Journal of the Nigerian Association of Mathematical Physics
***Volume 47* (*July, 2018 Issue*), pp173 – 182**

A GENERALIZED AKASH DISTRIBUTION AND ITS APPLICATION TO SURVIVAL DATA

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Abstract

In this paper, we proposed a new two-parameter lifetime distribution and some of its Mathematical properties such as the density function, cumulative distribution function, hazard rate function, mean residual life function, moment generating function, moments and the Renyi entropy are obtained. The maximum likelihood method was employed in estimating the parameters of the proposed distribution. Finally, an application of the proposed distribution to two real data sets are presented and compared with the fit attained by some existing lifetime distributions. It follows from our result that the proposed distribution was superior in terms of some measure of goodness of fit test-statistic.

***Keywords*:** Akash distribution, Lindley Distribution, Hazard rate, Renyi entropy, Moments

1.0 Introduction

The Lindley distribution was introduced in [1]. Itis a mixture of the exponential distribution and a special gamma distribution. This distribution has receivedconsiderable attention and have also attracted a wide range of applicability in the area of medicine, engineering, insurance, finance and many others. The properties of the Lindley distribution and its usefulness in analyzing lifetime data set was highlighted in [2]. The generalized Lindley distribution which is also an extension of the one-parameter Lindley distribution was introduced in [3] and its superiority over the popular Gamma, Weibull and Lognormal models were established. The two-parameter weighted Lindley distribution and its usefulness in modelling mortality data was introduced in [4]. The Power Lindley distribution, Quasi Lindley distribution and its application tosocial sciences data,an estimation of the reliability of a stress-strength system from Power Lindley distribution and the Lindley-Exponential distribution model with applications to biological data was introduced in [5-8] respectively. Variant of this distribution called Akash distribution was proposed in [9].

A random variable X follows the one-parameter Akash distribution if its probability density function (pdf) is defined by:

$$f(x,\lambda) = \frac{\lambda^3}{\lambda^2+2}\left(1+x^2\right)e^{-\lambda x} \quad ; \; x>0, \;\; \lambda>0 \qquad (1.1)$$

This distribution is a two-component mixtures of an exponential(λ) and gamma (3,λ) distributions with their mixing proportions $\frac{\lambda^2}{\lambda^2+2}$ and $\frac{2}{\lambda^2+2}$ respectively.

The corresponding cumulative distribution function (cdf) of (1.1) is given by

$$F(x,\lambda) = 1-\left[1+\frac{\lambda x(\lambda x+2)}{\lambda^2+2}\right]e^{-\lambda x} \; ; \; x>0, \;\; \lambda>0 \qquad (1.2)$$

and the first four raw moments of the Akash distribution have been obtained as

$$\mu_1' = \frac{\lambda^2+6}{\lambda\left(\lambda^2+2\right)} \quad \mu_2' = \frac{2(\lambda^2+12)}{\lambda^2\left(\lambda^2+2\right)} \quad \mu_3' = \frac{6(\lambda^2+20)}{\lambda^3\left(\lambda^2+2\right)} \quad \mu_4' = \frac{24(\lambda^2+30)}{\lambda^4\left(\lambda^2+2\right)}$$

The hazard rate function *h(x)* and the mean residual life *m(x)* of the Akash distribution are given by

$$h(x) = \frac{\lambda^3(1+x^2)}{\lambda x\left(\lambda x+2\right)\left(\lambda^2+2\right)} \qquad (1.3)$$

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and

$$m(x)=\frac{\lambda^2x^2+4\lambda x+(\lambda^2+6)}{\lambda\left[\lambda x(\lambda x+2)+(\lambda^2+2)\right]} \qquad (1.4)$$

These mathematical properties of the Akash distribution tend to be more flexible than the Lindley distribution and the Exponential distribution. In spite of the flexibility of Akash distribution over Lindley and Exponential distribution in modeling real lifetime data, there are situations where the one-parameter Akash distribution may not give a better fit when considered for modeling real lifetime data. Thus, the aim of this paper is to propose a new two-parameter lifetime distribution as an extension of the one-parameter Akash distribution. We shall call this distribution, "A Generalized Akash Distribution". The rest Sections in this paper is organized as follows: Section 2 reveals the density of the proposed distribution, in Section 3, we discussed the special cases of the proposed distribution. In Section 4, we obtained the Survival function, Hazard rate function and the Mean residual life function of the proposed distribution. Section 5-7 covers the Moments and related measures, Moment Generating function and Renyi entropy of the proposed distribution. In Section 8, we estimated the parameters of the distribution using the Maximum likelihood method. In Section 9, we fit the proposed distribution to two real data sets and compared with the fit attained by some existing lifetime distributions. Finally, Section 10 gives a concluding remark.

