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A MATHEMATICAL APPROACH RELATING PSYCHOSOCIAL THERAPY AS A VERITABLE PANACEA FOR BIPOLAR II *DISORDER* PATIENT

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Abstract

Bipolar II disorder is characterized by recurrent, alternating episodes of hypomania and depression. To examine the dynamical bases of this cyclical illness we use limit cycle oscillators to model bipolar II disorders based on the observation that the two poles of the disease are mutually exclusive with negatively damped oscillator. The psychosocial therapy that the patient is subjected to is modeled as the forcing function with the tendency of mood stabilization. We solve the model analytically using multiscale perturbation method. The numerical simulation suggests that if the world should go back to the era of extended family, the disorder will be curtailed drastically.

Keywords: Bipolar disorder; Mood variation; Perturbation method; Psychosocial therapy

1. Introduction

Bipolar disorder is a chronic recurrent mental illness [1]. The universal health problem of bipolar disorder is alarming: $1-4\%$ of adults live with the condition and present projection suggest that this mental illness accounts for up to 10% of the burden of other mental and substance use disorder [2]. Several indications show that over 60 million Nigerians have one form of mental disorder or the other with only about 20 per cent of persons in such category are seen to have the obvious forms of it, which includes what the ordinary Nigerian refers to madness, schizophrenia, and perhaps extreme case of drug or alcohol addiction; a reason that has largely made mental disorder in the remaining 80 per cent or 48 million Nigerians ignored or poorly understood [2]. Specifically, the World Health Organisation (WHO) in its 2017 World Health Day message says 7,079,815 Nigerians suffer from one of the most ignored and misunderstood form of mental disorder in the country – depression. This represents 3.9 per cent of the entire population; making Nigeria, according to the current prevalence rate, the most depressed country in Africa.Bipolar disorder is associated with pathological mood variation including episodes of both elevated mood called mania and extreme low moo called depression, interspersed with less severe but still problematic mood fluctuations or, in some people, relative mood stability [3]. Mania is a condition in which the patient probably experience racing thoughts, grandiose ideas, delusions and impulsive actions. Under these circumstances, individuals are liable to indulge in activities which can be damaging both to themselves and to those around them [4]. Depression is peculiar with low mood, poor concentration, insomnia; suicidal attempt etc. the annual years of healthy life lost per $100{,}000$ people from bipolar disorder in Nigeria has increased by 2.5% since1990, an average of 0.1% a year. For men, the health burden of bipolar disorder in Nigeria, as measured in years of healthy life lost per $100{,}000$ men peaks at age$18-29$. Women are harmed at the highest rate from bipolar disorder in Nigeria at age $16-29$. In 2013 the peak rate for women was higher than that of men (300.3 per $100{,}000$ women against 233.8 per $100{,}000$ men) [5]. Younger age of disease onset is associated with higher suicidal risk, with lifetime suicide attempt rates estimated between $20-47\%$ [6]. The prevailing alarming rate is as such that bipolar disorder has the highest rate of suicide across all psychiatric disorders [7]. The neurobiology of bipolar disorder understanding is greatly underpinned/low, and the long – term pharmacological interventions which are a standard treatment always result to poor clinical responses [3, 8]. There have been no significant clinical advances since the use of Lithium in the $1950s$ [9]. Fortunately, the good news is; bipolar disorder can be treated, and people with this illness can live fully productive lives [10, 11]. Observation has shown that bipolar can be rather erratic, there exist patterns of recurrence to the episodes. Progress has been made on the explanation of the mechanism and the development of treatments, the molecular and cellular bases of bipolar disorders remain unknown [12, 13]. Many studies explained the involvement of neural circuits located in the prefrontal cortex with connections in the thalamus [14 – 16].

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These psychopathological patterns are accompanied but apparently not correlated with variations of concentration of biological parameters such as the blood plasma cortisol level: the mood of depressive patients is better in the evening whereas cortisol usually exhibits a diurnal rhythm with a higher level in the morning than in the evening hours. This affective instability or disorder poses peculiar problems to temporal clinical practitioners; ranging from difficulty in diagnosing, stigma of mental illness, patient non – compliance to treatment and/or medication, and the toxicity of most of the drugs if taken individually [17]. Consequently, there is a deficient in data pertaining bipolar disorder, in part because of a lack in concordance on well – suited trial design. Hence, bipolar disorder is very difficult to study using clinical trials only [18]. For a patient with this affective disorder, there is a periodicity which governs the manic and depressive episodes [19, 20]. Understanding of the complex and dynamics temporal mood variability in bipolar disorder leads to the use of quantitative methods. The variability in mood provides a basis for developing mathematical approaches based on limit cycle oscillators [21 – 24] to appreciate the dynamics of this mental health disorder.
Our objective in this work is to understand better the dynamics of mood variation and the effect of psychosocial therapy on the mood swings; with the assumption that the observed behavior of bipolar disorder patient can be modeled as an oscillator. We assume that the treatment with the psychosocial therapy as the main consideration can be modeled using parametric polynomial forcing function. We use multiple scale perturbation method to determine the effect of psychosocial therapy to a bipolar II disorder patient.

