Journal of the Nigerian Association of Mathematical Physics
Volume 47 (July, 2018 Issue), pp111 – 116

NUMERICAL MODELING TO STUDY THE EFFECT OF SMALL BOTTOM IRREGULARITIES ON STEADY FLOW IN HOMOGENEOUS OCEAN

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Abstract

The numerical modeling of topographic waves is based on the continuity and momentum equations. The solutions of these equations for the vertical and horizontal velocities and surface elevation of the oceans were obtained through a FORTRAN-program that was run using different values of bottom slope (α), coefficient of bottom friction (r) at different intervals (n). Slopes of the plots of horizontal, vertical velocities and surface elevation of the ocean for flat bottom ocean and fixed values of bottom friction were obtained. Similarly, slopes of the plots of resultant velocity against surface elevation and surface elevation versus horizontal and vertical velocity were also obtained. The resultant velocity (c) against the surface elevation η of the ocean produced negative slopes. The low value of slopes is an indication that the ocean waves move slowly and steadily.

Keywords: Resultant Velocity, Bottom Slope, Bottom Friction Coefficient, Surface Elevation, Topographic Waves,

1.0 Introduction

The wave bottom boundary layer refers to the thin area of fluid that lies closest to turbulent shear flow near the ocean floor. The fluid motion in the wave bottom boundary layer caused by shoaling waves interacts with the seabed and causes a two-fold effect. In the other way, the wave field influences the seafloor by liberating sand particles from the bed when the local shear stress is strong enough. These particles can become entrained in the water column reducing water clarity and effectively transporting sediment to new locations. Under conditions of oscillatory potential flow external to the boundary layer caused by the passing of surface gravity waves, sand ripples can be formed as a result of the particle redistribution. As the ripples evolve, their presence feeds back into the fluid-sediment system and influences the behavior of the turbulent boundary layer. The coupling between fluid motion and particle redistribution continues to alter the state of the seabed until a quasi-steady state is achieved and the ripples maintain their shape. Practical aspects such as sediment transport associated with erosion processes, bottom drag on mean currents, and wave damping become important as the ripples influence the turbulent shear flow, and begin to migrate due to the combined effect of surface waves, tides, and currents in the near shore environment [1]. Variations in ripple amplitude and shape are explored along with known relationships between ripple dimensions and flow parameters, to simulate how specified flows behave during ripple formation. Through the examination of flow field dynamics and statistics, the wave energy dissipation rates are quantified and a better understanding of oscillatory flow over sand ripples in general is achieved [1]. Topographic waves are modeled using a primitive-equation ocean model [2].This model enabled the first general circulation studies with the primitive equations (i.e., without reliance on the quasi-geostrophic approximation, as it had been the case until then), but it eventually became apparent that a series of improvements were necessary. Many researchers have investigated the combined effects of different wave, current, and bed form situations and have found that the resulting flows, especially within the boundary layer, present highly nonlinear and complex relationships. Studies as early as observed connections between ripple evolution and vortex formation at the ripple crests in oscillatory flows. An important prediction of the eddy viscosity model was the indication that the mean current within the boundary layer may be distorted due to these complex nonlinear interactions in the presence of ripples. [3] Studied oscillatory turbulent flow near a rough seabed due to linearized surface waves and presented an analogy to steady turbulent flow. The analogy provides the basis for a time-varying eddy viscosity model, which was used to obtain approximate closed-form solutions to the boundary layer equations. [4] Used the κ-ω model of Wilcox to simulate waves plus a current over sand ripples and noted that the shape and steepness of the ripple was very important in obtaining a strong separation bubble at the crest. Recently, [5] quantified wave energy dissipation in turbulent boundary layers over a flat bed

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with direct numerical simulations in three dimensions utilizing a numerical model developed by [6]. The main aim of this work was to investigate the dynamics of boundary layer by adding features to the topography and comparing turbulence levels and dissipation rates to those of a smooth seabed.

2.0 Methodology

The momentum equations were used in the programming and the input parameters requisite stated. The program used was FORTRAN 95; data were input and run in order to achieve the output data.