2. The PDF of the proposed distribution

Let X be a continuous random variable, then X is said to follow a Generalized Akash distribution (GAD) with parameter (λ, β), if its density function is given by

$$f(x,\lambda,\beta)=\frac{\lambda^3}{\lambda^2+2\beta}(1+\beta x^2)e^{-\lambda x} \qquad x>0, \lambda>0, \beta>-\lambda \qquad (2.1)$$

The pdf in equation (2.1) is a two-component mixture of Exponential *(λ)* and Gamma *(3, λ)* distributions.

i.e $f(x,\lambda,\beta)=q\, f_1(x)+(1-q)\, f_2(x)$

$$q=\frac{\lambda^3}{\lambda^2+2\beta}, \qquad f_1(x)=\lambda\, e^{-\lambda x}, \quad f_2(x)=\frac{\lambda^3x^2e^{-\lambda x}}{2}$$

Where $q$is referred to as the mixing proportion of the distributions

Taking the natural logarithm of (2.1), its first derivative is given by

$$\frac{d}{dx}\log f(x) = \frac{2\alpha x}{1+\alpha x^2}-\lambda$$

it follows that,

(i) for $\lambda<\beta$, $\frac{d}{dx}\log f(x)=0$, implies that *f(x)* has a unique mode at x_0. Where

$$x_0=\frac{2\beta+\sqrt{4\beta(\beta-\lambda^2)}}{2\lambda\beta}$$

(ii) for $\lambda\geq\beta$, $\frac{d}{dx}\log f(x)\leq 0$, i.e,*f(x)* is decreasing in x

Thecorrespondingcdf is of the form

$$F(x,\lambda,\beta)=1-\left[1+\frac{\left(\beta(\lambda x)^2+2\lambda\beta x\right)}{\lambda^2+2\beta}\right]e^{-\lambda x},\ x>0, \lambda>0, \beta>-\lambda \qquad (2.2)$$

The graph of the density function of the GAD for different values of λ and β are shown in figure 1 below

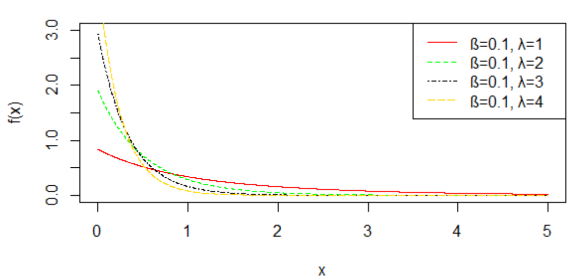

Figure 1: Probability density function of GAD for different values of λ and β

3. Special Case of the Generalized Akash Distribution

It is observed that, at β =1, the GAD reduces to the one-parameter Akash distribution with pdf in (1.1) and cdf in (1.2). Also at $\beta = 0$, the Generalized Akash Distribution reduces to an Exponential distribution (λ) distribution with pdf and cdf respectively given by

$$f(x,\lambda) = \lambda e^{-\lambda x} \qquad x>0,\ \lambda>0 \tag{3.1}$$

and

$$F(x,\lambda) = 1 - e^{-\lambda x} \tag{3.2}$$

4. Survival, Hazard and Mean Residual Life Function

Let Xbe a continuous random variable with density function $f(x)$ and cumulative distribution function $F(x)$. The survival (Reliability function), Hazard rate function (Failure rate function) and Mean residual life function of the random variable X are defined by

$$s(x) = P(X > x) = 1 - F(x) \tag{4.1}$$

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X - x + \Delta x \mid X > x)}{\Delta x} = \frac{f(x)}{1 - F(x)} \tag{4.2}$$

and

$$m(x) = E[X - x \mid X > x] = \frac{1}{1 - F(x)} \int_x^\infty (1 - F(t))dt \tag{4.3}$$

