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SOFT MULTISET ORDERING

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Abstract

Soft set and Soft multisets are important tools for modelling uncertainty in real life situation. In this paper, we implore an idea upon which Demorgan's laws hold in soft multiset. In addition, the concept of soft multiset ordering relation is introduced and some related properties are presented.

Key Words: Multiset, Soft Set, Soft Multiset, Ordering

1.0 Introduction

Soft set which is a mapping from a set of parameters to a power set of a universe was initiated with the goal of modelling vagueness and uncertainty in real life situation. The theory is applicable to many different fields such as decision making, medical diagnosis, data analysis, forecasting, integration, game theory etc. [1, 2, 3, 4, 5].

Multiset (mset, for short) which is an unordered collection of objects where unlike a standard (Cantorian) set, duplicates or multiples of objects are admitted is initiated with the aim of addressing repetition which is significant in real life situations.

Besides in mathematics, multisets are being extensively used in theoretical computer science, biosystems, economics, formal language theory, social sciences and so on [6, 7, 8, 9, 10].

Soft multisets (Soft msets, for short) was first introduce in [11] using the idea of universes. Since then various scholars studies the theory using different approaches. Using the idea of [11], in [12] the notion of complement of a soft multiset is reintroduce and further shows that the laws of exclusion and contradiction are satisfied, which fails in [11]. The notion of distance and similarity between two soft multisets were provided in [13]. In closely similar way, soft multisets was defined in [14] as a mapping from parameter set to whole multisubset of a universal multiset and points out different set theoretic operations on it. An application of soft multiset in decision making problem is also discussed.

As Demorgan's laws fails in soft multiset, in this paper, we implore an idea upon which Demorgan's laws hold. In addition, Soft multiset ordering is introduced together with some of its properties.

2.0 Soft set

Definition 2.1 [15, 16, 17]

Let U be an initial universe set and E a set of parameters or attributes with respect to U. Let $P(U)$ denote the power set of U and $A \subseteq E$. A pair (F, A) is called a *soft set* over U, where F is a mapping given by $F: A \longrightarrow P(U)$.

In other words, a soft set (F, A) over U is a parameterized family of subsets of U. For $e \in A,\ F(A)$ may be considered as the set of e-elements or e-approximate elements of the soft set (F, A). Thus (F, A) is defined as

$(F, A) = \{F(e) \in P(U): e \in E, F(e) = \emptyset \text{ if } e \notin A\}$.

Definition 2.2 Let (F, A) and (G, B) be two soft sets over a common universe U, we say that

(a) (F,A) is a **soft subset** of (G,B), denoted $(F,A) \tilde{\subseteq} (G,B)$, if

(i) $A \subseteq B$, and

(ii) $\forall\ e \in A, F(e) \tilde{\subseteq} G(e)$.

(b) (F, A) is **soft equal** to (G, B), denoted $(F, A) = (G, B)$, if $(F,A) \tilde{\subseteq} (G,B)$ and $(G,B) \tilde{\subseteq} (F,A)$.

Definitions 2.3 Let (F,A) and (G,B) be two soft sets over a common universe U.

The union of (F,A) and (G,B), denoted $(F,A) \tilde{\cup} (G,B)$, is a soft set (H,C) where $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), & e \in A - B \\ G(e), & e \in B - A \\ F(e) \cup G(e), & e \in A \cap B. \end{cases}$$

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(i) The **extended intersection** of (F,A) and (G,B) , denoted $(F,A)\tilde{\cap}(G,B)$, is a soft set (H,C) where $C = A \cup B$ and $\forall\ e \in C$,

$$H(e) = \begin{cases} F(e), \text{ if } e \in A - B \\ G(e), \text{ if } e \in B - A \\ F(e) \cap G(e), \text{ if } e \in A \cap B. \end{cases}$$

(ii) The **restricted intersection** of (F,A) and (G,B), denoted $(F,A) \cap_R (G,B)$, is a soft set (H,C) where $C = A \cap B$ and $\forall\ e \in C,\ H(e) = F(e) \cap G(e)$. If $A \cap B = \emptyset$ then $(F,A) \cap_R (G,B) = \tilde{\Phi}$

(iii) The **restricted union** of (F,A) and (G,B), denoted $(F,A) \cup_R (G,B)$, is a soft set (H,C) where $C = A \cap B$ and $\forall\ e \in C$, $H(e) = F(e) \cup G(e)$. If $A \cap B = \emptyset$ then $(F,A) \cup_R (G,B) = \tilde{\Phi}$.