2. Governing Equation

We shall make some necessary assumptions for the purpose of proposing a model for bipolar II disorder. The manic and depressive episodes are governed by periodic patterns and it is assumed that the disorder will escalate greatly if left untreated. Patients are more prone to rapid cycling especially if initially treated only with antidepressant [25]. In addition, the behavior of a single bipolar patient studied by [26] can be described qualitatively as a limit cycle oscillator. We then model the periodic mood variations of a bipolar II patient with a negatively damped oscillator. The model can be written thus

$$u'' - \delta u' + a_0 u = 0 \tag{1}$$

where δ is the damping coefficient, u is the emotional state of the patient, and a_0 is the natural frequency of the oscillator. Eqn. (1) is the episodes of the untreated patient. We suppose that treatment can be modeled using

$$f(\bar{t}, u) = \sum_{m=0}^{M} \left(P_m \cos w\bar{t} + Q_m \sin w\bar{t} \right) - \sum_{n=1}^{N} a_n u^{2n+1} \tag{2}$$

where P_m, Q_m, w and a are parameters. The treatment of bipolar II disorder is always the combination of mood stabilizers, antidepressant, antipsychotic or tranquilizers and psychotherapy. Treatments of bipolar II disorder are basically dependent upon the severity of the emotional state which explains why we choose the parametric function for the medication. The DampedDuffing Oscillator with multiple driving force is generally expressed as

$$u'' - \delta u' + \sum_{n=0}^{N} a_n u^{2n+1} = \sum_{m=0}^{M} \left(P_m \cos w\bar{t} + Q_m \sin w\bar{t} \right) \tag{3}$$

where δ is the damping coefficient, a_n, P_m, Q_m are arbitrary constant that translate the restoring forces and the periodic driving forces (day and night, HPA, Circadian rhythms). We consider the Duffing equation for $N = 3$ and $M = 3$. We have

$$u'' - \delta u' + a_0 u + a_1 u^3 + a_2 u^5 + a_3 u^7 = P \cos w\bar{t} + Q \sin w\bar{t} \tag{4}$$

where $P_0 + P_1 + P_2 + P_3 = P$, $Q_0 + Q_1 + Q_2 + Q_3 = Q$. Then we have

$$u'' - \delta u' + a_0 u + a_1 u^3 + a_2 u^5 + a_3 u^7 = \varepsilon K \sin\left(w\bar{t} + \beta \right), \tag{5}$$

Where

$$P = \varepsilon P_\alpha \text{ and } Q = \varepsilon Q_\alpha, \ \varepsilon K = \sqrt{P_\alpha^{\,2} + Q_\alpha^{\,2}} \text{ and } \tan\beta = \frac{P_\alpha}{Q_\alpha}, \ \bar{t} > 0, \ 0 < \delta \ll 1. \tag{6a}$$

$u' =$ Rate of change of mood, $a_0 =$ Natural frequency of the oscillator, $a_1 =$ Medication parameter that is a measure of medication adeherence, $a_2 =$ Psycosocial parameter, $a_3 =$ Other psycho - therapy parameter, $K =$ Resultant of external and internal forces, $\beta =$ Oscillation lag.

Assuming initial condition

$$u(0) = 0, \ u'(0) = 0. \tag{6b}$$

Equation $(6b)$represents the healthy state of an individual. The emotional state of the individual does not exhibit any noticeable variation.

3. Analytical approach

We now introduce a slow time scale τ such that

$$\tau = \delta t \tag{6c}$$

and

$$\hat{t} = t + \frac{1}{\delta}\left(\varepsilon\mu_1 + \varepsilon^2\mu_2 + \varepsilon^3\mu_3 + \cdots\right), \tag{7}$$

$$\bar{t} = t,\ \mu_i = \mu_i(\tau),\ \mu_i(0) = 0, \tag{8}$$

$$u(t,\tau) = v(\hat{t},\tau;\varepsilon,\delta). \tag{9}$$

Thus

$$\frac{du}{dt} = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial \hat{t}}\cdot\frac{\partial \hat{t}}{\partial t} + \frac{\partial v}{\partial \hat{t}}\cdot\frac{\partial \hat{t}}{\partial \tau}\cdot\frac{d\tau}{dt} + \frac{\partial v}{\partial \tau}\frac{d\tau}{dt}. \tag{10}$$

Substituting appropriately the values of the derivatives in (10)we have

$$\frac{du}{dt} = \frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}, \tag{11}$$

$$\frac{d^2u}{dt^2} = \frac{d}{dt}\left(\frac{du}{dt}\right) = \frac{d}{dt}\left(\frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}\right)$$
$$= \frac{\partial v'}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v'}{\partial \hat{t}} + \delta\frac{\partial v'}{\partial \tau}. \tag{12}$$

Substituting (11)into (12),

$$\frac{d^2u}{dt^2} = \frac{\partial}{\partial \hat{t}}\left(\frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3 + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}\right)$$
$$+\left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial}{\partial \hat{t}}\left(\frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}\right)$$
$$+\delta\frac{\partial}{\partial \tau}\left(\frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}\right),$$

$$\frac{d^2u}{dt^2} = \frac{\partial^2 v}{\partial \hat{t}^2} + 2\left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial^2 v}{\partial \hat{t}^2} + 2\delta\frac{\partial^2 v}{\partial \hat{t}\partial \tau} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)^2\frac{\partial^2 v}{\partial \hat{t}^2}$$
$$+2\left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\delta\frac{\partial^2 v}{\partial \hat{t}\partial \tau} + \delta^2\frac{\partial^2 v}{\partial \tau^2}. \tag{13}$$

Substituting (11) and (13) into (5), we have

$$\frac{\partial^2 v}{\partial \hat{t}^2} + 2\left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial^2 v}{\partial \hat{t}^2} + 2\delta\frac{\partial^2 v}{\partial \hat{t}\partial \tau} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)^2\frac{\partial^2 v}{\partial \hat{t}^2}$$
$$+2\left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\delta\frac{\partial^2 v}{\partial \hat{t}\partial \tau} + \delta^2\frac{\partial^2 v}{\partial \tau^2} + \delta\left(\frac{\partial v}{\partial \hat{t}} + \left(\varepsilon\mu_1' + \varepsilon^2\mu_2' + \varepsilon^3\mu_3' + \cdots\right)\frac{\partial v}{\partial \hat{t}} + \delta\frac{\partial v}{\partial \tau}\right)$$
$$+a_0u + a_1u^3 + a_2u^5 + a_3u^7 = \varepsilon K\sin\left(wt + \beta\right). \tag{14}$$

