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EFFECT OF SUCTION/INJECTION ON BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER A NONLINEARLY STRETCHING PERMEABLE SHEET IN A NANOFLUID

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Abstract

We analysed the effects of suction/injection on the boundary layer and heat transfer characteristics of a nanofluid over a nonlinearly stretching sheet. Similarity transformations are employed to transform the governing partial differential equations into nonlinear ordinary differential equations which are solved numerically using the fourth-order Runge-Kutta scheme with a shooting technique. The results obtained are presented for different values of the governing parameters for valuesof parameters of interest. It is observed that the rate of heat transfer increases sharply with suction and later decreases rapidly to a non-zero value while the rate of mass transfer decreases rapidly to the ambient value. On the other hand, the rate of heat transfer increases with injection, whereas, the rate of mass transfer decreases rapidly. The results are benchmarked with published and the agreement is good.

***Keywords**:* Boundary Layer, nanofluid, nonlinearly stretching sheet, suction/injection, thermal radiation.

1.0 Introduction

The flow and heat transfer over a nonlinear stretching sheet has important application in polymer processing, environmental pollution, biological processes, etc. Also, the diffusion of reactant contaminant in boundary layer flow around a stretching sheet is relevant practically in chemical industries and metal and polymer processing industries. It is very important to control the drag and the heat flux during the manufacturing process for better product quality. It is noted in [1] that there is a rise in the temperature with an increase in the thermophoresis parameter or Brownian motion ;the stretching parameter and the species concentration decrease with an increase in Brownian motion while the concentration increases for an increase in the values of the thermophoresis parameter. A rising value in *Nb* and the decreasing in *Nt* produce a decrease in the nanoparticle concentration as a result of increase in the Sherwood number. In their work on numerical solution of the forced convection boundary layer flow, [2] assumed that the stretching velocity and the surface temperature vary nonlinearly along with the distance from theforward stagnation point. The flow of an electrically conducting fluid in presence of uniform magnetic field over a stretching elastic sheet was studied by Pavlov [3]. By employing Darcy model for the porous medium in [4], the authors discussed the problem of natural convection past a vertical plate. They looked into the effect of Brownian motion and thermophoresis parameters on the velocity and temperature profiles. Also the author, [5] was the first to introduce the word nanofluid that represents the fluid in which nano scale particles whose diameter is less than 100 nm, [6] presented different theories on enhanced heat transfer characteristics of nanofluids and he concluded that thermal dispersion phenomenon cannot explain fully the high heat transfer coefficients in nanofluids. In [7], it was proved that stretching may not necessarily be a linear function therefore he extended his research to flow over a quadratic and nonlinearly stretching sheet respectively. In [8] the behavior of nanofluid molecules in thin channels was explained while [9] explained that the thermal conductivity of solid particles is severally better than that of the base or convectional fluids. The major obstacles facing the effectiveness and heat transfer characteristics of the base fluids in heat exchange was discussed by authors of [10] while [11]discussed the heat transfer characteristics of nanofluids by immersing the high conductivity nano particles in base fluids and they concluded that the effective thermal conductivity of the fluid increases appreciably and consequently enhances the heat transfer characteristics. Furthermore, analysis of the flow and heat transfer of a MHD Newtonian fluid over impulsively stretched plane surface was carried out using an analytical method known as homotophy analysis method by [12]. In [13] ,the viscosity and thermal conductivity on hydro magnetic flow over a nonlinear stretching sheet was discussed while the two dimensional mixed convection boundary layer MHD stagnation point flow in porous medium bounded by a stretching vertical plate was studied by [14].

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The transition effect of the boundary layer flow due to suddenly imposed magnetic field on the viscous flow past a stretching sheet and sudden withdrawal of a magnetic field on the viscous flow past a stretching sheet under a magnetic field is discussed in [15] and [16] also discussed radiation and chemical reaction effects of the MHD flow over a vertical plate; they found that in both cases the sheet stretches linearly along the direction of the fluid flow. In [17] the radiation effects on unsteady natural convective flow of a nanofluid past an infinite vertical plate was discussed while [18] extended this work by considering magnetic field and chemical reaction effects on convective flow of dusty viscous fluid. Recently, the authors in [19] analysed the heat and momentum transfer characteristics at different channels by immersing the micro or millimetre sized particles in to base fluids.
Nevertheless, to the best of the authors' knowledge, no attempt has been made to analyse the simultaneous effects of suction/injection on boundary layer flow of a nanofluid over a nonlinearly permeable stretching sheet which is now investigated.

