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COMPARATIVE STUDY OF THE PERFORMANCE OF MEWMA AND HOTTELLING'S T^2 CONTROL CHART

Nura Muhammad, Mansur Idi, Ameer Hassan, Aminu S. M, Abdurrahman Abdul Hamid and Kabiru M.

Department of Statistics, Kano State Polytechnic, Kano, Nigeria

Abstract

This research presents a comparative study of the performance of MEWMA, as well as Hotelling's T^2 Control Charts. The objective of this research is to verify when Mewma and Hotelling's T^2 Control Chart perform become the best, in terms of detecting small changes in the process average. Using the Data obtained from Bagco super sack plc, both two techniques were used to detect the out of control signal conditions.T^2 Found by each Chart was then compared. It was discovered that Mewma Control Chart practically out-performsHotelling's T^2in terms of detection ability of small and moderate shifts. Forthese variation levels, that is where Hotelling's T^2cannot detect a single shifts,Mewma was able to detect nine points' out-of-control signals, based on these results then Mewma Control Chart was more efficient in detecting small and moderate shifts than Hotelling's T^2control Chart.

Keywords: Statistical process control, Mewma Statistic, Hotelling's T^2

I.0 Introduction:

The main reason of Statistical process control is to improve the quality of goods produce by manufacturing industries. One of the tools used for quality improvement, is the Control Chart. Control Charts are more efficient instruments for checking changes in the processes. The choice of the Control Charts to be used depends on the characteristics to be measured in the process, as well as the way that these samples are taken. In order to monitor in-control mean vectorμ_{y0}, a standard approach is to monitor the observations on y_j with a control charts such as Hotelling$'s$ T^2 control chart and multivariate exponentially-weighted moving average, [**1**].In the control process, the data routinely collected are used and this information is employed in a practical way for the staff, engineers and managers to work on the Process improvement. In this way the cost of implementing these improvements in quality and productivity is almost insignificant. One of the procedures applied; in this kind of confirmation is control charts [**2**].The goal of statistical process control is to reduce the variability and the control charts are efficient tools to reduce this variability as much as possible [**2**].

Montgomery [**3**]highlighted five reasons for the control chart popularity:

i. Control charts are proven technique for the improving productivity. A program that uses control Charts can reduce waste and the rework, which harm productivity in any operation. In this way there is a production increase and a cost decrease.
ii. Control charts are efficient in preventing faults, helping to keep the process in control. It is the Philosophy of doing it right from the first time; it is more expensive to classify faulty and perfect items than manufacturing just good items. If there is not an efficient process, somebody is being paid to produce items of no good quality.
iii. Control charts prevent unnecessary process adjustments. A control chart can distinguish between a common cause and a special cause of variation. Unnecessary adjustments can result in a deterioration of the process development.
iv. Control charts provide diagnosis information. The point drawing shape that the control chart gets, will often contain information with diagnosis value for an experienced operator or engineer.
v. Control charts produce information about the process capacity, through the value of their parameters and stability about the time. This allows an estimate of the process capacity. This information is of an extraordinary use for product and process engineers.

Corresponding Author**:** Nura M., Email: muhdnuru@gmail.com, Tel: +2348065426933

1. Multivariate Ewma Control Chart:

Lowry et al. [1] proposed the Mewma Chart as natural extension of Ewma Chart. It is a popular Chart used to monitor a process with p quality characteristics for detecting small to moderate shifts. The in-control process mean is assume without loss of generality to be a vector of zero, and covariance matrix $\sum$.The MEWMA control statistic is defined as vectors,

$Z_{n=\lambda y_{n+(1-\lambda)Z_{n-1},}}$ $\quad n=1,2,\ldots\ldots\ldots\ldots$ [1]

Where,

$X_{0=0,} \quad 1\times p \ \ vevtor \ \ and \ \ \lambda=diag\,(\lambda_{1,}\lambda_{2,\ldots\ldots}\lambda_{p}),0<\lambda_i \leq 1$, I=1, 2.., p [2]