We now let

$$v(\hat{t},\tau) = \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} v^{ij}\left(\hat{t},\tau;\varepsilon,\delta\right)\varepsilon^i\delta^j, \tag{15}$$

where ij on v^{ij} denotes superscript and on ε,δ denotes powers. Hence

$$v(\hat{t},\tau,\varepsilon,\delta) = \varepsilon\left(v^{10} + \delta u^{11} + \delta^2u^{12} + \cdots\right) + \varepsilon^2\left(v^{20} + \delta u^{21} + \delta^2u^{22} + \cdots\right)$$
$$+\varepsilon^3\left(v^{30} + \delta u^{31} + \delta^2u^{32} + \cdots\right) + \cdots. \tag{16}$$

Substituting (16)into (14), and equate coefficients of $\varepsilon^i\delta^j$ we get

Terms of order ε :

$$\frac{\partial^2 v^{10}}{\partial \hat{t}^2} + \Omega^2 v^{10} = k\sin\left(wt + \beta\right). \tag{17}$$

Terms of order $\varepsilon\delta$:

$$\frac{\partial^2 v^{11}}{\partial \hat{t}^2}+\Omega^2 v^{11}=-2\frac{\partial^2 v^{10}}{\partial \hat{t}\partial \tau}-\frac{\partial v^{10}}{\partial \hat{t}}. \tag{18}$$

Terms of Order $\varepsilon\delta^2$:

$$\frac{\partial^2 v^{12}}{\partial \hat{t}^2}+\Omega^2 v^{12}=-2\frac{\partial^2 v^{11}}{\partial \hat{t}\partial \tau}-\frac{\partial v^{11}}{\partial \hat{t}}-\frac{\partial^2 v^{10}}{\partial \tau^2}. \tag{19}$$

Terms of order ε^2:

$$\frac{\partial^2 v^{20}}{\partial \hat{t}^2}+\Omega^2 v^{20}=-2\mu_1'\frac{\partial^2 v^{10}}{\partial \hat{t}^2}. \tag{20}$$

Terms of order $\varepsilon^2\delta$:

$$\frac{\partial^2 v^{21}}{\partial \hat{t}^2}+\Omega^2 v^{21}=-2\mu_1'\frac{\partial^2 v^{11}}{\partial \hat{t}^2}-2\mu_1'\frac{\partial^2 v^{10}}{\partial \hat{t}\partial \tau}-2\frac{\partial^2 v^{20}}{\partial \hat{t}\partial \tau}-2\frac{\partial v^{20}}{\partial \hat{t}}-\mu_1'\frac{\partial v^{10}}{\partial \hat{t}}. \tag{21}$$

Terms of order $\varepsilon^2\delta^2$:

$$\frac{\partial^2 v^{22}}{\partial \hat{t}^2}+\Omega^2 v^{22}=-2\mu_1'\frac{\partial^2 v^{12}}{\partial \hat{t}^2}-2\mu_1'\frac{\partial^2 v^{11}}{\partial \hat{t}\partial \tau}-2\frac{\partial^2 v^{21}}{\partial \hat{t}\partial \tau}-\frac{\partial v^{21}}{\partial \hat{t}}-\mu_1'\frac{\partial v^{11}}{\partial \hat{t}}-\frac{\partial v^{20}}{\partial \tau}-\frac{\partial^2 v^{20}}{\partial \tau^2}. \tag{22}$$

Terms of order ε^3:

$$\frac{\partial^2 v^{30}}{\partial \hat{t}^2}+\Omega^2 v^{30}=-\left(\mu_1'\right)^2\frac{\partial^2 v^{10}}{\partial \hat{t}^2}-2\mu_1'\frac{\partial^2 v^{20}}{\partial \hat{t}^2}-\mu_2'\frac{\partial v^{10}}{\partial \hat{t}^2}-\Omega^2\left(v^{10}\right)^3. \tag{23}$$

Terms of order $\varepsilon^3\delta$:

$$\begin{aligned}\frac{\partial^2 v^{31}}{\partial \hat{t}^2}+\Omega^2 v^{31}=&-2\left(\mu_1'\frac{\partial^2 v^{21}}{\partial \hat{t}^2}+\mu_2'\frac{\partial^2 v^{11}}{\partial \hat{t}^2}\right)-\left(\mu_1'\right)^2\frac{\partial^2 v^{11}}{\partial \hat{t}^2}-2\left(\mu_1'\frac{\partial^2 v^{20}}{\partial \hat{t}\partial \tau}+\mu_2'\frac{\partial^2 v^{10}}{\partial \hat{t}\partial \tau}\right)-2\frac{\partial^2 v^{30}}{\partial \hat{t}\partial \tau}\\&-\left(\frac{\partial v^{30}}{\partial \hat{t}}+\mu_1'\frac{\partial v^{20}}{\partial \hat{t}}+\mu_2'\frac{\partial v^{10}}{\partial \hat{t}}\right)-3\Omega^2\left(v^{10}\right)^2 v^{11}.\end{aligned} \tag{24}$$

Terms of order $\varepsilon^3\delta^2$:

$$\begin{aligned}\frac{\partial^2 v^{32}}{\partial \hat{t}^2}+\Omega^2 v^{32}=&-\left(\mu_1'\right)^2\frac{\partial^2 v^{12}}{\partial \hat{t}^2}-2\frac{\partial^2 v^{30}}{\partial \tau^2}-2\left(\mu_1'\frac{\partial^2 v^{22}}{\partial \hat{t}^2}+\mu_2'\frac{\partial^2 v^{12}}{\partial \hat{t}^2}\right)-2\frac{\partial^2 v^{31}}{\partial \hat{t}\partial \tau}\\&-2\left(\frac{\partial v^{31}}{\partial \hat{t}}+\mu_1'\frac{\partial v^{21}}{\partial \hat{t}}+\mu_2'\frac{\partial v^{11}}{\partial \hat{t}}+\frac{\partial v^{30}}{\partial \hat{t}}\right)-a_1\left(3\left(v^{10}\right)^2 v^{12}+\left(v^{11}\right)^2 v^{10}\right).\end{aligned} \tag{25}$$