2. Formulation of the problem

We considered that the flow is the steady laminar, incompressible and two dimensional boundary layer and heat transfer over a nonlinearly stretching sheet. The governing boundary layer equations of continuity, momentum, energy and chemical species concentration are (see [1]]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \ldots (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = v\frac{\partial^2 u}{\partial y^2} \quad \ldots (2)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\frac{\partial C}{\partial x}\left(\frac{\partial T}{\partial y}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right\} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \quad \ldots (3)$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\left(\frac{\partial^2 C}{\partial y^2}\right) + \frac{D_T}{T_\infty}\left(\frac{\partial^2 T}{\partial y^2}\right) \quad \ldots (4)$$

The boundary conditions for equations (1) – (4) are:

$u=\lambda U_w$, $v=V_w$, $T=T_w$, $C=C_\infty$ at $y=0$,

$U\to 0$, $T\to T_\infty$, $C\to C_\infty$, as $y\to\infty$ ….(5)

Where u and v are the velocity components in the x and y directions respectively, T is the temperature, C is the nanoparticle concentration, T_W is the surface temperature, T_∞ is the ambient temperature, C_W the nanoparticles volume fraction at the plate and is $C_\infty$the nanoparticles concentration far from the plate, the suction velocity is$v_w < 0$ while $v_w > 0$ is the velocity injection,$\tau = (\rho c)_p / (\rho c)_f$, where$(\rho c)_p$ is the effective heat capacity of the nanoparticles,$(\rho c)_f$ is the heat capacity of the base fluid, $\alpha = k_{nf}/(\rho c)_f$ is the thermal diffusivity of the nanofluid, v is the kinematic viscosity, D_B is the Brownian diffusion coefficient,D_T is the thermophoretic diffusion coefficient and λ is the stretching/shrinking parameter with $\lambda > 0$ for a stretching surface and $\lambda < 0$ for a shrinking surface . The constant n is the nonlinearity parameter with n=1for the linear case and $n \neq 1$ is for the nonlinear case. It is assumed that the surface is stretched or is shrunk with the velocity $U_w = ax^n$, where a > 0 is a constant. Thus, for a nonlinear flow of a fluid over a permeable stretching surface, by using Roseland approximation, the radiation heat flux q_r is given by:

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \quad \ldots (6)$$

Where σ^* and k^* are the Stefan- Boltzmann constant and the mean absorption coefficient respectively. Considering the temperature differences within the flow sufficiently small such that T^4 may be expressed as the linear function of temperature. Then expanding T^4 in Taylor series about T_∞ and neglecting higher-order terms takes the form:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \quad \ldots (7)$$

Equation (3) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left\{D_B\frac{\partial C}{\partial x}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial u}{\partial y}\left(\frac{\partial T}{\partial y}\right)^2\right\} + \frac{v}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 - \frac{1}{\rho c_p}\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \quad \ldots (8)$$

Equations (1) to (4) were subjected to the boundary conditions (5) .The governing partial differential equations can be solved by transforming them to ordinary differential equations by using similarity functions:

$$\eta = x^{\frac{n-1}{2}} y\sqrt{\frac{a(n+1)}{2v}}, \quad u = ax^n f'(\eta),$$

$$v = -\sqrt{\frac{av(n+1)}{2}}\, x^{\frac{n-1}{2}}\left(f(\eta) + \left(\frac{n-1}{n+1}\right)\eta f'(\eta)\right), \quad \theta(\eta) = \frac{T-T_\infty}{T_w - T_\infty}$$

$$\phi(\eta) = \frac{C-C_\infty}{C_w - C_\infty}, \quad R = \frac{16T_\infty^3\sigma^*}{3kk^*} \quad \ldots (9)$$

