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ANALYSIS OF OPTIMUM PROMOTION COST IN A MANPOWER SYSTEM USING MARKOV CHAIN.

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Abstract

A close review of several publications on manpower planning reveals that so far many aspects and approaches have been discussed in various literature sources pertaining to manpower planning. However, these models are of no use as long as they cannot be converted into effective tools usable within organizations. In this paper a fresh attempt has been made by modeling employee's eligibility for promotion by a Poisson arrival and lengths of waiting for promotion using an Erlang distribution. To highlight the importance of the model, a hypothetical example is used for illustration. It is observed that when the promotion cost is fixed per unit per time, the service cost and the waiting cost per unit per time which are contravention to each other varying, the values of the optimal promotion policy and the total optimal cost of promotion are obtained.

Keywords: Recruitment, Wastage and Promotion

1.0 Introduction

Manpower has been defined in [1] as human resource used in carrying out jobs in any organization. As contained in [2] manpower (personnel) flow in organizations having various grades/ranks of employees can be classified into the recruitment stream, promotion between grades and wastage-flow out of the system. Recruitment and promotion have be considered as the main activities of an organization.

The two major questions usually asked in manpower planning as stated in [3] and [4]are: (i) How many people are needed? and (ii) what sort of people are needed? As reported in [5], [6] and [7],there are three factors responsible for staff transition or migration in a manpower system: recruitment, promotion and wastage.

Recruitment is a process of absorbing employees into a manpower system of an organization. As contained in [7], there are two sources of manpower supply namely; external and internal supply. External supply has to do with recruitment of staff from outside the organization while internal manpower supply sources include transfer and redeployment of employees within the organization. Promotion is a process whereby a staff in an organization is moved from a lower grade to a higher one, [8]. Wastage refers to staff who leave an organization for various reasons such as resignation, retirement, retrenchment, dismissal, death etc.[9], and [10].

Various models applicable to manpower planning have been developed in the past by many well-known researchers such as Rao, Nirmala and Jeeva, Mutingi and Mbohwa etc. Dynamic programming for determining optimal recruitment policy was developed in [11]. A dynamic programming approach to manpower recruitment policies for a two grade system was developed in [12]. Wastage and promotion rates required to bring about any desired future personnel structure has be discussed in [13].A semi-Markov Model of a manpower system was developed in [14] with the interest focused on the total number of vacancies available in an entire organization. A study on training dependent promotions and wastage was also carried out in [15]. Queuing and inventory concepts have been applied to manpower planning problems in several literatures.

Promotional probabilities and recruitment vectors embedding Markovian theory with certain assumptions on the promotional polices of an organization such as promotions allowed to the next grade and no demotion without maintaining the grade structure over a period of timeis discussed in [16].As personnel are lost from the system through retirement, resignation, death, etc., workers are being promoted or recruited to fill the vacancies, [17].A model which takes into account the recruitment/wastage factors is developed in [17]. Figure 1 represents manpower flows as contained in [5].

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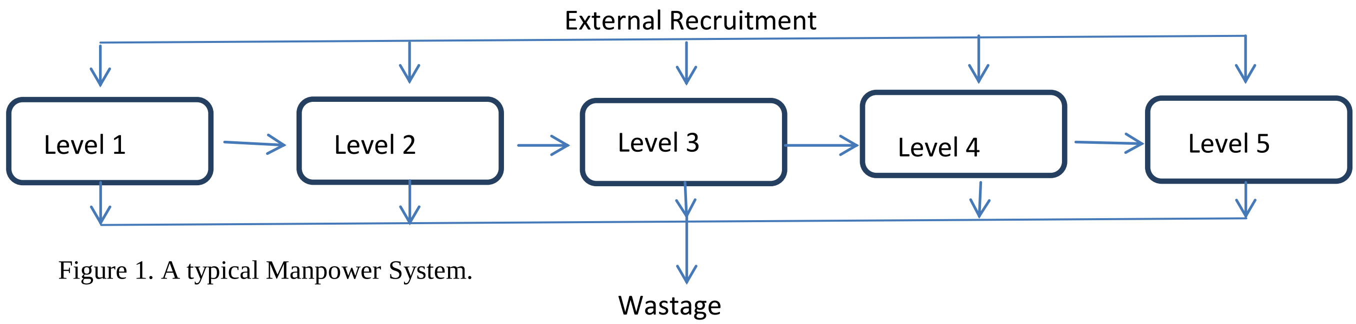

Figure 1. A typical Manpower System.

The rectangles represent ''stock'' while the horizontal "arrows" represent the movement of staff between the various ranks of the manpower system. The vertical "arrows" point into and out of the rectangle represent external recruitment and wastage respectively.

In this paper a fresh attempt has been made to analyze the promotion policy components of manpower planning by mapping the system to a queuing model where we describe employees eligibility for promotion by a Poisson arrival and lengths of waiting for promotion are model using an Erlang distribution.