The associated initial conditions can obtained by evaluating (11) and (16) at (0,0) and

equating the coefficient of different orders, we obtain

$$v^{ij}(0,0)=0, \tag{26}$$

$$O(\varepsilon):\frac{\partial v^{10}}{\partial \hat{t}}(0,0)=0, \tag{27}$$

$$O(\varepsilon\delta):\frac{\partial v^{11}}{\partial \hat{t}}(0,0)+\frac{\partial v^{10}}{\partial \tau}(0,0)=0, \tag{28}$$

$$O\left(\varepsilon\delta^2\right):\frac{\partial v^{12}}{\partial \hat{t}}(0,0)+\frac{\partial v^{11}}{\partial \tau}(0,0)=0, \tag{29}$$

$$O\left(\varepsilon^2\right):\frac{\partial v^{20}}{\partial \hat{t}}(0,0)+\mu_1'(0)\frac{\partial v^{10}}{\partial \hat{t}}(0,0)=0, \tag{30}$$

$$O\left(\varepsilon^2\delta\right):\frac{\partial v^{21}}{\partial \hat{t}}(0,0)+\mu_1'(0)\frac{\partial v^{11}}{\partial \hat{t}}(0,0)+\frac{\partial v^{20}}{\partial \tau}(0,0)=0, \tag{31}$$

$$O\left(\varepsilon^2\delta^2\right):\frac{\partial v^{22}}{\partial \hat{t}}(0,0)+\mu_1'(0)\frac{\partial v^{12}}{\partial \hat{t}}(0,0)+\frac{\partial v^{21}}{\partial \tau}(0,0)=0, \tag{32}$$

$$O\left(\varepsilon^3\right): \frac{\partial v^{30}}{\partial \hat{t}}(0,0) + \mu_1'(0)\frac{\partial v^{20}}{\partial \hat{t}}(0,0) + \mu_2'(0)\frac{\partial v^{10}}{\partial \hat{t}}(0,0) = 0, \tag{33}$$

$$O\left(\varepsilon^3\delta\right): \frac{\partial v^{31}}{\partial \hat{t}}(0,0) + \mu_1'(0)\frac{\partial v^{21}}{\partial \hat{t}}(0,0) + \mu_2'(0)\frac{\partial v^{11}}{\partial \hat{t}}(0,0) + \frac{\partial v^{30}}{\partial \tau}(0,0) = 0, \tag{34}$$

$$O\left(\varepsilon^3\delta^2\right): \frac{\partial v^{32}}{\partial \hat{t}}(0,0) + \mu_1'(0)\frac{\partial v^{22}}{\partial \hat{t}}(0,0) + \mu_2'(0)\frac{\partial v^{12}}{\partial \hat{t}}(0,0) + \frac{\partial v^{31}}{\partial \tau}(0,0) = 0. \tag{35}$$

We then solve (17) using the initial conditions, (26)and (27);recasting, we have

$$\frac{\partial^2 v^{10}}{\partial \hat{t}^2} + \Omega^2 v^{10} = h_1(\tau), \tag{36}$$

$$h_1(\tau) = k\sin(wt + \beta). \tag{37}$$

Solving (36), we get

$$v^{10}\left(\hat{t},\tau\right) = \alpha_{10}\cos\Omega\hat{t} + \beta_{10}\sin\Omega\hat{t} + h_1(\tau). \tag{38}$$

Using (28)on (38), we obtain

$$\beta_{10}(0) = 0$$

$$v^{10}\left(\hat{t},\tau\right) = \alpha_{10}\cos\Omega t + h_1(\tau). \tag{39}$$

To solve (18) we make appropriate substitution, we get

$$\frac{\partial^2 v^{11}}{\partial \hat{t}^2} + v^{11} = 2\alpha_{10}'\Omega\sin\Omega\hat{t} - 2\beta_{10}'\Omega\cos\Omega\hat{t} + \alpha_{10}\Omega\sin\Omega\hat{t} + \beta_{10}\Omega\cos\Omega\hat{t}. \tag{40}$$

To ensure uniformly valid asymptotic solution in $\hat{t}$, we equate to zero the coefficients of $\cos\Omega\hat{t}$ and $\sin\Omega\hat{t}$ in (40) and it results to

$$\alpha_{10}(\tau) = -h_1(0)e^{-\frac{\tau}{2}}, \tag{41}$$

$$\beta_{10}(\tau) = 0. \tag{42}$$

Therefore

$$v^{10}\left(\hat{t},\tau\right) = -h_1(0)e^{-\frac{\tau}{2}}\cos\Omega\hat{t} + h_1(\tau). \tag{43}$$

The remaining terms in (40) and its solution are

$$\frac{\partial^2 v^{11}}{\partial \hat{t}^2} + \Omega^2 v^{11} = 0, \tag{44}$$

$$v^{11} = \alpha_{11}\cos\Omega\hat{t} + \beta_{11}\sin\Omega\hat{t}. \tag{45}$$

Applying (26) and (28)to (45) we get

$$\alpha_{11}(0) = 0, \tag{46}$$

$$\beta_{11}(0) = -h_1'(0). \tag{47}$$

$$\therefore v^{11}\left(\hat{t},\tau\right) = \beta_{11}(0)\sin\Omega\hat{t}. \tag{48}$$

To solve (19), we make proper substitutions to get

$$\frac{\partial^2 v^{12}}{\partial \hat{t}^2} + \Omega^2 v^{12} = \Omega\left(2\alpha_{11}' + \alpha_{11}\right)\sin\Omega\hat{t} - \Omega\left(2\beta_{11}' + \beta_{11} + \alpha_{10}''\right)\cos\Omega\hat{t} - h_1''. \tag{49}$$