2.1 Rotating Shallow Water Equations

The fluid thickness is expressed in terms of mean layer depth, H , upper free surface displacement, $\eta(x, y, t)$, and topographic elevation of the solid bottom surface, $B(x,t)$; the layer has a thickness,

$$h = H + \eta - B \tag{1}$$

a depthindependent horizontal velocity, u , a free surface elevation anomaly, η and a bottom elevation, B . The mean positions of the top and bottom are $z = H$ and $z = 0$, respectively.

Consequently,

$$\frac{d\omega}{dz} = \frac{1}{h}\frac{D\eta}{Dt} - \frac{1}{h}u.\nabla B \tag{2}$$

$$\frac{d\omega}{dz} = \frac{1}{h}\frac{D(\eta - B)}{Dt} \tag{3}$$

$$\frac{d\omega}{dz} = \frac{1}{h}\frac{Dh}{Dt} \tag{4}$$

The linear Shallow Wave Equations are:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \tag{5}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \tag{6}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \tag{7}$$

For conservative, flat bottom, Shallow Water equation becomes:

$$\frac{Du}{Dt} - fv = -g\frac{\partial \eta}{\partial x} \tag{8}$$

$$\frac{Dv}{Dt} + fu = -g\frac{\partial \eta}{\partial y} \tag{9}$$

$$\frac{D\eta}{Dt} + (H + \eta)\frac{\partial u}{\partial t} = 0 \tag{10}$$

where u and v are the horizontal and vertical velocities respectively.

The linear Shallow Wave Equations used in the FORTRAN PROGRAM for the horizontal and vertical integrated momentum are:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x} \tag{11}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y} \tag{12}$$

A FORTRAN program was designed to solve the numerical problems and was used to generate the velocities, depth at different value of bottom slope and friction.

3 RESULTS AND DISCUSSION

Tables 1, 2, 3 and 4 show the results obtained from the program. On the tables, the horizontal velocity is represented by U_{topo} in m/s, vertical velocity by V_{topo} in m/s and surface elevation (H_{topo}) by $\boldsymbol{\eta}$ in m.

Table 1: Computed values of horizontal, vertical, resultant velocities and surface elevation when $\alpha = 1.0e-4, n = 720, r = 0.0003$

horizontal velocity $U_{topo}(m/s)$	vertical velocity $V_{topo}(m/s)$	Resultant Velocity $C=(U^2+V^2)^{0.5}(m/s)$	surface elevation $H_{topo}(\eta)(m)$
0.00	0.00	0.00	0.00
0.00	1.71	1.72	0.00
0.18	1.72	1.73	0.07
0.30	1.78	1.81	0.23
0.40	0.92	1.00	0.34
0.46	0.06	0.46	0.42
0.51	0.46	0.69	0.47
0.53	0.30	0.61	0.50
0.54	0.17	0.57	0.53
0.55	0.06	0.55	0.54

Table 2: Computed values of horizontal, vertical, resultant velocities and surface elevation when $\alpha = 1.0e-4, n = 3600$, $r = 0.0003$

horizontal velocity $U_{topo}(m/s)$	vertical velocity $V_{topo}(m/s)$	Resultant Velocity $C=(U^2+V^2)^{0.5}(m/s)$	surface elevation $H_{topo}(\eta)(m)$
0.00	0.00	0.00	0.00
0.00	7.13	7.13	0.00
0.70	7.13	7.16	0.33
1.21	5.16	5.30	0.91
1.57	3.71	4.03	1.33
1.83	2.63	3.20	1.63
2.01	1.80	2.70	1.84
2.14	1.16	2.43	1.98
2.21	0.65	2.30	0.21
2.23	0.21	2.24	0.21

Table 3: Computed values of horizontal, vertical, resultant velocities and surface elevation when $\alpha = 1.0e-4, n = 7200$, $r = 0.0003$

horizontal velocity $U_{topo}(m/s)$	vertical velocity $V_{topo}(m/s)$	Resultant Velocity $C=(U^2+V^2)^{0.5}(m/s)$	surface elevation $H_{topo}(\eta)(m)$
0.00	0.00	0.00	0.00
0.00	7.15	7.15	0.00
0.70	7.12	7.15	0.29
1.20	5.12	5.26	0.88
1.57	3.73	4.05	1.32
1.82	2.64	3.21	1.63
2.01	1.82	2.71	1.84
2.13	1.17	2.43	1.99
2.20	0.65	2.29	0.21
2.23	0.21	2.24	0.21

