Journal of the Nigerian Association of Mathematical Physics
Volume 47 (July, 2018 Issue), pp131 – 134

ANALYSIS OF RECRUITMENT IN MANPOWER PLANNING USING POISSON DISTRIBUTION

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Abstract

Humans are the most crucial, volatile and unpredictable resource which organizations use for production of goods and service. This paper examines the recruitment aspect of manpower planning as it relates to pure-birth process by using Poisson distribution to determine recruitment rate of personnel in an organization. The model describes interval lengths between recruitment and promotion periods which are independently and identically distributed with a continuous density function. In this model, the unpredictable and volatile aspect of personnel is being incorporated in the probability function.

***Keywords*: Recruitment, Wastage and Promotion**

1.0 Introduction

Organizations tend to be hierarchical with a finite number of ranks. Humans are considered in [1] as the most crucial, volatile and potentially unpredictable resource which an organization utilizes. As stated in [2], manpower demand is determined by random arrival of projects which makes managers to periodically increase or decrease manpower level. In [3], it is also remarked that seasonal fluctuation is another reason for unpredictable manpower demand. Factors such as recruitment, promotion and wastage which influence the migration of staff in a manpower system are discussed in [4].

Several researchers have addressed different aspects of manpower planning models. For example, a contingent manpower planning model which assigns workers with different skills to maximize production is discussed in [5]. Markov manpower models in discrete time are developed in [6]. A model which makes use of Markov chain to estimate projected manpower requirements over a period of ten to twenty years horizon is discussed in [7]. An air cargo terminal manpower planning model is discussed in [8]. A manpower planning model which determines the optimal number of employees to meet uncertain demand for manpower using a stochastic approach is developed in [9], while a manpower planning using dynamic approach is discussed in [10]. Raghavendra [6] has employed a Markov chain model in obtaining the transition probabilities for promotion in a bivariate manpower system. A Markov chain model which investigates the input policies subject to cost objective functions was developed in [11]. A Semi – Markov model in which a single grade system allows for wastage and recruitment is discussed in [12]. The semi – Markov processes are a generalization of Markov process in which the probability of leaving a state at a given point in time may depend on the length of time the state has been occupied (duration of stay) and on the next state.

Sethare [13] classified stochastic models of manpower systems into two categories: Markov chain models and Renewal models. In all these models, the manpower system is hierarchically graded into mutually extensive and exhaustive grades so that each member of the system may be in one and only one grade at any given time. These grades are defined in terms of any relevant state variables. Individuals move between the grades due to promotions or demotions and to the outside world due to retirement, retrenchment or dismissal. .

In this paper we present a manpower system which describes the recruitment and wastage with specified probability distribution of inter arrival times using Poisson distribution.

2.0 Model Assumptions and Notations

Notations

$f(t) \equiv$(density function for the time interval t between any two successive arrivals or recruitments)
where $t \geq 0$, and also *define*
$\frac{1}{\lambda}$ $\equiv$mean time between arrivals

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So that
λ =Arrival's rate per unit of time.
Note the verbal description of $1/\lambda$ and λ (lambda). We can determine λ from $f(t)$ by taking the mathematical expectation of t:

Assumptions

(a)Interval lengths between recruitment and promotion periods are independently and identically distributed, and described by a continuous density function. This sort of input is an example of what is called a **renewal process,** and the succession of recruitment and promotion demonstrate what is termed a sequence of **recurrent events.**
(b) An employee in a particular rank has a fixed probability of promotion in a given year.
(c) Employees receive promotions depending on the number of eligible staff. Manpower requirements are met by changes in promotions and recruitments.

$$\int_0^\infty tf(t)dt \equiv \frac{1}{\lambda} \qquad (1)$$

where equation (1) above is the mean time between recruitments intervals.
For example, if the unit of time is ayear and $(\lambda = 4)$ is the average number of arrivals per year, then $1/\lambda$ equals one –fourth (0.25) of a year between arrivals (that is, one arrival on the average in any quarter of year).
Random arrivals. The most important example of an interval time distribution is that associated with completely random arrivals. Complete randomness means that the probability of an arrival (i.e recruitment) occurring in any small interval of time $(T, T + h)$ depends only on the length of the interval h and not on the interval's starting point T or on the specific history of arrivals prior to T. In other words, the arrival/recruitment process is both stationary, or as it is often called, **homogenous**. In the below, we indicate how the assumptions of **completely random arrivals correspond to postulating**

$$f\,(t) = \lambda e^{-\lambda t}, \quad t \geq 0, \qquad (2)$$

Where equation (2) is the negative exponential distribution
mean $= \frac{1}{\lambda}$, variance $= 1/\lambda^2$, and $e = 2.71828\ldots$.
To check $homogeity\, property$ of the exponential distribution, suppose $t = 0$ represents the system's starting point in time. Then the probability that no arrival (recruitment) occurs in the interval (0, T) is the same as the probability that the first arrival occurs after T.