To ensure uniformly valid asymptotic solution in $\hat{t}$, we equate to zero the coefficients of $\cos\Omega\hat{t}$ and $\sin\Omega\hat{t}$ in (49) we obtain

$$\beta_{11}(\tau) = e^{-\frac{\tau}{2}}\left(\int_0^{\tau} e^{\frac{s}{2}}\alpha_{10}''(s)\,ds - h_1'(0)\right), \tag{50}$$

$$\alpha_{11}(\tau) = \alpha_{11}(0)e^{-\frac{\tau}{2}} = 0. \tag{51}$$

$$\therefore\ v^{11}\left(\hat{t},\tau\right)=\beta_{11}\left(\tau\right)e^{-\frac{\tau}{2}}\sin\Omega\hat{t}. \quad (52)$$

The remaining terms in (49) and its solution is

$$\frac{\partial^2 v^{12}}{\partial\hat{t}^2}+\Omega^2 v^{12}=h_1'', \quad (53)$$

$$v^{12}=\alpha_{12}\cos\Omega\hat{t}+\beta_{12}\sin\Omega\hat{t}+h_1''. \quad (54)$$

Applying the corresponding initial value conditions, we have

$$\alpha_{12}\left(0\right)=h_1''(0), \quad (55)$$

$$\beta_{12}\left(0\right)=0,$$

$$\therefore\ v^{12}\left(\hat{t},\tau\right)=h_1''(0)\cos\Omega\hat{t}-h_1''(\tau). \quad (56)$$

To solve (20), (21), and (22), we make necessary substitutions and apply the secularity condition we obtain that

$$v^{20}\left(\hat{t},\tau\right)=v^{21}\left(\hat{t},\tau\right)=v^{22}\left(\hat{t},\tau\right)=0. \quad (57)$$

Using binomial expansion on (23) we get

$$\frac{\partial^2 v^{30}}{\partial\hat{t}^2}+\Omega^2 v^{30}=2\mu_2'\alpha_{10}\Omega^2\cos\Omega\hat{t} \quad (58)$$
$$-a_1\left[\left(h_1^3+\frac{3\alpha_{10}^2h_1}{2}\right)+\left(\frac{3\alpha_{10}^2}{4}+3\alpha_{10}^2h_1^2\right)\cos\Omega\hat{t}+\frac{3\alpha_{10}^2h_1}{2}\cos2\Omega\hat{t}+\frac{\alpha_{10}^3}{4}\cos3\Omega\hat{t}\right].$$

To ensure uniformly valid asymptotic solution in $\hat{t}$, we equate to zero the coefficients of $\cos\Omega\hat{t}$ in (58), we get

$$\mu_2'\left(0\right)=\frac{3a_1}{2\Omega^2}=\frac{3a_1}{2\Omega^2}\left(\frac{\left(h_1\left(0\right)\right)^2}{4}+\left(h_1\left(0\right)\right)^2\right)=\frac{15a_1\left(h_1\left(0\right)\right)^2}{8\Omega^2}. \quad (59)$$

The remaining equation in (58) is

$$\frac{\partial^2 v^{30}}{\partial\hat{t}^2}+\Omega^2 v^{30}=-a_1\left[\left(h_1^3+\frac{3h_1\alpha_{10}}{2}\right)+\frac{3h_1\alpha_{10}^2}{2}\cos2\Omega\hat{t}+\frac{\alpha_{10}^3}{4}\cos3\Omega\hat{t}\right]. \quad (60)$$

Let

$$l_1\left(\tau\right)=-\left(h_1^3+\frac{3h_1\alpha_{10}}{2}\right)a_1, \quad (61)$$

$$l_2\left(\tau\right)=-\frac{3h_1\alpha_{10}^2}{2}a_1, \quad (62)$$

$$l_3\left(\tau\right)=-\frac{\alpha_{10}^3}{4}a_1. \quad (63)$$

Therefore the solution to(59)becomes

$$v^{30}=\alpha_{30}\cos\Omega\hat{t}+\beta_{30}\sin\Omega\hat{t}+l_1-\frac{l_2}{3}\cos2\Omega\hat{t}-\frac{l_3}{8}\cos3\Omega\hat{t}. \quad (64)$$

Using the appropriate initial conditions we have

$$\alpha_{30}\left(0\right)=\frac{65\left(h_1\left(0\right)\right)^3}{32}a_1, \quad (65)$$

$$\beta_{30}\left(0\right)=0,$$

$$\therefore v^{30}\left(\hat{t},\tau\right)=\alpha_{30}\left(0\right)\cos\Omega\hat{t}+l_1+\frac{l_2}{3}\cos2\Omega\hat{t}-\frac{l_3}{8}\cos3\Omega\hat{t}. \quad (66)$$

Moving further, (24)becomes

$$\frac{\partial^2 v^{31}}{\partial\hat{t}^2}+\Omega^2 v^{31}=2\Omega^2\mu_2'\beta_{11}\sin\Omega\hat{t}-2\Omega\mu_2'\alpha_{10}'\sin\Omega\hat{t}+2\Omega\alpha_{30}'\sin\Omega\hat{t}+2\Omega\beta_{11}\cos\Omega\hat{t}+\frac{4l_2'}{3}\Omega\sin2\Omega\hat{t}$$
$$+\frac{3l_3'\Omega}{4}\sin3\Omega\hat{t}+2\mu_2'\Omega\alpha_{10}\sin\Omega\hat{t}+\alpha_{30}\Omega\sin\Omega\hat{t}+\beta_{30}\Omega\cos\Omega\hat{t}+\frac{2l_2\Omega}{3}\sin\Omega\hat{t}+\frac{3\Omega l_3}{8}\sin3\Omega\hat{t}$$
$$-3a_1\left[\left(\frac{\alpha_{10}^2}{4}+h_1^2\right)\beta_{11}\sin\Omega\hat{t}+h_1\alpha_{10}\beta_{11}\sin\Omega\hat{t}+\frac{\alpha_{10}^2\beta_{11}}{4}\sin3\Omega\hat{t}\right]. \quad (67)$$