3.0 Multisets (msets, for short)

Definition 3.1 [18, 19] An mset M drawn from the set X is represented by a function *Count M* or C_M defined as $C_M: X \longrightarrow \mathbb{N}$.
Let M be a multiset from X with x appearing n times in M, this is denoted by $x \in^n M$. $M = \{k_1/x_1, k_2/x_1, \ldots, k_n/x_n\}$ where M is a multiset with x_1 appearing k_1 times, x_2 appearing k_2 times and so on.
Definitions 3.2 Let M and N be two msets drawn from a set X. Then
(a) $M \subseteq N$ iff $C_M(x) \leq C_N(x)$ for all $x \in X$.
(b) $M = N$ if $C_M(x) = C_N(x)$ for all $x \in X$.
(c) $M \cup N = max\{C_M(x), C_N(x)\}$ for all $x \in X$.
(d) $M \cap N = min\{C_M(x), C_N(x)\}$ for all $x \in X$.
(e) $M - N = max\{C_M(x) - C_N(x), 0\}$ for all $x \in X$.
Let M be a multiset drawn from a set X. The support set of M denoted by M^* is a subset of X given by $M^* = \{x \in X: C_M(x) > 0\}$.
Note that $M \subseteq N$ iff $M^* \subseteq N^*$.
The power multiset of a given mset M, denoted by $P(M)$ is the multiset of all submultisets of M, and the power set of a multiset M is the support set of $P(M)$, denoted by $P^*(M)$.

4.0 Soft Multiset (Soft mset, for short)

Definition 4.1 [20, 21] Let U be a universal multiset, E be a set of parameters and $A \subseteq E$. Then a pair (F,A) or F_A is called a soft multiset where F is a mapping given by $F: A \longrightarrow P^*(U)$.
Example 4.2 Let the universal mset $U = \{3/w, 2/x, 4/y, 1/z\}$, the parameter set $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, $A = \{e_1, e_2, e_3\}$ and the mapping $F: A \longrightarrow P^*(U)$ be defined as
$F(e_1) = \{2/w, 1/y, 1/z\}, F(e_2) = \{1/w, 2/x, 3/y\}$ and $F(e_3) = \{1/x, 2/y\}$.
Thus, $(F,A) = \{(e_1, \{2/w, 1/y, 1/z\}), (e_2, \{1/w, 2/x, 3/y\}), (e_3, \{1/x, 2/y\})\}$.
Definition 4.3 Let (F,A) and (G,B) be two soft multisets over U. Then
(a) (F,A) is a soft submultiset of (G,B) written $(F,A) \sqsubseteq (G,B)$ if
i. $A \subseteq B$
ii. $C_{F(e)}(x) \leq C_{G(e)}(x), \forall x \in U^*, \forall e \in A$.
$(F,A) = (G,B) \Leftrightarrow (F,A) \sqsubseteq (G,B)$ and $(G,B) \sqsubseteq (F,A)$.
Also, if $(F,A) \sqsubset (G,B)$ and $(F,A) \neq (G,B)$ then (F,A) is called a proper soft submset of (G,B) and (F,A) is a whole soft submset of (G,B) if $C_{F(e)}(x) = C_{G(e)}(x), \forall x \in F(e)$.
Example 4.4
Considering (F,A) in Example 4.2, if $B = \{e_1, e_2, e_3\}$ and $C = \{e_1, e_3\}$, then
$(G,B) = \{(e_1, \{2/w, 1/z\}), (e_2, \{1/w, 2/x, 2/y\}), (e_3, \{1/x, 2/y\})\}$ is a proper soft submultiset of (F,A), and
$(H,C) = \{(e_1, \{2/w, 1/y, 1/z\}), (e_3, \{1/x, 2/y\})\}$ is a whole soft submultiset of (F,A).
(b) **Union:**
$(F,A) \sqcup (G,B) = (H,C)$ where $C = A \cup B$ and $C_{H(e)}(x) = max\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(c) **Intersection:**
$(F,A) \sqcap (G,B) = (H,C)$ where $C = A \cap B$ and $C_{H(e)}(x) = min\{C_{F(e)}(x), C_{G(e)}(x)\}, \forall e \in C, \forall x \in U^*$.
(d) **Difference:** $(F,E)\backslash(G,E) = (H,E)$ where $C_{H(e)}(x) = max\{C_{F(e)}(x) - C_{G(e)}(x), 0\}, \forall x \in U^*$.
(e) **Null:** A soft multiset (F,A) is called a Null soft multiset denoted by Φ if $\forall e \in A, F(e) = \emptyset$.
Complement: The complement of a soft multiset (F,A), denoted by $(F,A)^c$, is defined by $(F,A)^c = (F^c, A)$ where $F^c: A \longrightarrow P^*(U)$ is a mapping given by $F^c(e) = U \backslash F(e), \forall e \in A$ where $C_{F^c(e)}(x) = C_U(x) - C_{F(e)}(x),\ \forall x \in U^*$.
Remark 4.5 It is observe that, Demorgan's laws fails. For example,