Where prime denotes differentiation with respect to η. To have similarity solution of equations (1) to (5), we assume: $v = -\sqrt{\frac{av(n+1)}{2}}\, x^{\frac{n-1}{2}} S$, where the constant parameter S corresponds to suction (S < 0) and injection (S>0) By applying these similarity variables on the governing partial differential equations, transformed conservation equations and boundary conditions , thus obtained as follows ([1]):

$$f''' + ff'' - \frac{2n\ (f')^2}{n+1} = 0 \quad \ldots\ldots (10)$$

$$\theta'' + Pr(N_b\theta'\phi' + N_t(\theta')^2 + f\theta' + Ec(f'')^2)/(1 + PrR) = 0 \quad \ldots\ldots (11)$$

$$\phi'' + \frac{N_t}{N_b}\theta'' + Lef\phi' = 0. \quad \ldots\ldots (12)$$

The boundary Conditions are:

$$f(0) = S, f'(0) = \lambda,\ \theta(0) = 1,\ \phi(0) = 1 \text{ as } \eta \to 0$$

$$f'(\eta) \to 0, \theta(\eta) \to 0, \phi(\eta) \to 0 \qquad \text{as } \eta \to \infty \quad \ldots\ldots (13)$$

Where dimensionless parameters are defined as:

$Pr = \frac{\nu}{\alpha}$(Prandtl number), $Le = \nu/D_b$(Lewis number),

$N_b = \frac{\tau(C_w - C_\infty)D_b}{\nu}$ (Brownian motion parameter),

$N_t = \frac{\tau(T_w - T_\infty)D_t}{\nu T_\infty}$(Thermophoresis parameter),

$Ec = \frac{(u_w)^2}{2c_p(T_w - T_\infty)}$ (Eckert number)

$R = \frac{16T_\infty^3\sigma^*}{(\rho c)_f 3\nu k^*}$ (Thermal radiation parameter) . . . . (14)

The quantities of physical and engineering interest are the skin friction coefficient $f''(0)$,the reduced Nusselt number $-\theta'(0)$ and the reduced Sherwood number - $\phi'(0)$. From the knowledge of the Nusselt number, the local convection coefficient may be found and the local heat flux may then be computed. The reduced Sherwood number on the other hand is the parameter that defines the dimensionless concentration gradient at the surface, and it provides a measure of the convection mass transfer occurring at the surface.The skin friction coefficient can be used to compute stresses at the wall.

$$C_f = \frac{\tau_w}{\rho(U_w)^2} = \frac{\mu\frac{\partial u}{\partial y}}{\rho(U_w)^2} = Re_x^{-1/2}\sqrt{\frac{(n+1)}{2}}f''(0)$$

$$C_f = Re_x^{-1/2}\sqrt{\frac{(n+1)}{2}}f''(0) \quad \ldots\ldots (15)$$

The local heat transfer rate (local Nusselt) number is given by

$$Nu_x = \frac{xq_w}{k(T_w - T_\infty)} = -\left(\frac{Re_x\nu}{a}\right)^{1/2}\sqrt{\frac{a(n+1)}{2\nu}}\theta'(0)$$

$$Nu_x\ Re_x^{-1/2} = -\sqrt{\frac{(n+1)}{2}}\theta'(0). \quad \ldots\ldots (16)$$

The local Sherwood number is;

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} = -\left(\frac{Re_x\nu}{a}\right)^{1/2}\sqrt{\frac{a(n+1)}{2\nu}}\phi'(0)$$

$$Sh_x\ Re_x^{-1/2} = -\sqrt{\frac{(n+1)}{2}}\phi'(0). \quad \ldots\ldots (17)$$

where q_w and q_m denotes the wall heat and mass flux rates respectively.

3. Numerical solution

The set of ordinary differential equations (10)-(12) has been solved numerically, using the boundary value problem (bvp) solver in Matlab Software, subject to the boundary conditions stated in equation (13). Solutions for the dimensionless parameters such as Prandtl number (Pr), Lewis number (Le), suction/injection parameter (S), radiation parameter (R) and stretching parameter (λ), has been obtained. The effect of thermal radiation on boundary layer flow and heat transfer over a nonlinearly permeable stretching sheet in a nanofluid with suction/injection are studied for different values of the governing parameters**.**

The results of our numerical procedure are compared for $-\theta'(0)$ with that of previously published work and is found to be in good agreement.

Table 1: Comparison of the values of $-\theta'(0)$ with those of Rana and Bhargava for Ec=0,R=0, Nt=Nb=0 and by setting $f(0) = 0$, $f'(0) = 1$ for the boundary conditions for various values of Pr and n shown.