To ensure uniformly valid asymptotic solution in $\hat{t}$, we equate to zero the coefficients of $\cos\Omega\hat{t}$ and $\sin\Omega\hat{t}$ in (67) and we obtain

$$\alpha_{30}(\tau)=e^{-\frac{\tau}{2}}\left(\int_0^{\tau}h_2(s)e^{\frac{s}{2}}ds+\alpha_{30}(0)\right). \tag{68}$$

Hence

$$\frac{\partial^2 v^{31}}{\partial\hat{t}^2}+\Omega^2 v^{31}=\left(\frac{4l_2'}{3}+\frac{2l_2}{3}+h_1\alpha_{10}\beta_{11}\right)\sin 2\Omega\hat{t}+\left(\frac{3l_3'}{4}+\frac{3l_3}{8}+\frac{\alpha_{10}^2}{4}\beta_{11}\right)\sin 3\Omega\hat{t}. \tag{69}$$

We set

$$\left.\begin{aligned} l_4(\tau)&=\frac{4l_2'}{3}+\frac{2}{3}l_2+h_1\alpha_{10}\beta_{11},\\ l_5(\tau)&=\frac{3}{4}l_3'+\frac{3}{8}l_3+\frac{1}{4}\alpha_{10}^2\beta_{11}.\end{aligned}\right\} \tag{70}$$

The solution to (70)is

$$v^{31}(\hat{t},\tau)=\alpha_{31}\cos\Omega\hat{t}+\beta_{31}\sin\Omega\hat{t}-\frac{l_4}{3}\sin 2\Omega\hat{t}-\frac{l_5}{8}\sin 3\Omega\hat{t}. \tag{71}$$

Using the appropriate initial conditions, we obtain

$$\alpha_{31}(0)=0, \tag{72}$$

$$\beta_{31}(0)=\frac{2}{3}l_4(0)+\frac{3}{8}l_5(0)-\mu_2'(0)\beta_{11}(0)-\alpha_{30}'(0)-l_1'(0)+\frac{1}{3}l_2'(0)-\frac{1}{8}l_3'(0). \tag{73}$$

Furthermore, (25) can be reduced to

$$\begin{aligned}\frac{\partial^2 v^{32}}{\partial\hat{t}^2}+\Omega^2 v^{32}=&-\alpha_{30}''\cos\Omega\hat{t}-\beta_{30}''\mu_2'\Omega^2\sin\Omega\hat{t}-l_1''-\frac{l_2''}{3}\cos 2\Omega\hat{t}-\frac{l_3''}{8}\cos 3\Omega\hat{t}+2\mu_2'\Omega^2\alpha_{12}\cos\Omega\hat{t}\\ &+2\mu_2'\Omega^2\beta_{12}\sin\Omega\hat{t}-2\mu_2'\Omega\beta_{11}\cos\Omega\hat{t}+2\alpha_{31}'\Omega\sin\Omega\hat{t}-2\beta_{31}'\Omega\cos\Omega\hat{t}+\frac{4\Omega l_4'}{3}\cos 2\Omega\hat{t}\\ &+\frac{3\Omega l_5'}{4}\cos 3\Omega\hat{t}+\alpha_{31}\Omega\sin\Omega\hat{t}-\beta_{31}\Omega\cos\Omega\hat{t}+\frac{4\Omega l_4}{3}\cos 2\Omega\hat{t}+\frac{3\Omega l_5}{4}\cos 3\Omega\hat{t}\\ &-2\mu_2'\Omega\beta_{11}\cos\Omega\hat{t}+2\alpha_{30}\Omega\sin\Omega\hat{t}-2\beta_{30}\Omega\cos\Omega\hat{t}-\frac{4\Omega l_2}{3}\sin 2\Omega\hat{t}-\frac{3\Omega l_3}{4}\sin 3\Omega\hat{t}\\ &+\frac{1}{2}h_1\beta_{11}+2\alpha_{10}h_1h_1''-\frac{1}{2}\alpha_{10}^2h_1''-h_1^2h_1''+\left(\frac{1}{4}\alpha_{10}\beta_{11}+\frac{3}{4}\alpha_{10}^2h_1''+h_1^2h_1''-2\alpha_{10}h_1''\right)\cos\Omega\hat{t}\\ &+\left(\alpha_{10}h_1h_1''-\frac{1}{2}h_1\beta_{11}-\frac{1}{2}\alpha_{10}^2h_1''\right)\cos 2\Omega\hat{t}+\left(\frac{1}{4}\alpha_{10}^2h_1''-\frac{1}{4}\alpha_{10}\beta_{11}\right)\cos 3\Omega\hat{t}.\end{aligned} \tag{74}$$

To ensure uniformly valid asymptotic solution in $\hat{t}$, we equate to zero the coefficients of $\cos\Omega\hat{t}$ and $\sin\Omega\hat{t}$ in (74)

$$\beta_{31}(\tau)=e^{-\frac{\tau}{2}}\left(\int_0^{\tau}h_3(s)e^{\frac{s}{2}}ds+\beta_{31}(0)\right), \tag{75}$$

$$\alpha_{31}(\tau)=e^{-\frac{\tau}{2}}\left(\int_0^{\tau}h_4(s)e^{\frac{s}{2}}ds+\alpha_{31}(0)\right). \tag{76}$$