2.0 Model Assumptions and Mathematical Notations.

a. It is assumed that the employees in grade 1 become eligible at a rate which is randomly distributed according to a Poisson distribution.

b. Employees proceed to be serviced on a first come first out basis (FIFO).

c. It is further assumed that the interval between two consecutive instances of a vacancy arising in grade (i-I) is exponentially distributed such that the expected number of vacancies arising during unit time is μ_1 with the traffic intensity

$$\frac{\lambda_1}{\mu_1} < 1$$

d. The promotion time distribution is assumed to be an Erlang distribution with mean $\frac{1}{k\mu}$ where μ is the parameter of the exponential distribution.

Mathematical Notations

i. λ = mean value rate of promotion

ii. C_o = fixed cost of promotion per unit of time for any organization

iii. C_1 = Promotion cost (service cost) per unit per time.

iv. C_2 = holding (waiting cost) per unit per time for the model.

v. P_n = Probability of n employees in the system n, = o, 1,

vi. L = queue length (expected number of employees in the system).

vii. C_3 = Per unit cost per phase associated with fluctuations in the expected queue length of the system.

3.0 Model Description

This model presents a single channel in which there is no limit placed on the number of employees applying for promotion. The employees applying for promotion are kept on the waiting list and considered for promotion as and when vacancies arise. Hence the manpower model in this case is based on queuing system. Since eligibility (arrivals) follows a random distribution, fluctuations will occur in expected queue length for the promotion in the manpower planning system. Using the Erlangian distribution with mean, $\frac{1}{k\mu}$ the total expected queue length of the system, average number of phases and per phase fluctuations in the system are obtained as follows:

Expected queue length in the system

$$L = \frac{(k+1)\lambda}{2k\mu(\mu-\lambda)} \quad \ldots\ldots(3.1)$$

Average number of phases

$$A_i = \frac{k(k+1)P}{2(1-kP)} \quad \ldots\ldots(3.2)$$

where $P = \frac{\lambda}{k\mu}$ for $i = 1, 2, \ldots n$

Per phase fluctuations in the queue length of the system:

$$= \sum_{n=0}^{\infty}(n-L)^2 p_n = (I-P)\sum_{n=0}^{\infty}\left(n^2 p^n - L^2\right) \quad \ldots\ldots(3.3)$$

The model developed here considers three phases (k=3). The first phase is used for basic screening such as minimum number of years of service put in, minimum qualification and required training for promotion. The second phase is used for evaluation of the performance towards target and quality achievement. The third and final phase is considered for interviewing of staff.

The total cost incurred by the organization for implementing the promotion policy consists of the sum of the fixed cost of promotion, the promotion cost, the cost of waiting for a vacancy to be created multiplied by the average number of phases and the hamper (fluctuation cost) per unit multiplied by per variability in the queue length (i.e. number of employees) in the system. The cost function as total optimal cost (TOC) is defined as:

$$TOC = C_0 + C_1\mu + C_2\frac{(k+1)\lambda}{2k\mu(\mu-\lambda)} + C_3\frac{k(k+1)P}{2(1-kP)}(I-P)\sum_{n=0}^{\infty}\left(n^2p^n - L^2\right) \quad (3.4)$$

After simplification, the above equation (3.4) reduces to:

$$\text{TOC} = Co + C_1\mu + C_2\frac{(K+1)\lambda}{2K\mu(\mu-\lambda)} + C_3\frac{\mu K(K+1)P^2}{2(\mu-\lambda)(1-P)^2} \quad (3.5)$$

$$\text{Let TOC} = Co + A_1 + A_2 + A_3 \quad (3.6)$$

Where $A_1 = C_1\mu$ $A_2 = C_2\frac{(k+1)\lambda}{2k\mu(\mu-\lambda)}$ and $A_3 = C_3\frac{\mu k(k+1)\lambda 2}{2(\mu-\lambda)(k\mu-\lambda)2}$

For the optimum promotion policy (μ), equation (3.5) yields a non-linear equation in μ after taking the first derivative of the same which is solved by making use of the fast converging Newton-Raphson method.

4.0 Numerical Illustration and Discussion of the Results

In the numerical illustration, since the model under consideration is studied for the steady state the costs of the model are considered to vary in such a way that at least one cost must be contradictory to other costs. This is a basic requirement for the formation of the queue. Moreover, the selection of the arrival rate is also considered as per the steady condition (i.e. $\lambda < 3\mu$). If the aforesaid conditions are violated, then the model shows erroneous output by giving a negative total optimal cost of the system. The Table 4.1 illustrates the optimal promotion policy (μ*) and the optimal cost of the manpower system for the promotion. The values of parameters (*) in row 9 of Table 4.1 shows the optimal promotion and total optimal cost of the system corresponding to various parameters.