Using equation (2.1) and (2.2) in the above equation (4.1) – (4.3), the survival, hazard rate and mean residual life function of the GeneralizedAkash distribution are obtained as follows

$$s(x) = \left[1 + \frac{(\beta(\lambda x)^2 + 2\lambda\beta x)}{\lambda^2 + 2\beta}\right] e^{-\lambda x} \tag{4.4}$$

$$h(x) = \frac{\lambda^3(1 + \beta x^2)}{((\lambda^2 + 2\beta) + \beta(\lambda x)^2 + 2\lambda\beta x)} \tag{4.5}$$

and

$$m(x) = \frac{[(\lambda^2 + 6\beta + \beta(\lambda x)^2 + 4\lambda\beta x)]}{\lambda[(\lambda^2 + 2\beta + 2\lambda\beta x + \beta(\lambda x)^2)]} \tag{4.6}$$

The graph of the Hazard rate function of the GAD for different values of $\lambda$and β is given in figure 2.

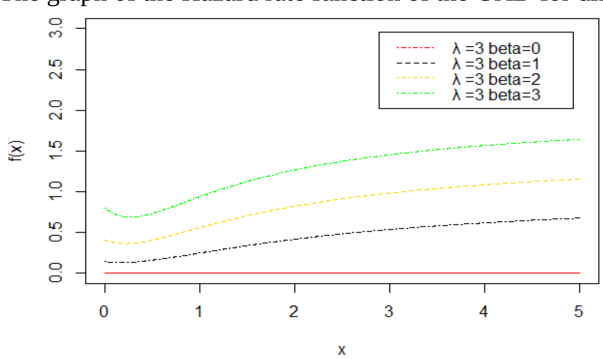

Figure 2: Hazard rate function of the TPAD for different values of λ and β

Remarks

We can observe that the Generalized Akash distribution exhibits an increasing failure rate property at some various choice of the parameters and also a constant failure rate property when $\beta=0$, which is the case of the Exponential distribution.

5. Moments and Related Measures based on Moments

Theorem 1: let $X \sim$ GAD (x, λ, β), then the r^{th}moment about the origin of X is given by

$$\mu_r'(x) = \frac{\Gamma(r+1)\lambda^2 + \Gamma(r+3)\beta}{\lambda^r(\lambda^2 + 2\beta)} \tag{5.1}$$

Proof:

Let X be a random variable following a Generalized Akash distribution with parameter β and λ, then

$$\mu_r'(x) = E(X^r) = \int_{-\infty}^{\infty} x^r f(x,\lambda,\beta)dx$$

$$= \int_0^{\infty} x^r \frac{\lambda^3(1+\beta x^2)}{\lambda^2+2\beta} e^{-\lambda x} dx$$

$$= \frac{\lambda^3}{\lambda^2+2\beta} \int_0^{\infty} x^r (1+\beta x^2) e^{-\lambda x} dx$$

$$= \frac{\lambda^3}{\lambda^2+2\beta}\left[\frac{\Gamma(r+1)}{\lambda^{r+1}} + \frac{\Gamma(r+3)\beta}{\lambda^{r+3}}\right]$$

$$= \frac{1}{\lambda^2+2\beta}\left[\frac{\Gamma(r+1)\lambda^2 + \Gamma(r+3)\beta}{\lambda^r}\right]$$

$$= \frac{\Gamma(r+1)\lambda^2 + \Gamma(r+3)\beta}{\lambda^r(\lambda^2+2\beta)}$$

This completes the proof.