The remaining equation (74) is

$$\frac{\partial^2 v^{32}}{\partial\hat{t}^2}+\Omega^2 v^{32}=l_6+l_7\cos 2\Omega\hat{t}+l_8\sin 2\Omega\hat{t}+l_9\cos 3\Omega\hat{t}+l_{10}\sin 3\Omega\hat{t}, \tag{77}$$

where

$$l_6=\left(\frac{1}{2}h_1\beta_{11}+2\alpha_{10}h_1h_1''+\frac{1}{2}\alpha_{10}^2h_1''-h_1^2h_1''\right)a_1, \tag{78}$$

$$\left.\begin{aligned} l_7 &= \alpha_{10} h_1 h_1'' - \frac{1}{2} h_1 \beta_{11} - \frac{1}{2}\alpha_{10}^2 h_1'' - \frac{l_2''}{3} + \frac{4l_4'}{3} + \frac{4l_4}{3} \\ l_8 &= -\frac{4l_2}{3} \end{aligned}\right\} \tag{79}$$

$$\left.\begin{aligned} l_9 &= \frac{\alpha_{10}^2 h_1''}{4} - \frac{\alpha_{10}\beta_{11}}{4} - \frac{l_3''}{8} + \frac{3l_5'}{4} + \frac{3l_5}{4}, \\ l_{10} &= -\frac{3l_3}{4}. \end{aligned}\right\} \tag{80}$$

The solution to (77) is

$$v^{32}\left(\hat{t},\tau\right) = \alpha_{32}\cos\Omega\hat{t} + \beta_{32}\sin\Omega\hat{t} + l_6 - \frac{l_7}{3}\cos 2\Omega\hat{t} - \frac{l_8}{3}\sin 2\Omega\hat{t} - \frac{l_9}{8}\cos 3\Omega\hat{t} - \frac{l_{10}}{8}\sin 3\Omega\hat{t}. \tag{81}$$

Using the appropriate associated initial conditions, we get

$$\alpha_{32}(0) = \frac{l_7(0)}{3} + \frac{l_9(0)}{8} - l_6(0), \tag{82}$$

$$\beta_{32}(0) = \frac{2l_8(0)}{3} + \frac{3l_{10}(0)}{8} - \alpha_{31}'(0). \tag{83}$$

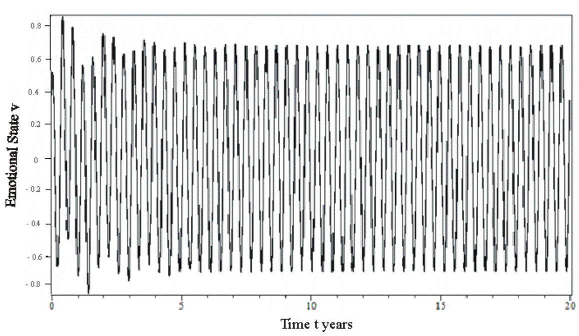
Hence

$$\begin{aligned} v^{32}\left(\hat{t},\tau\right) &= \alpha_{32}(0)\cos\Omega\hat{t} + \beta_{32}(0)\sin\Omega\hat{t} + l_6 - \frac{l_7}{3}\cos 2\Omega\hat{t} - \frac{l_8}{3}\sin 2\Omega\hat{t} \\ &\quad - \frac{l_9}{8}\cos 3\Omega\hat{t} - \frac{l_{10}}{8}\sin 3\Omega\hat{t}. \end{aligned} \tag{84}$$

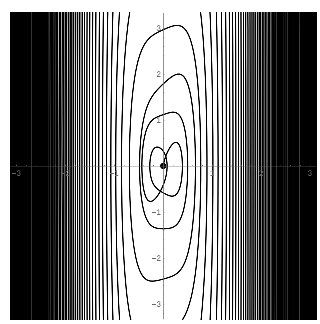
Therefore

$$\begin{aligned} v\left(\hat{t},\tau,\varepsilon,\delta\right) &= \varepsilon\left[h_1(0)e^{-\frac{\tau}{2}}\cos\Omega\hat{t} + h_1(\tau) + \delta\left(h_1'(0)e^{-\frac{\tau}{2}}\sin\Omega\hat{t}\right) + \delta^2\left(h_1''(0)\cos\Omega\hat{t} - h''(\tau)\right) + \cdots\right] \\ &\quad + \varepsilon^3(\alpha_{30}(0)\cos\Omega\hat{t} + l_1 + \frac{l_2}{3}\cos 2\Omega\hat{t} - \frac{l_3}{8}\cos 3\Omega\hat{t} + \delta(\beta_{31}(0)\sin\Omega\hat{t} - \frac{l_4}{3}\sin 2\Omega\hat{t} - \frac{l_5}{8}\sin 3\Omega\hat{t}) \\ &\quad + \delta^2\left(\alpha_{32}(0)\cos\Omega\hat{t} + \beta_{32}(0)\sin\Omega\hat{t} + l_6 - \frac{l_7}{3}\cos 2\Omega\hat{t} - \frac{l_8}{3}\sin 2\Omega\hat{t} - \frac{l_9}{8}\cos 3\Omega\hat{t} - \frac{l_{10}}{8}\sin 3\Omega\hat{t}\right) + \cdots). \end{aligned} \tag{85}$$

The graphing of (85) confirms the analytical result. Graphing (85) for the initial conditions around the equilibrium point (0, 0) we obtain

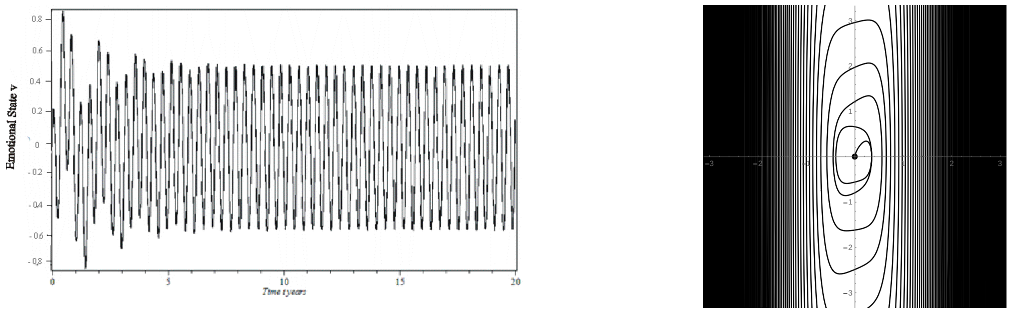


(a)



(b)

Fig. 1. Patient that is receiving treatment and placed on psychosocial therapy with parameter $\delta = 0.5, F = 0.9,$ $w = 1.0, a_0 = 1, a_1 = 20, a_2 = 10, a_3 = 50$: (a) transition between an unstable larger limit cycle to a stable smaller limit cycle and (b) the phase plane corresponding to (a).