The Mewma Chart gives an out-of-control signal as soon as

T=$Z_i^1 \sum_{zi}^{-1} Z_i$>h [3]

Where h (>0) is chosen to achieve a specified in control ARL and $\sum_{xn}$ is the covariance matrix of X_n given by $\sum_{xn}$= {λ/(2-λ)}Σ,under equality of weights of past observations for all p characteristics;λ_1=λ_2=…=λ_p.If one or more points fall beyond h, the process is assumed to be out-of-control. The magnitude of the shift is reflected in the non-centrality parameter$\mu_1^{'}\sum^{-1}\mu_1$.they conclude that an assignable causes result in a shift in the process mean from$\mu_0$to$\mu_{1.}$

2. Over-view of Hotelling's T^2 _Chart:

Indeed in Multivariate SPC, a signal can be caused by variety of situations. For example, an observation on one of the p quality characteristics may be out of control. Similarly, the signal may be due to a relationship between two or more of the variables. Worse yet, a signal may be produced by combination of these two situations, with some variables being out of control and others have counter relationships. [4].One fundamental assumptions of using Hotelling's$T^2$is to describe the nature of statistical distance that is observation vector must follow a multivariate normal distribution. Using the assumption ($X_{1,}$, X_2) can be considered jointly using matrix notation:

$$(X-\mu)^{'}\sum^{-1}(X-\mu)=SD^2$$

Where $X^{'}=(x_1,x_2)$, $\mu^{'}=(\mu_1,\mu_2)$ and $\sum^{-1}$ is the inverse of the covariance matrix. This formula is the quadratic form of vector $(X-\mu)$ that represents the statistical distance. Therefore the equation is known as Hotelling'sT^2 statistic.

$$T^2=n(\bar{x}-\mu)^{'}S^{-1}(\bar{x}-\mu)=SD^2$$

Hotelling's$T^2$is a well-known control statistic for monitoring and control of a Multivariate process. The statistic has some characteristics. Among its characteristics is its dependency on the correlation, however, the need to construct shewart charts monitoring with a T^2 statistic is quite enough, if the correlation exists, T^2can be decomposed into orthogonal components. These components reveal how each variable is associated with the remaining variables [4]. The knowledge of these relationships is very helpful in interpreting multivariate control chart-signals.

3. Description of the methodology used:

To reach the main purpose of this research, the data were generated from Bagco super sack plc.Sharara Phase II Kano, Nigeria. The data is then used togenerate a real data set collected at every point where there is a variation; four variables were monitored for twelve observations were recorded. The four readings of latex, polyester, cyclone and coolant were taken at different interval from a machine with twelve observations each.

II. Illustration

Let$y_1 = latex$, y_2= polyester, y_3= cyclone and y_4= coolant.

Table 1: Data for Bagco Super Sack

No. of observation	y_1	y_2	y_3	y_4
1	750	746	732	750
2	746	740	734	744
3	744	740	742	742
4	746	740	740	746
5	744	744	744	720
6	744	740	738	740
7	740	742	732	746
8	740	750	736	742
9	740	750	734	740
10	742	754	730	740
11	740	754	732	742
12	740	750	736	744
Total	8920	8950	8830	8996
Average	743.5	745.8	735.8	741.3

The estimated covariance matrix of the data above is given by:

$$S = \begin{bmatrix} 10.2727 & -7.3636 & 1.7273 & 2.9091 \\ -7.3636 & 30.8788 & -13.4848 & -1.2121 \\ 1.7273 & -13.4848 & 19.2424 & -18.3030 \\ 2.9091 & -1.2121 & -18.3030 & 54.0606 \end{bmatrix}$$, with the inverse,

$$S^{-1} = \begin{bmatrix} 0.1199 & 0.0329 & 0.0102 & -0.0023 \\ 0.0329 & 0.0713 & 0.0691 & 0.0232 \\ 0.0102 & 0.0691 & 0.1482 & 0.0512 \\ -0.0023 & 0.0232 & 0.0512 & 0.0365 \end{bmatrix}$$

$$UCL = \frac{p(n+1)(n-1)}{n(n-p)} F_{0.05,4,8} = \frac{4(5)(3)}{12(8)}(3.84) = 15.25$$