Using equation (5.1), the first four raw moments of the GeneralizedAkash distribution are obtained as follows

$$\mu_1' = \frac{\lambda^2+6\beta}{\lambda(\lambda^2+2\beta)} = \mu \qquad \mu_2' = \frac{2\lambda^2+24\beta}{\lambda^2(\lambda^2+2\beta)} \qquad \mu_3' = \frac{6\lambda^2+120\beta}{\lambda^3(\lambda^2+2\beta)} \qquad \mu_4' = \frac{24\lambda^2+720\beta}{\lambda^4(\lambda^2+2\beta)}$$

The central moments of the GeneralizedAkash distribution are obtained as follows:

$$\mu_k = E\{(X-\mu)^k\} = \sum_{r=0}^{k}\binom{k}{r}\mu_r'(-\mu)^{k-r}$$

In particular,

$$\mu_2 = \frac{\lambda^4+16\lambda^2\beta+12\beta^2}{\lambda^2(\lambda^2+2\beta)^2}$$

$$\mu_3 = \frac{2(\lambda^6+30\lambda^4\beta+36\lambda^2\beta^2+24)}{\lambda^3(\lambda^2+2\beta)^3}$$

$$\mu_4 = \frac{3(3\lambda^8+128\lambda^6\beta+408\lambda^4\beta^2+576\lambda^2\beta^3+240)}{\lambda^4(\lambda^2+2\beta)^4}$$

Using the central moments above, the coefficient of variation (γ), coefficient of skewness ($\sqrt{\gamma_1}$) and the coefficient of kurtosis (γ_2) of GAD are obtained as follows

$$\gamma = \frac{\sigma}{\mu} = \frac{\sqrt{\lambda^4+16\lambda^2\beta+12\beta^2}}{\lambda^2+6\beta}$$

$$\sqrt{\gamma_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{2(\lambda^6+30\lambda^4\beta+36\lambda^2\beta^2+24)}{(\lambda^4+16\lambda^2\beta+12\beta^2)^{3/2}}$$

$$\gamma_2 = \frac{3(3\lambda^8+128\lambda^6\beta+408\lambda^4\beta^2+576\lambda^2\beta^3+240)}{(\lambda^4+16\lambda^2\beta+12\beta^2)^2}$$

Remarks

1. $h(0) = f(0) = \dfrac{\lambda^3}{\lambda^2+2\beta}$
2. $m(0) = \mu_1' = \dfrac{(\lambda^2+6\beta)}{\lambda(\lambda^2+2\beta)}$

The Hazard rate function of the Generalized Akash distribution is an increasing function in β ,λ and *x*, while the mean residual life function is a decreasing function in β ,λ and *x*. The Hazard rate function and mean residual life function of the Generalized Akash distribution shows its flexibility over the one-parameter Akash distribution, one-parameter Lindley distribution and two-parameter Lindley distribution.

6. Moment Generating Function

The moment generating function of the GAD is given by

$$M_X(t) = E[e^{tx}] = \int_0^\infty e^{tx} f(x)dx \tag{6.1}$$

$$= \int_0^\infty e^{tx} \frac{\lambda^3(1+\beta x^2)}{\lambda^2+2\beta} e^{-\lambda x} dx$$

$$= \frac{\lambda^3}{\lambda^2+2\beta} \int_0^\infty (1+\beta x^2) e^{-(\lambda-t)x} dx$$

$$= \frac{\lambda^3}{\lambda^2+2\beta} \left[\frac{1}{(\lambda-t)} + \frac{2\beta}{(\lambda-t)^3} \right]$$

$$= \frac{\lambda^3}{\lambda^2+2\beta} \left[\frac{1}{\lambda} \sum_{k=0}^{\infty} \left(\frac{t}{\lambda}\right)^k + \frac{2\beta}{\lambda^3} \sum_{k=0}^{\infty} \binom{k+2}{k} \left(\frac{t}{\lambda}\right)^k \right]$$

$$= \frac{\lambda^3}{\lambda^2+2\beta} \left[\sum_{k=o}^{\infty} \left(\frac{1}{\lambda} + \frac{(k+2)(k+1)\beta}{\lambda^3} \right) \left(\frac{t}{\lambda}\right)^k \right]$$

$$= \sum_{k=0}^{\infty} \frac{t^k \left(\lambda^2 + (k+2)(k+1)\beta\right)}{\lambda^k(\lambda^2+2\beta)} \tag{6.2}$$