Table 4: Computed values of horizontal, vertical, resultant velocities and surface elevation when $\alpha = 1.0e-4,\ n = 10800\ ,\ r = 0.0003$

horizontal velocity $U_{topo}(m/s)$	vertical velocity $V_{topo}(m/s)$	Resultant Velocity $C = (U^2 + V^2)^{0.5}(m/s)$	surface elevation $H_{topo}(\eta)(m)$
0.00	0.00	0.00	0.00
0.00	7.18	7.18	0.00
0.70	7.18	7.21	0.28
1.20	5.21	5.35	0.88
1.56	3.76	4.07	1.34
1.82	2.67	3.23	1.66
2.00	1.84	2.72	1.88
2.12	1.19	2.43	2.01
2.20	0.67	2.30	2.11
2.22	0.21	2.23	2.16

Figures 1 to 12 shows the variations of the resultant velocity (C) against the surface elevation, Surface elevation against horizontal velocity and surface elevation against vertical velocity for various values obtained as shown (tables 1 to 4). This shows the effect of bottom slope (α) and bottom frictionon steady flow.

Figures 1 – 3 represents the plots of table 1

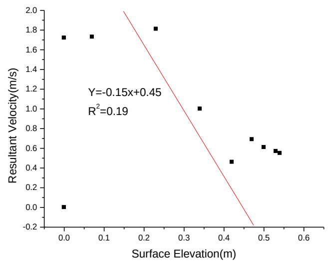

Figure 1: Resultant Velocity (m/s) against Surface Elevation (m).

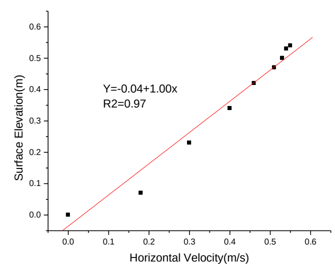

Figure 2: Surface Elevation (m) against Horizontal Velocity (m/s).

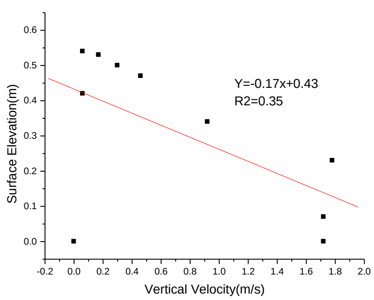

Figure 3: Surface Elevation (m) against Vertical Velocity (m/s).

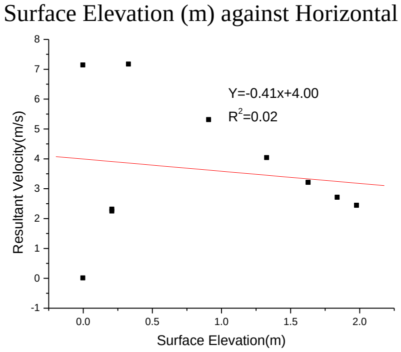

Figure 4: Resultant Velocity (m/s) against Surface Elevation (m).

Figures 4 – 6 represents the plots of table 2

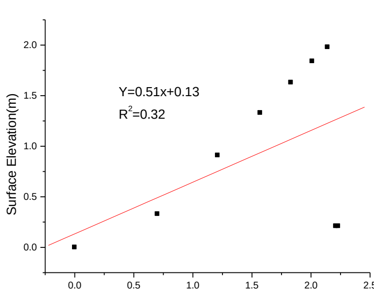

Figure 5: Surface Elevation (m) against Horizontal Velocity (m/s).

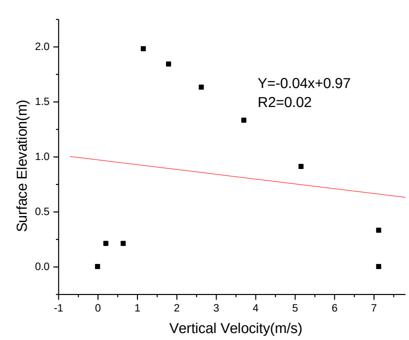

Figure 6: Surface Elevation (m) against Vertical Velocity (m/s).