The computation of the individual T^2-value was obtained using STATGRAPHICS and is as shown in the table 2 below:

Table 2: Computation of Hotelling's T-square

No. of observation	Hotelling's T^2-value	Critical value F control function
1	5.8870	15.25
2	3.0367	
3	3.2973	
4	3.1585	
5	8.7068	
6	1.3363	
7	6.0043	
8	2.0468	
9	2.1050	
10	3.1707	
11	2.4999	
12	2.7579	

From the table (2) above, it is obvious that all twelve values fall within the control limit. Hence, we concluded that the production process was in-control. Below is the graph of Hotelling's T^2 for individual observation; where the individual T^2 Values were plotted against the number of observations, and the dotted line represents the control limit.

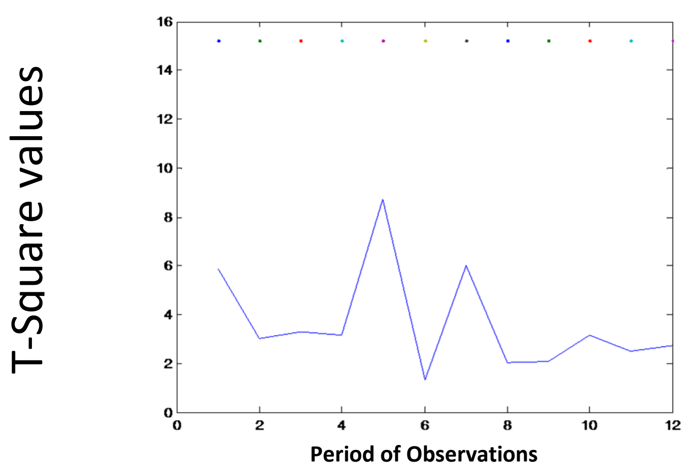

Figure 1: Chart of Hotelling's T-square Statistic

While the computation of Mewma Statistic was also done with aids of STATGRAPHICS with the same set of data and result was shown as:

Table 3: Computation of Mewma Statistic

Observations	T-squared	Critical value F Control function
1	21.44* *	8.84
2	27.00* *	
3	27.63* *	
4	40.66* *	
5	46.44* *	
6	48.65* *	
7	26.20* *	
8	7.24	
9	6.89	
10	2.32	
11	21.82* *	
12	31.04* *	

From table 3 above, it is obvious that nine values went beyond the control limit. Hence, we concluded that the production process was out-of-control. Below is the graph of Mewma statisticfor individual observation; where the individual Values were plotted against the number of observations, and the red straight line represents the control limit.

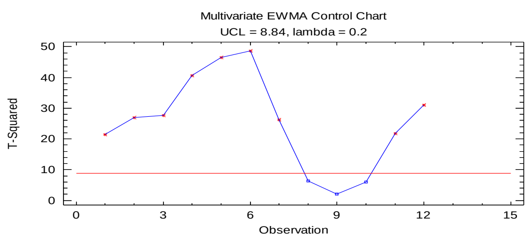

Figure 2: MEWMA Control Chart

4. Comparison of Results: Table 2 and 3 summarized our main findings: the Mewma control Chart was able to identified nine points above the control limits while Hotelling's T^2 did not detect a single point as out-of-control signal, based on this we concluded that Mewma statistic is better alternative to Hotelling's T^2. Sincerely speaking when we began this research, we fully expected to identify scenarios in which the Hotelling's T^2 Performed extremely better than the MEWMA but the reverse is the case, because there is strong advantage in using Mewma method over the Hotelling's T^2.

III. Conclusion

The systematic use of Control Chart is a very good way of minimizing the variability in the production process. The study has been able to show that Mewma is truly superior alternative to Hotelling's T^2especially in the detection of small and moderate shifts. Therefore, when the interest is to detect small and moderate variation Mewma is recommended to be use notHotelling'sT^2.

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