7. Renyi Entropy

According to [10], entropyof a random variable *X* is a measure of variation of uncertainty. Renyi entropy is defined by

$$\tau_R(\xi) = \frac{1}{1-\xi} \log\left[\int f^{\xi}(x)dx \right] \qquad \xi > 0, \quad \xi \neq 1 \tag{7.1}$$

Thus, the Renyi entropy of GAD is given by

$$\tau_R(\xi) = \frac{1}{1-\xi} \log\left[\int_0^\infty \frac{\lambda^{3\xi}(1+\beta x^2)^{\xi}}{(\lambda^2+2\beta)^{\xi}} e^{-\lambda\xi x} dx \right]$$

$$= \frac{1}{1-\xi} \log\left[\frac{\lambda^{3\xi}}{(\lambda^2+2\beta)^{\xi}} \int_0^\infty (1+\beta x^2)^{\xi} e^{-\lambda\xi x} dx \right]$$

Recall that,

$$(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$$

$$= \frac{1}{1-\xi} \log\left[\frac{\lambda^{3\xi}}{(\lambda^2+2\beta)^{\xi}} \sum_{r=0}^{\infty} \binom{\xi}{r} \int_0^\infty (\beta x^2)^r e^{-\lambda\xi x} dx \right]$$

but

$$\int_0^\infty (\beta x^2)^r e^{-\lambda\xi x} dx = \frac{\Gamma(2r+1)\beta^r}{(\lambda\xi)^{2r+1}}$$

Thus,

$$\tau_R(\xi) = \frac{1}{1-\xi} \log\left[\sum_{r=0}^{\infty} \binom{\xi}{r} \frac{\lambda^{3\xi}}{(\lambda^2+2\beta)^{\xi}} \frac{\Gamma(2r+1)\beta^r}{(\lambda\xi)^{2r+1}} \right]$$

$$\tau_R(\xi) = \frac{1}{1-\xi} \log\left[\sum_{r=0}^{\infty} \binom{\xi}{r} \frac{\lambda^{3\xi-2r-1}}{(\lambda^2+2\beta)^{\xi}} \frac{\Gamma(2r+1)\beta^r}{(\xi)^{2r+1}} \right] \qquad (7.2)$$

8. Estimation of Parameter via maximum likelihood

Let $(x_1, x_2, \ldots, x_n)$ be a random sample from GAD with pdf defined in equation (2.1), then the likelihood function of equation (2.1) is of the form

$$L(x,\lambda,\beta) = \prod_{i=1}^{n} \left[\frac{\lambda^3 (1+\beta x^2) e^{-\lambda x}}{\lambda^2 + 2\beta} \right] \qquad (8.1)$$

The log-likelihood function based on the random sample of size n from the Generalized Akash distribution is given by

$$\ell(x,\lambda,\beta) = 3n\log\lambda + \sum_{i=1}^{n} \log(1+\beta x_i^{\,2}) - n\log(\lambda^2+2\beta) - n\lambda\overline{X} \qquad (8.2)$$

and the corresponding gradients are found to be

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^{n} \frac{x_i^{\,2}}{(1+\beta x_i^{\,2})} - \frac{2n}{(\lambda^2+2\beta)} \qquad (8.3)$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{3n}{\lambda} - n\overline{X} - \frac{2n\lambda}{\lambda^2+2\beta} \qquad (8.4)$$

Where $\overline{X}$ is the sample mean. From equation (8.4), it can be verified that the sample mean is equal to the population mean obtained from the first raw moments.

In estimation of the parameters λ and β, the equation $\frac{\partial \ell}{\partial \lambda} = 0$ and $\frac{\partial \ell}{\partial \beta} = 0$ cannot be resolved explicitly, thus we resort to a popular iterative scheme known as Fisher's scoring method. The second derivative of equation (8.2) is given by

$$\frac{\partial^2 \ell}{\partial \lambda' \partial \lambda} = \frac{-3n}{\lambda^2} - \frac{2n(2\beta - \lambda^2)}{(\lambda^2+2\beta)^2}$$

$$\frac{\partial^2 \ell}{\partial \lambda \partial \beta} = \frac{\partial^2 \ell}{\partial \beta \partial \lambda} = \frac{4n\lambda}{(\lambda^2+2\beta)^2}$$

$$\frac{\partial^2 \ell}{\partial \beta' \partial \beta} = \frac{4n}{(\lambda^2+2\beta)^2} - \sum_{i=1}^{n} \frac{x_i^{\,4}}{\left[(1+\beta x_i^{\,2})\right]^2}$$