Pr	n	Cortell [21]	Rana and Bhargava [20]	Present results
1.0	0.2	0.610262	0.6113	0.61022
	0.5	0.595277	0.5967	0.59522
	1.5	0.574537	0.5768	0.57477
	10	0.554960	0.5578	0.55495
5.0	0.2	1.607175	1.5910	1.60778
	0.5	1.586744	1.5839	1.58678
	1.5	1.557463	1.5496	1.55769
	10	1.528573	1.5260	1.52893

4. Results and Discussion

Figures 1-2 depict the effect of suction (S > 0) and injection (S < 0)on dimensionless velocity profiles $f'(\eta)$. It can be seen from the graph that the suction slows down the velocity of flow showing that suction reduces the boundary layer thickness and thus stabilizes the boundary layer growth, while injection increases the velocity in the boundary layer indicating that injection helps the flow to penetrate more into the fluid. In figures 3 and 4 the effect of suction and injection on the local skin friction coefficient are illustrated. Increasing the suction parameter (S > 0) increases the local skin friction coefficient up to the point where η=0.3, thereafter, the local skin coefficient increases steadily. On the other hands, injection of more fluid into the boundary layer increases the local skin friction coefficient up to the point where η=1.2 before it decreases to non-zero value. Figures 5 and 6, show that the concentration distribution decreases for increasing values of the suction parameter while the concentration increases for decreasing values of the injection parameter. With figures 7 and 8, we see temperature distribution decreasing for increasing the value of suction parameter while the temperature distribution increases for increasing injection parameter. This means that larger suction leads to faster cooling of the wall surface, which is of utmost importance in many engineering applications. But for larger values of injection parameter (S < 0) the temperature increases at the beginning until it achieves a certain value then decreases to the ambient value. This means that large values of the injection parameter the wall temperature gradient is positive which can be physically interpreted to mean that the heat is transferred from fluid to the surface. We also note this in figures 9 and 10, that the local Nusselt number increases rapidly at the initial point and later decreases at a point where (η<0.5) to a non-zero value for increasing values of suction parameter. For the different values of injection parameter the local Nusselt number increases until a rapid increase occurs at a point where (η≤1.8) and then increases rapidly to non-zero values. Figures 11 and 12, showthat the rate of mass transfer drops rapidly to near zero for increasing suction and increases gradually only to drop rapidly at about (η=1.4) to non-zero values for decreasing values of injection parameter

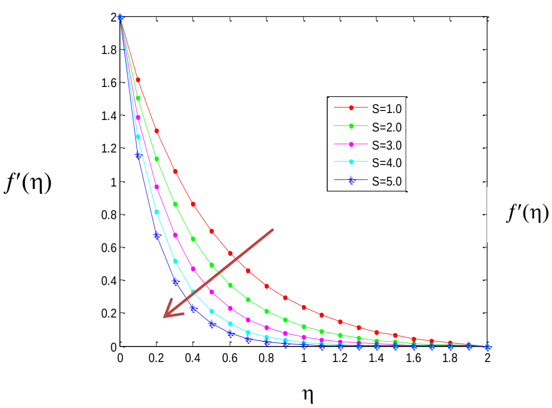

Figure 1: velocity profiles for various values of suction parameters.

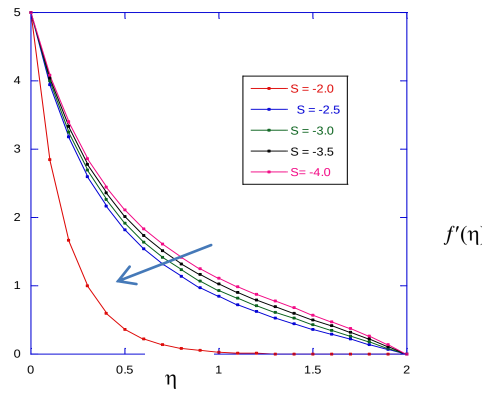

Figure 2: velocity profiles for various values of injection parameters.

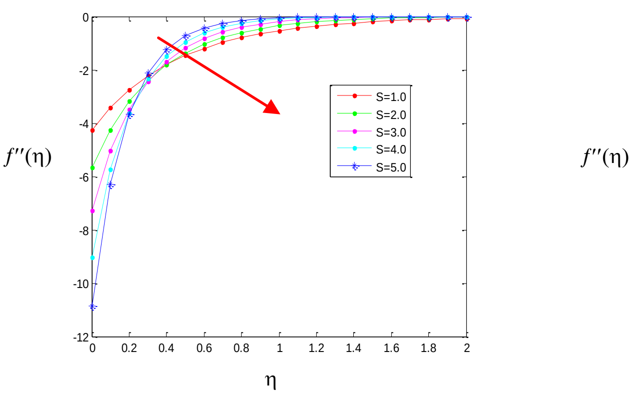

Figure 3: Skin friction for various values of suction parameters.