Let $U = \{3/w, 7/x, 9/y, 8/z\}$, $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$, $A = \{e_1, e_2, e_3\}$, $B = \{e_3, e_4\}$ and
$(F, A) = \{\ (e_1, \{2/x, 3/y\}), (e_2, \{1/x, 2/z\}), (e_3, \{2/w, 5/x, 3/y\})\}$
$(G, B) = \{\ (e_3, \{2/x, 4/y\}), (e_4, \{5/x, 5/y, 7/z\})\}$
Then,
$(F, A)^c = \{\ (e_1, \{3/w, 5/x, 6/y, 8/z\}), (e_2, \{3/w, 6/x, 9/y, 6/z\}), (e_3, \{1/w, 2/x, 6/y, 8/z\})\}$
$(G, B)^c = \{\ (e_3, \{3/w, 5/x, 5/y, 8/z\}), (e_4, \{3/w, 2/x, 4/y, 1/z\})\}$.
$(F, A) \sqcup (G, B) = \{\ (e_1, \{2/x, 3/y\}), (e_2, \{1/x, 2/z\}), (e_3, \{2/w, 5/x, 4/y\}), (e_4, \{5/x, 5/y, 7/z\})\}$.
$\left((F, A) \sqcup (G, B)\right)^c = \{(e_1, \{3/w, 5/x, 4/y, 8/z\}), (e_2, \{3/w, 6/x, 9/y, 6/z\}), (e_3, \{1/w, 2/x, 5/y, 8/z\}), (e_4, \{3/w, 2/x, 4/y, 1/z\})\}$.
Moreover,
$(F, A)^c \sqcap (G, B)^c = \{\ (e_3, \{1/w, 2/x, 5/y, 8/z\})\} \neq \left((F, A) \sqcup (G, B)\right)^c$.
Similarly, $(F, A)^c \sqcup (G, B)^c \neq \left((F, A) \sqcap (G, B)\right)^c$.
Thus, the following are provided from which the Demorgan's laws hold.

Definition 4.6 The power soft multiset of a given soft multiset F_A denoted $P(F_A)$ or $P(F, A)$ is defined as the multiset of all soft submultisets of (F, A).

