$A\lambda^2 + B\lambda + C = 0$$
$$A = 1$$
$$B = \eta e^{-t\tau} + \delta e^{-2t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau}$$
$$C = \omega^2 e^{-2t\tau} + \eta\delta e^{-2t\tau} - \omega ze^{-2t\tau} - \eta\omega e^{-2t\tau} - \eta ze^{-t\tau}$$

Using formula method of quadratic equation

$$\lambda = -\frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\lambda = -\Big((\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau}) \pm \sqrt{(\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau})^2 - 4(\omega^2\delta e^{-2t\tau} + \eta\delta e^{-2t\tau} - \omega ze^{-2t\tau} - \eta\omega e^{-2t\tau} - \eta ze^{-2t\tau})}\Big)/2$$

But

$$B^2 = (\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau})(\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau})$$
$$= \eta^2 e^{-2t\tau} + \eta\delta e^{-2t\tau} - 2\omega\eta e^{-2t\tau} - \eta ze^{-2t\tau} + \eta\delta^2 e^{-2t\tau} + \delta^2 e^{-2t\tau} - 2\omega\delta e^{-2t\tau} - z\delta e^{-2t\tau} - 2\omega\eta e^{-2t\tau} - 2\omega\delta e^{-2t\tau} + 4\omega^2 e^{-2t\tau} + 2\omega e^{-2t\tau}$$
$$B^2 = \eta^2 e^{-2t\tau} + 2\eta\delta e^{-2t\tau} - 4/\omega\eta e^{-2t\tau} - \eta ze^{-2t\tau} + \delta^2 e^{-2t\tau} - 4\,\omega/\delta e^{-2t\tau} - z\delta e^{-2t\tau} + 4\,\omega^2/e^{-2t\tau} + 2\omega ze^{-2t\tau}$$
$$-4c = -4\,\omega^2/e^{-2t\tau} - 4\eta\delta e^{-2t\tau} + 4\omega ze^{-2t\tau} + 4\,\omega/\delta e^{-2t\tau} + 4\eta\,\omega/e^{-2t\tau} + 4\eta ze^{-2t\tau}$$
$$B^2 - 4AC = \eta^2 e^{-2t\tau} - 2\eta\delta e^{-2t\tau} + 3\eta ze^{-2t\tau} + b\omega ze^{-2t\tau} + \delta^2 e^{-2t\tau}$$
$$\Rightarrow \lambda = \frac{-(+\eta e^{-2t\tau} + \delta e^{-2t\tau} - 2\omega e^{-2t\tau} - ze^{-2t\tau}) \pm \sqrt{\eta^2 e^{-2t\tau} + \delta^2 e^{-2t\tau} + 3\eta\delta e^{-2t\tau} + 6\omega ze^{-2t\tau}}}{2}$$

Let $V = \sqrt{\eta^2 e^{-2t\tau} + \delta e^{-2t\tau} - 2\omega e^{-2t\tau} + 3\eta ze^{-2t\tau} + 6\omega ze^{-2t\tau}}$

Then,

$$\lambda = \frac{-(\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau}) \pm V}{2}$$

Therefore,

$\lambda_2 = \frac{-(\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau}) \pm V}{2}$ and $\lambda_3 = \frac{-(\eta e^{-t\tau} + \delta e^{-t\tau} - 2\omega e^{-t\tau} - ze^{-t\tau}) - V}{2}$

The researcher assumed that the value of V is positive real number

Proposition 1: $\lambda_3 < 0$ and $\lambda_2 < 0$ if and only if $V > B$. Thus, from the proposition above, $\lambda_1 > 0$, $\lambda_2 > 0$ and $\lambda_3 < 0$. The birth spacing – free equilibrium is unstable.

Proposition 2: $\lambda_2 < 0$ and $\lambda_3 < 0$ if and only if $V < B$. The birth spacing – free equilibrium is asymptotically stable.

Implementation of Birth Space Model Equations

Here, the researcher implement the model equation by using quantitative data obtained by sample of 965 married women aged 18-49 years that referred to medical centre in Taraba State, where we used fourth order Runge-Kutta scheme for ordinary Differential Equation to solve the system.

Table 1, 2, 3, 4 show the composed of quantitative data of the interactions of birth space variables generated from medical centre Jalingo Taraba State.

Table 1 Description of Quantitative Variables

women's (men) view about no. of child	Number
Women's (men) view about birth interval (BI)	Month
Women's age at 1st marriage	Years
Description of categorical variables	
Variables	**Categories**
Birth interval	No interval (ref), < 18 months
	18-35 months, > 35 months
Contraceptive use	Natural method (ref), supply method, none
Women's education level	University (ref), high-school, secondary, primary, none
Women's (men) ethnicity	Jukun (ref), Tiv, Hausa, Others

Table 2 Estimated ratio of 1st birth interval

Variables	**1st birth interval (months)**		
	<18	**18-35**	**>35**
Traditional norms and cultural practices	0.43	0.44	0.35
Women's education	1.82	1.13	0.63
Women's age at marriage	0.16	0.17	0.27
Women's view about no. of child	0.24	0.11	0.001
Women's view about BI	0.004	0.17	0.012
Men's view about no. of child	0.32	0.43	0.41
Men's view about BI	0.45	0.50	0.38
Contraceptive used	0.89	0.70	1.26