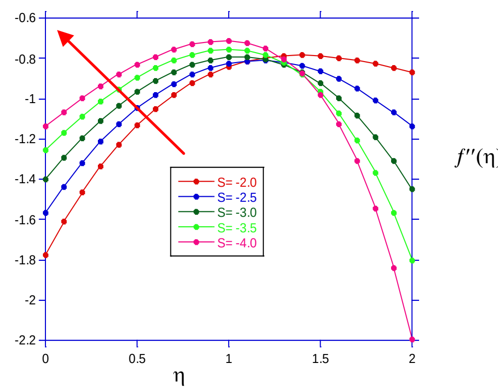

Figure 4: Skin friction for various values of injection parameters.

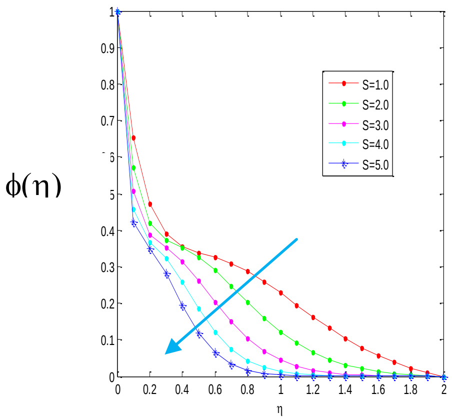

Figure 5: Concentration distribution for various values of suction parameters.

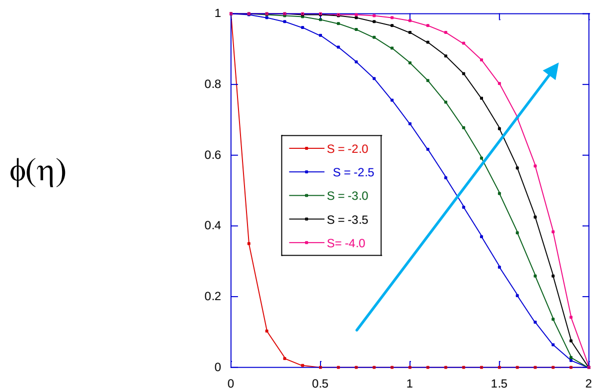

Figure 6: Concentration distribution for various values of injection parameters.

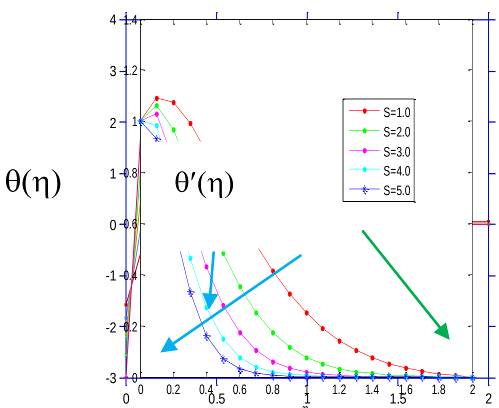

Figure 7: Temperature distribution for various values of suction parameters.

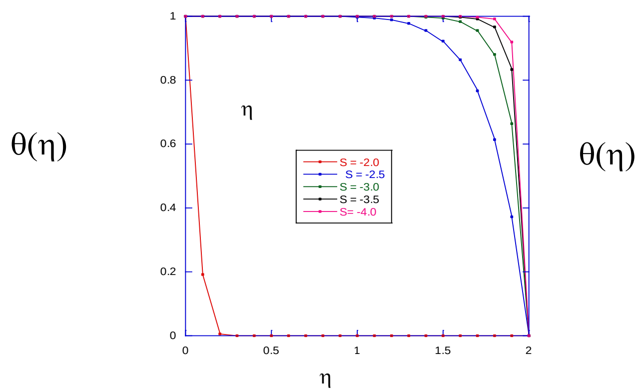

Figure 8: Temperature distribution for various values of injection parameters.

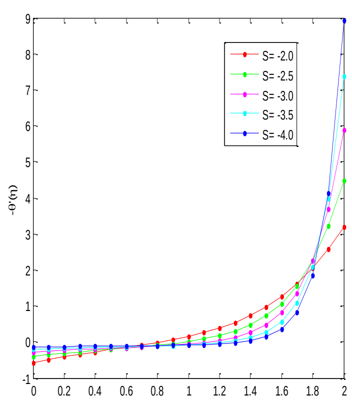

Figure 9: Rate of heat transfer for various values of suction parameters.