Table 4.1:Relationship between TOC and optimal promotion policy, μ when C_0 is fixed

λ	C_0(₦1000s)	C_1C_0(₦1000s)	C_2(₦1000s)	C_3C_0(₦1000s)	k	μ*	TOCC_0(₦1000s)
1	700	50	25	15	3	8.95	1147.89
2	700	53	24	14	3	8.91	1173.46
3	700	67	23	13	3	8.89	1298.40
4	700	69	22	12	3	8.89	1318.83
5	700	74	21	11	3	8.85	1365.12
6	700	77	20	10	3	8.8	1397.16
7	700	81	19	9	3	8.75	1444.76
8	700	88	18	8	3	8.78	1589.46
9*	700*	90*	17*	7*	3*	8.81*	912.73*
10	700	92	16	6	3	8.81	1401.61
11	700	100	15	5	3	8.83	1515.70
12	700	103	14	4	3	8.87	1563.69

Further, assuming that the promotion cost C_1 to be constant, which sometimes happens to organizations when they have budgetary constraints, then the resultant trend between the different costs and total optimal cost are shown in Table 4.2 below

Table 4.2: Relationship between TOC and Optimal Promotion Policy μ when both C_0 and C_1 are Fixed

λ	C_0(₦1000s)	C_1(₦1000s)	C_2(₦1000s)	C_3(₦1000s)	k	μ*	TOC(₦1000s)
1	700	177	19	5	3	8.95	2285.04
2	700	177	42	8	3	8.91	2290.29
3	700	177	76	10	3	8.89	1018.46
4	700	177	80	15	3	8.89	2106.30
5	700	177	91	23	3	8.85	1940.41
6	700	177	111	28	3	8.8	1537.73

In Table 4.3, it is assumed that waiting and hamper costs are constant while assessing the change in the total optimal cost with the change in the promotion cost.

Table 4.3: Relationship Between TOC and Optimal Promotion Policy μ when both C_2 and C_3 are Fixed

λ	C_0(₦1000s)	C_1(₦1000s)	C_2(₦1000s)	C_3(₦1000s)	k	μ*	TOC(₦1000s)
1	700	207	47	104	3	8.95	2554.14
2	700	194	47	104	3	8.91	2434.82
3	700	189	47	104	3	8.89	2397.13
4	700	175	47	104	3	8.89	2293.95
5	700	142	47	104	3	8.85	2038.51
6	700	129	47	104	3	8.8	2012.48

In the Table 4.4 we looked at the special case when $\lambda = \mu$. In this case employee's eligibility for the job and the expected number of vacancies that arise occur at the same rate. We notice that the optimal policy is achieved when $\lambda = \mu = 1$.

Table 4.4: Special case when $\lambda = \mu$

λ	C_0 (₦1000s)	C_1(₦1000s)	C_2(₦1000s)	C_3(₦1000s)	k	μ*	TOC(₦1000s)
1	700	50	25	15	3	1	772.50
2	700	53	24	14	3	2	827.00
3	700	67	23	13	3	3	920.50
4	700	69	22	12	3	4	994.00
5	700	74	21	11	3	5	1086.50
6	700	77	20	10	3	6	1177.00
7	700	81	19	9	3	7	1280.50
8	700	88	18	8	3	8	1416.00
9	700	90	17	7	3	9	1520.50
10	700	92	16	6	3	10	1629.00
11	700	100	15	5	3	11	1807.50
12	700	103	14	4	3	12	1942.00

While analyzing the variation over different parameters in Table 4.1, it is interesting to note that when C_0 is fixed and the other two costs which are in contravention to each other are varying, the values of the optimal promotion policy and total optimal cost of the promotion are obtained and this trend of variation in various parameters is worth noticing in an organization.
In Table 4.2 where C_0 and C_1 are fixed and other costs are varying. It is noticeable that the variation in the optimal cost is significant. Table 4.3 shows significant variation in TOC when C_0 , C_2 and C_3 are fixed.

5.0 Conclusion
Manpower planning is about ensuring that the right types of employees are available at the right place at the right time. The success of the manpower planning is paramount to the survival of the organization and the complexities associated with the planning process and environment. Quantitative techniques such as queuing theory applied in this study can enhance problem – solving abilities and hence improve decision – making effectiveness of an organization.
The most practical implication is that of controlling the internal structure through hiring, promotions, internal transfers, redundancies and retirement planning. The problem is to precisely plan and control these interrelated organization activities in order to achieve a stable organization capable of meeting its objectives.
Application of manpower planning techniques means organization effectiveness. i.e. it may maximize the overall effectiveness of promotion policies to retain the best skilled employees. As a result of using this model and trying alternative manpower policies, one can discover and explore the cost performance that exists.
Lastly, management may implement the human resource planning models in their functional areas of business to develop policies on recruitment and selections, training and development, hiring, promotion and retention benefits to foster the spirit of organizational citizenship.

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