The Fisher's Scoring method for estimating parameter $\hat{\lambda}$ and $\hat{\beta}$ is given by

$$\begin{bmatrix} \frac{\partial^2 \ell}{\partial \lambda' \partial \lambda} & \frac{\partial^2 \ell}{\partial \lambda \partial \beta} \\ \frac{\partial^2 \ell}{\partial \beta \partial \lambda} & \frac{\partial^2 \ell}{\partial \beta' \partial \beta} \end{bmatrix}_{\substack{\hat{\lambda}=\lambda_0 \\ \hat{\beta}=\beta_0}} \begin{bmatrix} \hat{\lambda} - \lambda_0 \\ \hat{\beta} - \beta_0 \end{bmatrix} = \begin{bmatrix} \frac{\partial \ell}{\partial \lambda} \\ \frac{\partial \ell}{\partial \beta} \end{bmatrix}_{\substack{\hat{\lambda}=\lambda_0 \\ \hat{\beta}=\beta_0}}$$

Where λ_0 and β_0 are the initial values of λ and β respectively. These equations are solved iteratively till sufficiently closed values of $\hat{\lambda}$ and $\hat{\beta}$ are obtained.

9. Applications of the Generalized Akash Distribution

In this Section, we fit the Generalized Akash distribution to two real data sets and compare its fitting with the fits attained by some existing lifetime distributions which includes; Akash distribution with pdf $f(x,\lambda) = \frac{\lambda^3}{\lambda^2+2}(1+x^2)e^{-\lambda x}$, Lindley distribution with pdf $f(x,\lambda) = \frac{\lambda^2}{\lambda+1}(1+x)e^{-\lambda x}$ and Two-parameter Lindley Distribution (TPLD) with pdf $f(x,\lambda,\beta) = \frac{\lambda^2}{\lambda+\beta}(1+\beta x)e^{-\lambda x}$. The estimates of the parameters of the distribution, *-2log(L)*, Akaike Information Criterion

[AIC = *2k-2log(L)*], Bayesian Information Criterion [BIC = *klog(n)-2log(L)*] and Kolmogorov-Smirnov(*K-S*) Statistic were considered for the comparison. Where *n* is the number of observations, *k* is the number of estimated parameters and *L* is the value of the likelihood function evaluated at the parameter estimates.

Data 1: The first data set represents the relief times of twenty patients receiving an analgesic. This data set was reported in [11] and its presented in Table 1 below

Table 1. Relief times of twenty patients.

1.1	1.4	1.3	1.7	1.9	1.8	1.6	2.2	1.7	2.7	4.1
	1.8	1.5	1.2	1.4	3.0	1.7	2.3	1.6	2.0	

Data 2: The Third data set represents the survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, reported in [12]. Table 2 below presents the data set.

Table 2:The survival times (in days) of 72 guinea pigs

0.1	0.33	0.44	0.56	0.59	0.72	0.74	0.77	0.92	0.93	0.96	1
1	1.02	1.05	1.07	1.07	1.08	1.08	1.08	1.09	1.12	1.13	1.15
1.16	1.2	1.21	1.22	1.22	1.24	1.3	1.34	1.36	1.39	1.44	1.46
1.53	1.59	1.6	1.63	1.63	1.68	1.71	1.72	1.76	1.83	1.95	1.96
1.97	2.02	2.13	2.15	2.16	2.22	2.3	2.31	2.4	2.45	2.51	2.53
2.54	2.54	2.78	2.93	3.27	3.42	3.47	3.61	4.02	4.32	4.58	5.55

It should be noted that the purpose of analyzing these data sets is not to project a complete statistical modeling or inferences for the data sets. Tables 3 and 4 shows the summary statistic of the estimate of the parameters, *-2logL, AIC, BIC* and the kolmogorov-smirnov (*k-s*) statistic of the distributions for each data set.