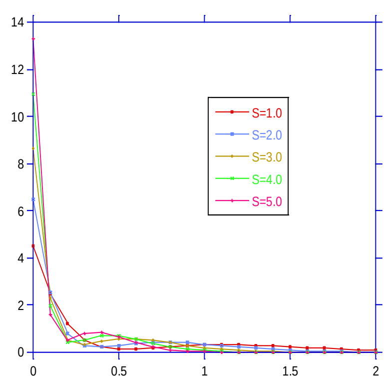

Figure 10: Rate of heat transfer for various values of injection parameters.

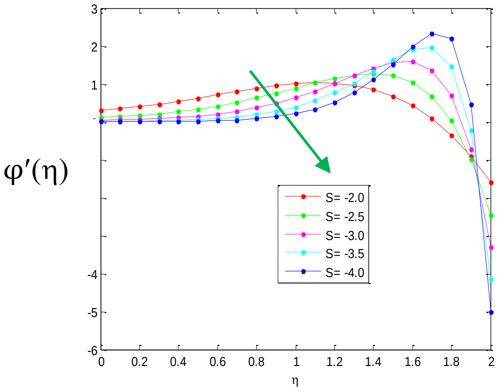

Figure 11: Rate of mass transfer for various values of suction parameters.

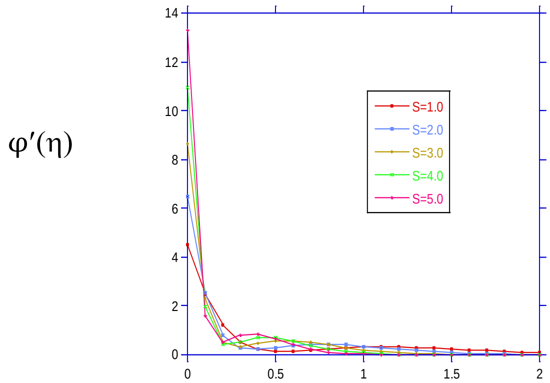

Figure 12: Rate of mass transfer for various values of injection

5. Conclusion

In conclusion, it is observed that large suction increases the local skin friction while large injection, though increases the local skin friction at first, nevertheless it decreases the local skin friction to a non-zero value. Furthermore, the heat transfer rate increases with suction initially it later decreases to a non-zerovalue towards the end of the boundary layer. The mass transfer rate drops rapidly to near zero value for increasing suction parameter, while it drops rapidly to non-zero values for decreasing injection parameter

List of symbols

S suction/injection parameter
f $reduced\ stream\ function$
T $temperature$
u, v $velocity\ components$
x, y $Rectangular\ Cartesian\ coordinates$
D_b $Brownian\ diffusion\ coefficient$
D_t $thermophoretic\ diffusion\ coefficient$
K_{eff} $effective\ thermal\ conductivity$
Le $Lewis\ number$
N_b $Brownian\ motion\ parameter$
N_t $Thermophoresis\ parameter$
Nu $Nusselt\ number$
Sh $Sherwood\ number$
R $Radiation\ parameter$
q $wall\ heat\ flux$
Re $Reynolds\ number$
T_w $Temperature\ at\ the\ wall$
T_∞ $Ambient\ temperature$
u_∞ $free\ stream\ velocity$

Greek symbols

α $thermal\ difussivity$
η $dimensionless\ distance$
θ $dimensionless\ Temperature\ distribution$
ϕ $dimensiopnless\ concentration\ distribution$
λ $stretching\ parameter$
μ $dynamic\ viscosity\ of\ fluid$
ρ_f $density\ of\ fluid$
ρ_s $density\ of\ nano\ particle$

$(\rho c)_f$ $heat\ capacity\ of\ fluid$
$(\rho c)_p$ $effective\ \ heat\ capacity\ of\ nano\ particle$
τ $of\ the\ effective\ heat\ capacity\ of\ the\ nano\ particle\ and\ that\ of\ the\ fluid$
ν kinematic viscosity of fluid

Subscripts
w $refers\ toconditions\ at\ the\ wall$
∞ $refers\ to\ conditions\ at\ the\ free\ stream$
nf $refers\ to\ nanoflu$