Example 4.7
Let $(F, A) = \{(e_1, \{1/x\}), (e_2, \{2/x, 2/y\})\}$ then
$P(F_A) = \{\Phi, \{(e_1, \{1/x\})\}, \{(e_2, \{1/x\})\}, \{(e_2, \{1/x\})\}, \{(e_2, \{1/y\})\}, \{(e_2, \{1/y\})\}, \{(e_2, \{2/x\})\}, \{(e_2, \{2/y\})\}, \{(e_2, \{1/x, 1/y\})\}, \{(e_2, \{1/x, 1/y\})\}, \{(e_2, \{1/x, 1/y\})\}, \{(e_2, \{1/x, 1/y\})\}, \{(e_2, \{2/x, 1/y\})\}, \{(e_2, \{2/x, 1/y\})\}, \{(e_2, \{1/x, 2/y\})\}, \{(e_2, \{1/x, 2/y\})\}, \{(e_2, \{2/x, 2/y\})\}, \{(e_1, \{1/x\}), (e_2, \{2/x, 2/y\})\}\}$.
Moreover, the power soft set of (F, A) is
$\{\Phi, \{(e_1, \{1/x\})\}, \{(e_2, \{1/x\})\}, \{(e_2, \{1/y\})\}, \{(e_2, \{2/x\})\}, \{(e_2, \{2/y\})\}, \{(e_2, \{1/x, 1/y\})\}, \{(e_2, \{2/x, 1/y\})\}, \{(e_2, \{1/x, 2/y\})\}, \{(e_2, \{2/x, 2/y\})\}, \{(e_1, \{1/x\}), (e_2, \{2/x, 2/y\})\}\}$.

Definition 4.8 Let $(G, B) \in P(F, A)$, the relative complement of (G, B) in $P(F, A)$ denoted $\overline{(G, B)} = (\bar{G}, B)$ with $\bar{G}: B \longrightarrow P^*(U)$ defined as $\bar{G}(e) = F(e) - G(e)$ where $C_{\overline{G(e)}}(x) = C_{F(e)}(x) - C_{G(e)}(x), \forall e \in A, \forall e \in U^*$.
Clearly, for every $(G, B) \in P(F, A), C_{G(e)}(x) \leq C_{F(e)}(x), \forall e \in B, \forall e \in U^*$.

Theorem 4.9
(i) $\overline{\overline{(G, B)}} = (G, B)$
(ii) $\overline{(G, B)} \sqcup \overline{(G, B)} = \overline{(G, B)}$
(iii) $\overline{(G, B)} \sqcap \overline{(G, B)} = \overline{(G, B)}$
(iv) $\overline{\left((G, B) \sqcup (H, C)\right)} = \overline{(G, B)} \sqcap \overline{(H, C)}$ and $\overline{\left((G, B) \sqcap (H, C)\right)} = \overline{(G, B)} \sqcup \overline{(H, C)}$.

Proof
The proofs of (i), (ii) and (iii) are trivial.
(iv) Suppose $(G, B), (H, C) \in P(F, A)$.
Let $(G, B) \sqcup (H, C) = (J, D)$ and $(G, B) \sqcap (H, C) = (K, L)$ where $D = B \cup C$ and $L = B \cap C$.
Then, for all $e \in D, J(e) = G(e) \cup H(e)$.
Moreover, $\overline{\left((G, B) \sqcup (H, C)\right)} = \overline{(J, D)} = (\bar{J}, D) \Longrightarrow \forall e \in D$ we have
$\bar{J}(e) = F(e) - J(e) = F(e) - [G(e) \cup H(e)] = [F(e) - G(e)] \cap [F(e) - H(e)]$
$= \bar{G}(e) \cap \bar{H}(e) = \overline{(G, B)} \sqcap \overline{(H, C)}$.
i.e., $\overline{\left((G, B) \sqcup (H, C)\right)} \sqsubseteq \overline{(G, B)} \sqcap \overline{(H, C)}$ …………..(1)
Also, $\overline{(G, B)} \sqcap \overline{(H, C)} = (\bar{G}, B) \sqcap (\bar{H}, C) = (\bar{K}, L) \Longrightarrow \forall e \in L$ we have
$\bar{K}(e) = \bar{G}(e) \cap \bar{H}(e) = [F(e) - G(e)] \cap [F(e) - H(e)]$
$= F(e) - [G(e) \cup H(e)] = F(e) - J(e) = \bar{J}(e) = \overline{(J, D)} = \overline{\left((G, B) \sqcup (H, C)\right)}$
i.e., $\overline{(G, B)} \sqcap \overline{(H, C)} \sqsubseteq \overline{\left((G, B) \sqcup (H, C)\right)}$ …………..(2)
From (1) and (2), $\overline{\left((G, B) \sqcup (H, C)\right)} = \overline{(G, B)} \sqcap \overline{(H, C)}$.
For the other part, we have for each $e \in L, K(e) = G(e) \cap H(e)$ and as
$\overline{\left((G, B) \sqcap (H, C)\right)} = \overline{(K, L)} = (\bar{K}, L)$ we have $\forall e \in L$
$\bar{K}(e) = F(e) - K(e) = F(e) - [G(e) \cap H(e)] = [F(e) - G(e)] \cup [F(e) - H(e)]$
$= \bar{G}(e) \cup \bar{H}(e) = (\bar{G}, B) \sqcup (\bar{H}, C) = \overline{(G, B)} \sqcup \overline{(H, C)}$.
i.e., $\overline{\left((G, B) \sqcap (H, C)\right)} \sqsubseteq \overline{(G, B)} \sqcup \overline{(H, C)}$ …………..(3)
Now, $\overline{(G, B)} \sqcup \overline{(H, C)} = (\bar{G}, B) \sqcup (\bar{H}, C) = (\bar{J}, D)$ and by definition, $\forall e \in L$ we have
$\bar{J}(e) = \bar{G}(e) \cup \bar{H}(e) = [F(e) - G(e)] \cup [F(e) - H(e)]$
$= F(e) - [G(e) \cap H(e)] = F(e) - K(e) = \bar{K}(e) = \overline{(K, L)} = \overline{\left((G, B) \sqcap (H, C)\right)}$
i.e., $\overline{(G, B)} \sqcup \overline{(H, C)} \sqsubseteq \overline{\left((G, B) \sqcap (H, C)\right)}$ …………..(4)