Table 3 Estimated ratio of 2nd birth interval

Variables	**2nd birth interval (months)**		
	<18	**18-35**	**>35**
Traditional norms and cultural	0.65	0.93	0.53
Women's education	0.18	0.35	0.56
Women's age at marriage	0.40	0.36	0.44
Women's view about no. of child	0.65	0.93	0.53
Women's view about BI	0.004	0.17	0.012
Men's view about no. of child	0.92	0.89	0.53
Men's view about BI	0.55	0.60	0.42
Contraceptive used	0.47	0.76	0.92

Table 4 Estimated ratio of 3rd birth interval

Variables	**3rd birth interval (months)**		
	<18	**18-35**	**>35**
Traditional norms and cultural practices			
Women's education	0.38	0.82	-0.5
Women's age at marriage	0.30	0.40	0.38
Women's view about no. of child	0.83	0.81	0.78
Women's view about BI	0.004	0.17	0.012
Men's view about no. of child	0.65	0.69	0.50
Men's view about BI	0.60	0.70	0.61
Contraceptive used	0.68	0.74	1.01

$T_0 = 50, \omega = 0.8, \pi = 0.4,\ t = 0.3,\ \hbar = 0.1$

$\theta_0 = 50.5, \quad z = 0.6,\ E = 50,\ \tau = 1$

$\gamma_0 = 60, \eta = 0.5,\ \rho = 0.5,\ \delta = 0.4$

We coded the fourth-order Runge-Kutta scheme of the model equation using Octave programming language. The results shown in Table 5

Table 5:

YEARS	T	θ	γ
1	50.0000	50.5	60
2	52.1767	53	66
3	54.4481	55.5	73
4	56.8184	58.2	81
5	59.2918	61.1	90
6	61.8730	64.1	99
7	64.5665	67.2	109
8	67.3773	70.4	121
9	70.3105	73.9	134
10	73.3713	77.5	148
11	76.5654	81.2	163
12	79.8985	85.2	180
13	83.3767	89.3	199
14	87.0064	93.7	220
15	90.7940	98.3	243
16	94.7466	103	269
17	98.8712	108.1	297
18	103.1754	113.3	328
19	107.6669	118.8	363
20	112.3540	124.6	401
21	117.2451	130.7	443
22	122.3492	137.1	490
23	127.6754	143.7	541

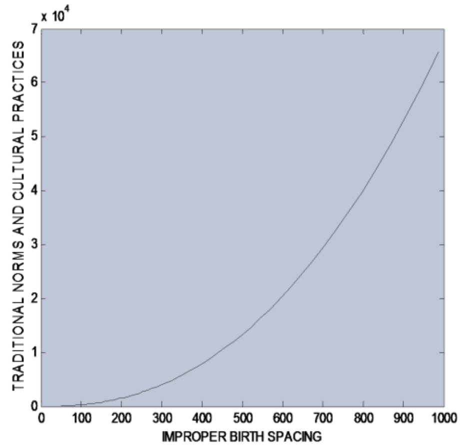

Figure 1: Graph of Traditional Norms and Cultural Practices Vs Improper Birth Spacing Model

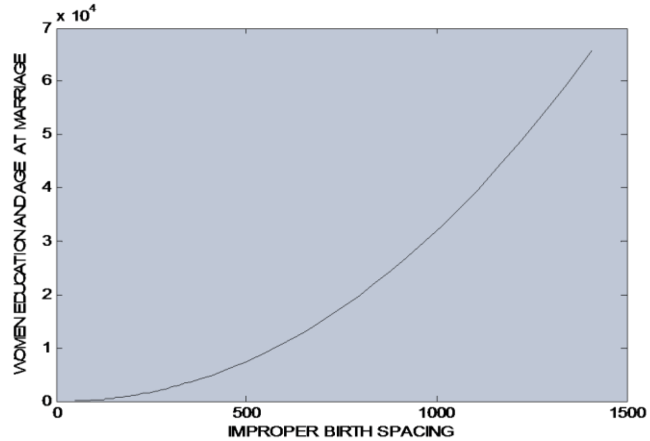

Figure 2: Graph of Women's Education and Age at Marriage Vs Improper Birth Spacing Model

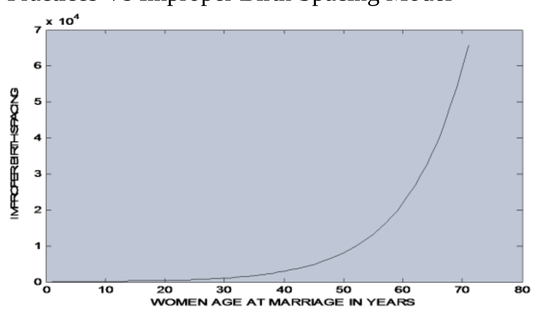

Figure 3: Graph of Improper Birth Spacing Vs Women's Age at Marriage

DISCUSSION AND CONCLUSION

In Figure 1 as traditional norms and cultural practices increases, improper birth spacing increases. In Figure 2 as women education and age at marriage increases, also improper birth spacing increases. It is in agreement with work done by ([1], [4]). In Figure 3, at 20 years improper birth spacing was minimal or no improper birth spacing, at 30 years and mid 30 years improper birth spacing increase a little, at 40 and 50 years improper birth spacing increases but at 50 and above improper birth spacing increases speedily. It is in line with work done by [10]. In conclusion women need to be encouraged by parents, societies, government and non-government organization to acquire early education in life and marry at least at age 20 and 30 years. Also, highly traditional norms and cultural practices that discourage women early education and marriages should be discouraged by parents, societies, government and non-government organizations. So that, to enable couples to get their desirable family size they wants. Moreover, our model can be used to predict women fruitfulness.

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