Table 3: Comparison Criterion for Data set 1

Models	*Estimates*	*-2logL*	*AIC*	*BIC*	*K-S*
GAD	$\lambda = 1.5788$ $\beta = 106289$	45.7748	49.7749	51.7663	0.2525
TPLD	$\lambda = 1.0527$ $\beta = 11953240$	52.3264	56.3264	58.3178	0.3221
LINDLEY	$\lambda = 0.8161$	60.4992	62.4991	63.4948	0.3911
AKASH	$\lambda = 1.1569$	59.5226	61.5226	62.5183	0.3705

Table 4: Comparison Criterion for Data set 2

Models	*Estimates*	*-2logL*	*AIC*	*BIC*	*K-S*
GAD	$\lambda = 1.6781$ $\beta = 84.9127$	188.0386	192.0386	196.592	0.1007
TPLD	$\lambda = 1.1310$ $\beta = 938567$	195.0482	199.0482	203.6016	0.1680
LINDLEY	$\lambda = 0.8682$	213.857	215.8569	218.1336	0.2467
AKASH	$\lambda = 1.2159$	214.6776	216.6777	218.9543	0.2345

The Best fitted distribution is considered by investigating the distribution with the minimum*-2logL, AIC, BIC and K-S Statistic* value. Table 3 and 4 above, indicates that the GeneralizedAkash distribution gives the best fit and thus demonstrates more flexibility over the examined lifetime distributions in modeling real lifetime data. Furthermore, the fits of the Probability-Probability (P-P) Plots, density and cumulative distribution fitsof the distributions for the two real data sets given in figures 3-6 validates the fact that the proposed distribution exhibits more flexibility among the examined lifetime distributions.