From (3) and (4), we have $\overline{((G,B) \sqcap (H,C))} = \overline{(G,B)} \sqcup \overline{(H,C)}$.

5.0 Soft Multiset Ordering

Definition 5.1 Let $S(U,E)$ denotes the set of all soft multisets over the universe U and the parameter set E.

Definition 5.2 Let $\ll$ be an ordering relation on $S(U,E)$ and $(F,A),(G,B) \in S(U,E)$. Then $(F,A) \ll (G,B)$ if $A = B$ and $F(e) \subseteq G(e), \forall e \in A$.

Example 5.3

Let$(F,A) = \{(e_1,\{1/x,2/y,3/z\}),(e_2,\{4/w\})\}$ and $(G,B) = \{(e_1,\{2/x,3/y,4/z\}),(e_2,\{4/w\})\}$. Then $(F,A) \ll (G,B)$.

Definition 5.4 Let $\lll$ be an ordering relation on $S(U,E)$ and $(F,A),(G,B) \in S(U,E)$. Then $(F,A) \lll (G,B)$ if $A \subseteq B$ and $F(e) = G(e), \forall e \in A$.

Example 5.5

Let$(F,A) = \{(e_1,\{1/x,2/y,3/z\}),(e_2,\{4/w\})\}$ and $(G,B) = \{(e_1,\{2/x,3/y,4/z\}),(e_2,\{4/w\}),(e_3,\{2/w,3/y,1/z\})\}$. Then $(F,A) \lll (G,B)$.

Theorem 5.6 Let $\preccurlyeq$ be an ordering relation on $(S(U,E), \sqcup, \sqcap)$, where $\sqcup$, $\sqcap$ denotes soft multiset union and intersection respectively, and suppose $(F,A),(G,B) \in S(U,E)$. Then $(F,A) \preccurlyeq (G,B)$ iff $A \subseteq B$ and $F(e) \subseteq G(e), \forall e \in A$.

Proof

Let $(F,A) \preccurlyeq (G,B)$. Then $(F,A) \sqcup (G,B) = (G,B)$ and $A \cup B = B$ by definition, thus $A \subseteq B$.

By $F(e) \cup G(e) = G(e), \forall e \in A$, it follows that $F(e) \subseteq G(e)$.

Conversely, let $A \subseteq B$ and $F(e) \subseteq G(e), \forall e \in A$. Obviously, $(F,A) \sqcup (G,B) = (G,B)$, and thus, $(F,A) \sqcup (G,B) = (G,B)$ and hence, $(F,A) \preccurlyeq (G,B)$.

Conclusion

The paper deals with soft multiset and some of its algebras, thus the concepts discussed here seem to have an application in decision making pattern recognition coding theory and many areas in mathematics and computer science.

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