REFERENCES

[1] A. Falana O. A. Ojewale Adeboje T. B. (2016).Effect of Brownian motion and Thermophoresis on a Nonlinearly Stretching Permeable Sheet in a Nanofluid, Advances in Nanoparticles, (5) 123-134Published Online February 2016 in Sci Res. http://www.scirp.org/journal/anphttp://dx.doi.org/10.4236/anp.2016.51014

[2] Salleh, M.Z.; Nazar, R.; Ahmad, S. (2008). Numerical solutions of the forced convection boundary layer flow at a forward stagnation point, Eur. J. Sci. Res. 19: 644-653

[3] Palov, K.B. (1974). Magnaeto hydrodynamic flow of an incompressible viscous fluid caused by deformation of a plane surface, *Magnitnaya Gidrodinamika* (USSR) 4, 146-147.

[4] Cheng, P., Minkowycz, W. J. (1977). Free convection about a vertical flat plate embedded in a porous medium with application to heat transfer from adike. *J. of Geophys. Res*. 82, 2040-2044.

[5] Choi, S.U.S (1995) .Enhancing thermal conductivity of fluids with nano particles. *Devels. Appls. Non Newtonian flows.* 66, 99-105.

[6] Buongiorno, J. (1996). Convective transport in nanofluids. *J. of Heat Transfer*. 128, 240-250.

[7] Elbashbeshy, E.M.A. (2001). Heat Transfer over an exponentially stretching continuous surface with suction. *Arch Mech*. 53, 641-651.

[8] Khanafer, K., Vafai, K., Lightstone, M. (2003). Buoyancy driven heat transfer enhancement in a two dimensional enclosure utilizing nanofluids. *Int. J. of Heat Mass Transfer,* 3639- 3653.

[9] Das, S.K., Choi, S. U. S. Yu, Pradeep, T. (2007). Nanofluids science and technology, *Willey New Jersey.*

[10] Daungthongsuk, W. and Wongwises, S. (2007). A critical review of convective heat transfer nanofluids. *Renew. Sust. Energy Rev*. 11, 797-817.

[11] Oztop, H.F, E. Abu-Nada, (2008). Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *Int. J. of Heat Fluid Flow*, 29, 1326-1336.

[12] Kumari, M., Nath, G. (2009). Analytical solution of unsteady three-dimensional MHD boundary layer flow and heat transfer due to impulsively stretched plane surface. *Comm. In Nonlinear Science and NumericalSimulation*. 14, 3339-3350.

[13] Prasad, K.V., Vajravelu, K., Datti, P.S. (2010). The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet. *Int. J. of Thermal Sciences.* 49, 603-610.

[14] Hayat, T., Abbas, Z., Pop, I., Asghar, S. (2010). Effects of radiation and magnetic field on the mixed convection stagnation point flow over a vertical stretching sheet in a porous medium. *Int. J. Heat and MassTransfer* 53, 466-474.

[15] Kumaran, V., Kumar, A. V., Pop, I. (2010). Transmission of MHD boundary layer flow past a stretching sheet. *Comm. in Nonlinear Science and Numerical Simulation.* 15, 300-311.

[16] Sandeep, N., Sugunamma, V., Mohankrishna, P. (2013). Effects of radiation on an unsteady natural convective

[17] Sandeep, N. and Sugunamma, V. (2013). Effect of inclined magnetic field on unsteady free convection flow of a dusty viscous fluid between two infinite flat plates filled by porous medium. *Int. J.App.Math.Modeling*. 1, 16-33.

[18] Mohan Krishna, P., Sugunamma, V. and Sandeep, N. (2013). Magnetic field and chemical reaction effects on convective flow of dusty viscous fluid. *Communications in App.Sciences*, 1 ,161-187.

[19] Ramana Reddy, J.V., Sugunamma, V., Mohan Krishna, P., Sandeep, N. (2014). Aligned magnetic field, Radiation and chemical reaction effects on unsteady dusty viscous flow with heat

[20] Rana, P., Bhargava, R. (2012). Flow and heat transfer of a nanofluid over a nonlinearly stretching sheet: A numerical study. *Commun. Nonlinear Sci. Numer. Simulation* 17, 212-226.

[21] Cortell, R. Viscous flow and heat transfer over a nonlinearly stretching sheet. Appl. Math. Comput. 184, 864–873 (2007).