$$P[t \geq T] = \int_T^\infty \lambda\, e^{-\lambda t}\; dt = e^{-\lambda T} \qquad (3)$$

Now, the $conditional$ probability that no arrival or recruitment occurs in the interval (0, $T + h$) $given$ that no arrival occurs in the interval (0, T) is, by definition,

$$\frac{P[t \geq T+h]}{P[t \geq T]} = \frac{e^{-(T+h)}}{e^{-\lambda T}} = e^{-\lambda h} = P[t \geq h] \qquad (4)$$

which depends $only$ on h. According to (4), the probability of no arrival in the interval $(T, T + h)$ is the same $regardless$ of whether there is no recruitment in $(0, T)$ or whether recruitment occurs at T and thereby "renews" the arrival process.
There is another way of describing the completely random nature of the exponential input process. The idea is given here in rough terms, but can easily be made exact. Suppose there are n arrival in the interval $(0, T)$. Then if the **inter arrival times** are exponentially distributed, the n arrival times are independently and uniformly distributed over the interval $(0, T)$. This observation provides the basis for several statistical tests to determine whether an exponential distribution adequately describes an actual input process.
A complementary insight into the assumption of exponential interarrivals times is gained by expressing $e^{-\lambda h}$ in its Taylor series expansion

$$P\begin{bmatrix}\text{number of staff}\\ \text{in any intervals}\\ \text{of length } h\end{bmatrix} = e^{-\lambda h} = 1 - \lambda h + \frac{(-\lambda h)^2}{2!} + \frac{(-\lambda h)^3}{3!} + \ldots \qquad (5)$$

For a very small, but positive value of h, the term $1 - \lambda h$ in (5) is $relatively$ large as compared to the remaining terms in summation. Therefore, this value can be used to approximate the probability in (5) when h is $very$ small. We use the symbol $(\doteq)$ to denote such an approximation. So we have, for $very$ small $h > 0$,

$$P\begin{bmatrix}\text{single arrival}\\ \text{in any nterval}\\ \text{of length } h\end{bmatrix} \doteq 1 - \lambda h \qquad (6)$$

A verbally inexact, but nevertheless helpful, way to explain the mathematical manipulation below is to state that at most only one arrival occurs for a time interval $h \geq 0$ sufficiently small. Since the approximate probability of no arrival occurring in the interval of length h is given by (6), the corresponding approximate probability of one arrival occurring is

$$P\begin{bmatrix}\text{single arrival}\\ \text{in any interval}\\ \text{of length } h\end{bmatrix} \doteq \lambda h \qquad (7)$$

A more precise way of expressing the reasoning would be to display the exact probability of a single arrival, in manner similar to (5),and then show that for a$very$ small h, the term λh is $relatively$ large as compared to the remaining terms. The symbol $(\doteq)$, means that a quantity of $relatively$ negligible magnitude is being ignored in the approximation.

To illustrate, suppose ($\lambda = 4$) arrivals per unit time. Then the probability of no arrival occurring in an interval of $h = 0.01$ hour is exactly 0.96079 from (5), and approximately 1 – 0.4 = 0.96 from (6); the probability of an arrival is approximately 0.4 from (7).
Given that the density function for interarrival times is exponential (2), an immediate consequence is that the density function of the $total$ arrival time y for any n consecutive arrivals is

$g(y) = \frac{\lambda(\lambda y)^{n-1}e^{-\lambda y}}{(n-1)!}$, $y \geq 0$ ……………………………………………………..………………(8)

Equation (8) is the gamma distribution where$n$ is a positive integer. We may interpret$y$ as the sum of n independent values drawn from the same exponential density (2). Then

$$P\begin{bmatrix} totalintervalfor \\ anyconsecutive \\ arrivals \leq T \end{bmatrix} = \int_0^T g(y)dy = 1 - \sum_{j=0}^{n-1} \frac{(\lambda T)^j e^{-\lambda T}}{j!} \quad \text{………………. (9)}$$

This can be verified by repeatedly applying integration by parts.
Finally, you should note that assuming exponential interarrivals is tantamount to postulating that the probability distribution of the number of arrivals n in any interval of length T is Poisson:

$$P\begin{bmatrix} n \text{ arivals in} \\ \text{any interval} \\ \text{of length } T \end{bmatrix} = \frac{(\lambda T)^n e^{-\lambda T}}{n!}, \; n = 0, 1, 2, \text{ ……………………………………………..…….(10)}$$

Where equation (10) is the Poissondistribution with
$E[n\,|T] = \lambda T$ and Var $[n\,|T] = \lambda T$…………………………………..(11)
Where equation (11) is the Poisson - Interval of length T.
Hence, if $\lambda = 4$ arrivals per year, the expected number of arrivals in $T = 2$ years is 8, and in $T = \frac{3}{4}$ year (i.e 9 months) is 3.