Figures 7 – 9 represents the plots of table 3

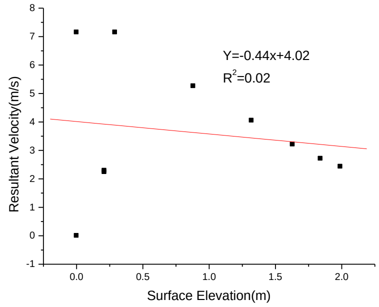

Figure 7: Resultant Velocity (m/s) against surface Elevation (m).

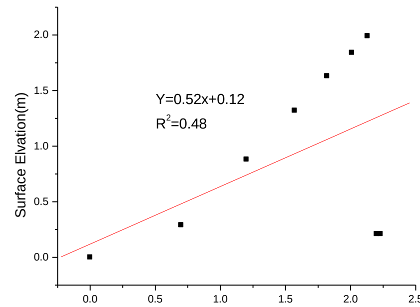

Figure 8: Surface Elevation (m) against Horizontal Velocity (m/s).

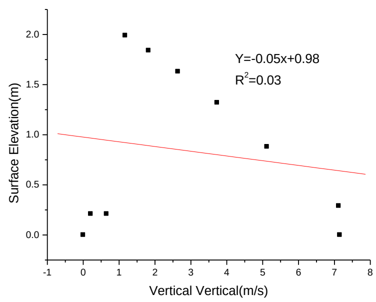

Figure 9: Surface Elevation (m) against Vertical Velocity (m/s).

Figures 10 – 12 represents the plots of table 4

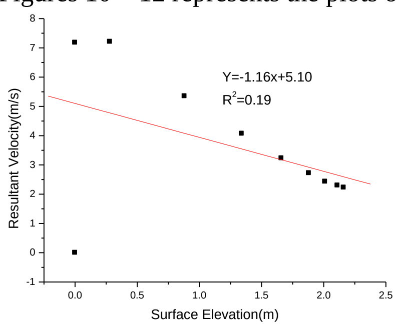

Figure 10: Resultant Velocity (m/s) against Surface Elevation (m)

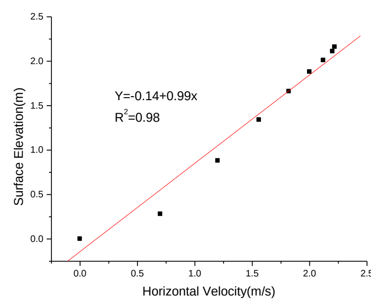

Figure 11: Surface Elevation (m) against Horizontal Velocity (m/s)

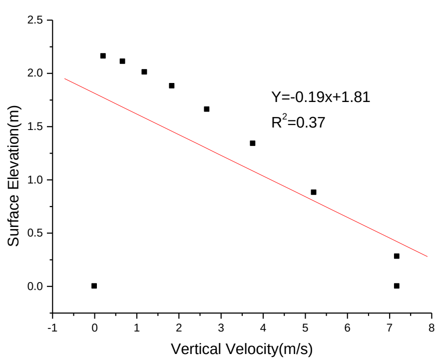

Figure 12: Surface Elevation (m) against Vertical Velocity (m/s)

In figures 1, 4, 7 and 10, the negative slopes mean the bottom slope is very small, and these will result to slow movement of the ocean waves.
Similarly, in figures 2, 5, 8, and 11 the bottom slopes were also low, meaning steady flow of ocean waves. The same was observed in figures 3, 6, 9, and 12.

5.0 CONCLUSION

The values of bottom slope, coefficient of bottom friction at different intervals are the determining factors of oceans waves. It is thus necessary to consider friction for correct flow modeling. Topography or bottom slope is another important parameter that has an effect on steady flows in homogenous ocean. Hence, low bottom slope will result to slow movement of ocean waves. In case of high bottom slope, the turbulence levels of the ocean wave will be high which may result to unsteady state.

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