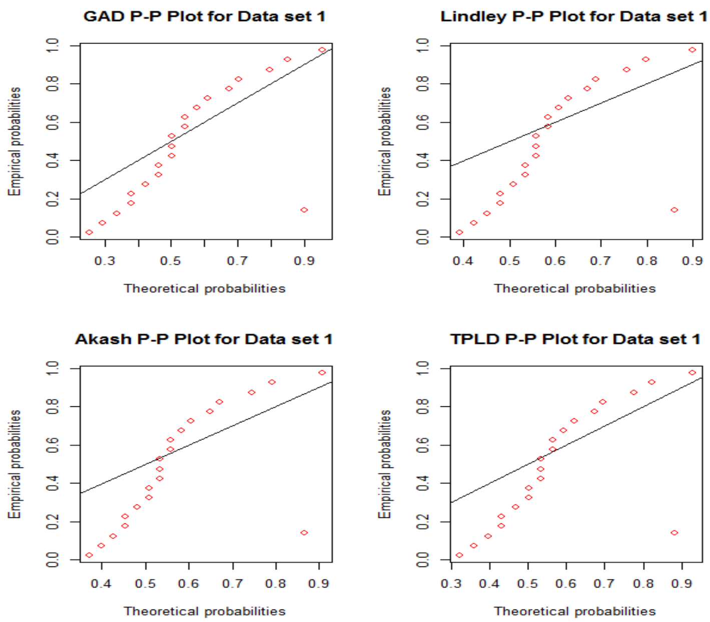

Figure 3: The fitted P-P plots of the distributions for Data set 1

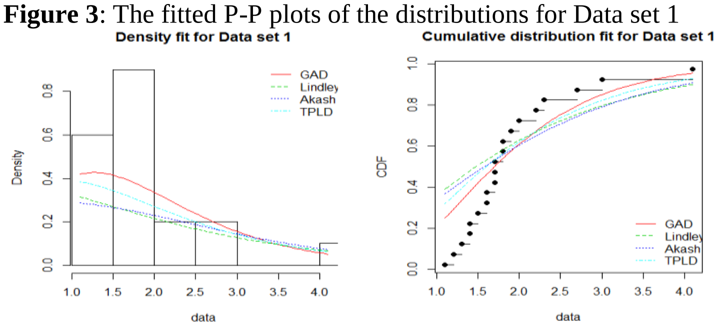

Figure 4: Density and Cumulative distribution fits for Data set 1

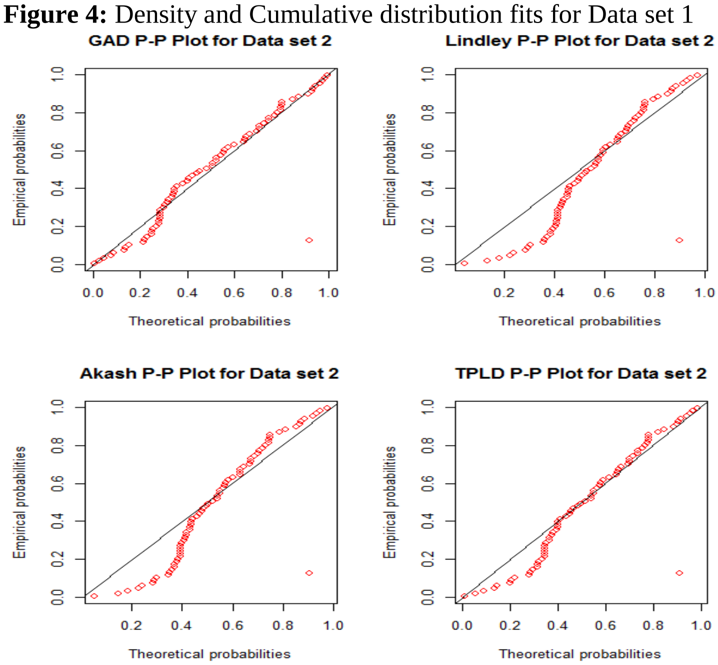

Figure 5: The fitted P-P plots of the distributions for Data set 2

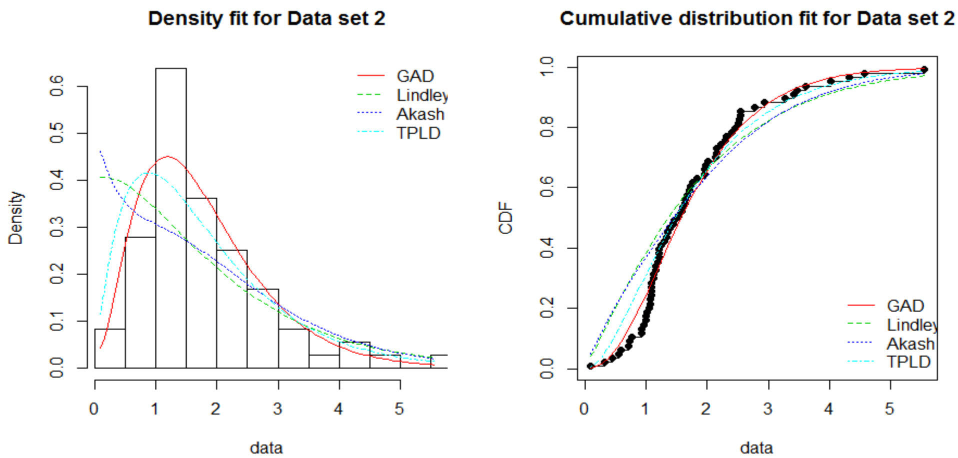

Figure 6: Density and Cumulative distribution fits for Data set 2

10. Conclusion

In this paper, a new two parameter lifetime distribution called the Generalized Akash distribution is introduced and the mathematical properties such as the shape of the density, hazard rate function, mean residual life function, moment generating function, moments, skewness, kurtosis measures and the Renyi entropy have been discussed. The Maximum likelihood method was employed in estimation of its parameters. The application of the proposed distribution to two real data sets (Biological data) alongside with Akash distribution, Lindley distribution and Two-Parameter Lindley distribution reveals that the proposed distribution fits better in modeling real lifetime data. It was observed from the results obtained in Table 3 and 4 that at 5% level of precision, the komolgorov-smirnov test statistic only accepted the proposed distribution as the model that best fit the two data sets. It should be noted that the komolgorov-smirnov test statistic only accepts distributions whose *(k-s)* value for a given data set is less than $\frac{1.36}{\sqrt{n}}$ and rejects distributions whose *(k-s)* value are greater than $\frac{1.36}{\sqrt{n}}$ at 5% level of precision.

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