(a) (b)

Fig. 2.Patient that is receiving treatment and placed on psychosocial therapy with parameter $\delta = 0.5, F = 0.9,$ $w = 1.0, a_0 = 1, a_1 = 20, a_2 = 100, a_3 = 50$: (a) transition between an unstable larger limit cycle to a stable smaller limit cycle and (b) the phase plane corresponding to (a).

4. Amplification of the Limit Cycles

Supposing the particular solution is given as

$$u_p(t) = P\cos wt + Q\sin wt. \tag{86}$$

Hence (3) becomes

$$\begin{aligned} &-Pw^2\cos wt - Qw^2\sin wt - \delta\left(-Pw\sin wt + Qw\cos wt\right) + a_0\left(P\cos wt + Q\sin wt\right) \\ &\quad + a_1\left(P\cos wt + Q\sin wt\right)^3 + a_2\left(P\cos wt + Q\sin wt\right)^5 \\ &\quad + a_3\left(P\cos wt + Q\sin wt\right)^7 = K\sin wt. \end{aligned} \tag{87}$$

Neglecting the terms of higher order; because individuals with BD regardless of their chaotic mood variation still have a check in the swings. Appling the solvability condition and using Cramer's rule, we have

$$P = \frac{K\delta w}{\left(a_0 - w^2\right)^2 + \delta^2 w^2}, \qquad Q = \frac{K\left(a_0 - w^2\right)}{\left(a_0 - w^2\right)^2 + \delta^2 w^2}. \tag{88}$$

To study the amplitude as a function of w, we write the particular solution as

$$u_p(t) = A\sin(wt - \beta), \tag{89}$$

where A is the amplitude and β is the phase lag given by

$$A = \sqrt{P^2 + Q^2} = \frac{K}{\sqrt{\left(w_0 - w^2\right)^2 + \delta^2 w^2}} \tag{90}$$

$$\tan\beta = \frac{\delta w}{w_0^2 - w^2}. \tag{91}$$

To obtain the maximum amplitude we differentiate (90) to obtain

$$A'(w) = \frac{Kw\left(\delta^2 - 2\left(w_0^2 - w^2\right)\right)}{\left(\sqrt{\left(w_0^2 - w^2\right)^2 + \left(\delta w\right)^2}\right)^3}.$$

The critical amplitude is $A'(w) = 0$, hence

$$w_{\max}^2 = w_0^2 - \frac{\delta^2}{2}. \tag{93}$$

Substituting (92) into(90), we have

$$A\left(w_{\max}\right)=\frac{2K}{\delta\sqrt{4w_0^2-\delta^2}}. \quad (94)$$

This yields the condition that must be met in order that the limit cycles will exist. These conditions are based on the values of a_0, δ and F . When the quantity $4w_0^2-\delta^2$ is large enough for the positive root, and small enough for negative roots, when there are no parametric excitation; the resultant limit cycle is always unstable. The limit cycle is stable if and only if $4w_0^2>\delta^2$ and the larger the difference the larger the limit cycle. When $a_0=w_0=0$ the limit cycle is unstable and the amplitude of the oscillation blows up when $\delta=0$ which is consider as the worst case of a bipolar patients.

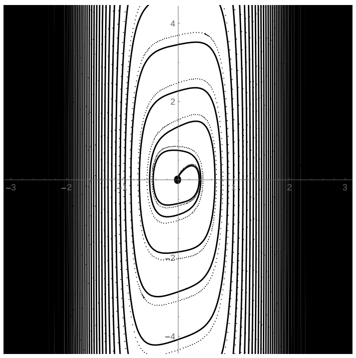


Fig. 3 Phase plane diagram: Circles are the numerical solution and crosses represent the approximate solution.

The numerical calculations suggest that the amplification of K has impact on the amplitude of these limit cycles but not on the stability of these limit cycles. Our results show that the amplitude of the parametric excitation is directly proportional to the amplitude of the stable limit cycles. Fig. 1 and Fig. 2 are in support of the above assertion.
The method of multiple scale perturbation that we used as a to approximate the solutions of non – linear oscillator with small dissipation coefficients with the numerical calculation as an act of verification. We choose the following set of parameters $w=1.0, a_0=1, a_1=20, a_2=100, a_3=50\ \delta=0.5, F=0.9,$.

In Fig. 3 we show in the phase plane a comparison between the analytic approximation solution (85) and the numerical solution.

5. Discussion and Conclusion

The main goal of this work is the mathematical modeling of a mental illness which is characterized by mood swings of bipolar II disorder individuals. This minimalist model is essential because there is insufficient data at this time of the quantitative construction. The patient should be on medications which permit stability of the oscillations. The medication function we choose comprises of the drugs, psychosocial therapy and other forms of therapies that help in the stability of the mood. Assuming there is a great adherence to the prescribed drugs, we are particular on the effect of psychosocial therapy to the stability. The results presented by Figs. 1 and 2 illustrate two different situations. Fig., 3 illustrates the case of an individual on whom the drugs are administered with great adherence and psychosocial therapy (FFT) is not neglected and Fig. 2 illustrates the total adherence to medication and fully implementation of FFT. The results show that stability of the mood swings of a bipolar II disorder patient is reached faster when the psychosocial therapy is fully implemented. This kind of study of mental illness could improve the comprehension of the dynamical characteristics of psychosocial disorders and the medication approach.
In this paper, we considered a mathematical framework for the modeling of bipolar II disorder patient, which is studied using analytical approach. Under the assumption of small damping, the method of perturbation is employed and phase of solutions has been derived. Finally, analytic approximate solutions have been numerically checked. In future works, we will consider adding a time – delay function to the treatment function.

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