3.0 Analysis of the Model

A synonym for the term **exponential arrivals** is **Poisson input.** (Sometimes the term **Markovian** is also used, and abbreviated by the symbol M.)
From (9) and (10), it follows that

$$P\begin{bmatrix} \text{total arrivals for any } n \\ \text{consecutive arrivals} \leq T \end{bmatrix} = P\begin{bmatrix} \text{number of arrivals in} \\ \text{any interval } T \geq n \end{bmatrix} \text{ ...………………….(12)}$$

To illustrate, the probability that the total interval for any 7 consecutive arrivals does not exceed $T = 2$years is the same value as the probability that the number of arrivals in $T = 2$ years is at least 7 persons. If $\lambda = 4$ arrivals per year and $T = 2$ years, this probability is 0.687. Actually, (12) is valid for any recurrent input process, not merely the Poisson, if the interval starts right after an arrival.
It is straightforward to apply the foregoing results to a **pure – birth model.** Consider a system that starts at Time 0 with no customer. Assume that customer arrivals obey a Poisson process, and that customers never depart from the system after they have arrived. Then at time T, the Poisson distribution in (10) gives the probability of n customers in the system. Similarly, (9) gives the density function for the total arrival tie of the first n customers.
The following discussion outlines how assuming certain properties of the arrival process leads to the derivation of exponentially distributed interarrival times and Poisson input.
We postulate that

(A) The time intervals between successive arrivals are independently and identically distributed; further, the probability of an arrival occurring in the time interval between T and $T + h$ depends only on the length h of the interval and not on T.
The corresponding interarrival density function is designated as $f(t)$.

(B) In any interval of time $h > 0$, there is a positive probability of an arrival

(C) In any sufficiently small interval of time, at most only one arrival can occur.
Suppose for simplicity that the system starts at Time 0 and the first arrival occurs at Time t, where $t > 0$.
Therefore $f(t)$ represents the density function for both the length of the arrival intervals as well as the actual time for the first arrival.

Define

(i) $r(T) \equiv 1 - \int_0^T f(t)dt,$
So that

(ii) $r(T) = P[\text{first arrival occurs after Time T}]$
Then the given postulates (A) and (B),

(iii) $r(T + h) = r(T)r(h)$ for all $T, H > 0,$
And it can be proved that the only function that satisfies (iii) is

(iv) $r(T) = e^{-\lambda T},$
Where λ is a positive constant. Therefore

(v) $e^{-\lambda T} = 1 - \int_0^T f(t)dt,$
So that

(vi) $f(t) = \lambda e^{-\lambda t}$,

As was to be shown.

Define

(vii) $P_n(T) \equiv P[n \text{ arrivals occur in the interval } (0, T)]$.

Let $t = x$ be thetime of the first arrival event, and according to postulate (C), only one customer enters at x. By postulate (A), we can write

(viii) $P_n(T) = \int_0^T P_{n-1}(T - x)\, f(x)\, dx = \int_0^T P_{n-1}(y) f(T - y) dy$

for $n = 1, 2, \ldots.,$

where $y = T - x$. Using (vi),

(ix) $P_n(T) = \int_0^T P_{n-1}(y) \lambda e^{-\lambda(T-y)} dy.$

Differentiating with respect to T yields

(x) $\frac{dP_n}{dT} = \lambda P_n(T) + \lambda P_{n-1}(T)$ for $n = 0, 1, 2, \ldots..$

Since $P_0(T) = r(T) = e^{-\lambda T}$, he equations in (x) can be solved recursively, starting with $n = 1$, as each is a linear first – order differential equation with constant coefficients. The complete solution is simply the Poisson distribution,

(xi) $P_n(T) = \frac{(\lambda T)^n e^{-\lambda T}}{n!}$, for $n = 0, 1, 2, \ldots$

This solution does in fact satisfy (x) as can be seen by differentiating $P_n(T)$ in (xi). You can apply the same line of reasoning to any interval T, thus yielding the result that is given in (10).

4.0 Conclusion

The above model represents a manpower system which allows a periodic recruitment of staff. The word arrival is interchangeably used with recruitment in this paper. The model is applicable to a pure – birth process in Poisson. The analysis of manpower systems have become very important component of planned economic development of any organization or nation. However, manpower planning depends on the highly unpredictable human behavior and the uncertain social environment in which the system functions. Hence the probabilistic or stochastic models of manpower system are very essential.

Markov model starts with a given group of employees that exist in a level of organization, given the flows in and out of each level, (i.e recruitment and promotion form outside the system together with wastage) they estimate the population of the level in the future. This type of models is particularly useful when the knowledge of existing employees is available together with the probabilities of flows between succeeding years and the required future manpower is not known. Markov models are based on the assumption that future employees in any level of the organization are determined not so much by the number required in that level but by the promotions and recruitment encouraging movement up through the system. Because of the characteristic of "pushing", Markov models are often called "push" models.

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