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DETERMINATION OF THE ECONOMIC ORDER QUANTITY FOR DELAYED DETERIORATING ITEMS WITH LINEAR TRENDED DEMAND AND PARTIAL BACKLOGGING

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Abstract

In this paper, the Economic Order Quantity model for Delayed deteriorating items with linear trended demand and partial backlogging is developed. The model, determines the cycle length or when to place an order for the inventory and how much to order and considers the assumption that items in stock have a delayed deterioration, meaning that they do not start deteriorating immediately they are stocked until after some time. The demand rate before deterioration sets in is a linear function of time, whereas when deterioration sets in, the demand rate is constant and it is assumed to remain so up to when the inventory reaches zero. Numerical examples on the model's application are provided.

***Keywords*:** Inventory, Delayed Deterioration, Trended Demand, partial backlogging

1.0 Introduction

The depletion of inventory could be as a result of demand or deterioration over a period of time. Items Such as vegetables, fruits and other food stuff deteriorate through direct spoilage. Highly volatile liquids like petrol, alcohol and so on are depleted physically over time through evaporation. Radioactive materials, electronics and so on deteriorate through the gradual loss of potential or utility with the passage of time. Many inventory models were developed with the assumption of a constant demand rate but in a more realistic situation, the demand for an inventory is a function of time.

The development of inventory replenishment policy model with a linear trend in demand was pioneered by Donaldson[1] who constructed an inventory replenishment policy model for a linear trend in demand. Abubakar and Usman [2] developed an Ordering policies of Delayed Deteriorating items with unconstrained retailer's capital and linear Trend in Demand. Musa and Sani [3] constructed an Inventory ordering policies of delayed deteriorating items under permissible delay in payments. Riche, [4] constructed the EOQ model for linear increasing demand. Cox, et,al [5] developed a model on the determination of order quantities with a linear trend in demand. Goyal, [6] constructed an Economic Replenishment Intervals model for linear trend in demand. Chakrabarty, et,al[7] developed an EOQ model for items with weibul distribution deterioration, shortages and trended demand. Chang and Dye [8] developed an EOQ model for deteriorating item with time varying demand and partial backlogging. Bar-lev, et,al [9] developed an EOQ Model with Inventory-Level-Dependent Demand Rate and Random Yield. Ahmad and Musa [10] Constructed An Economic Order Quantity for Delayed Deteriorating items Inventory with Time Dependent Exponential Declining Demand and shortages.

In this paper, an EOQ model for delayed deteriorating items with linear trend in demand is developed. The model, determines the cycle length for the inventory and amount to order.It is also built on the assumption that items in stock have a delayed deterioration meaning that they do not start deteriorating immediately they are stocked until after some time.

The demand rate before deterioration sets in is a linear function of time and when deterioration sets in, the demand rate is constant and it is assumed to remain so up to when the inventory reaches zero.

1.1 Assumptions and Notation

The following assumptions and notation are employed in the development of the model.

1.1.1 Assumptions

(i) Shortages are not allowed (ii) Instantaneous replenishment (iii) Linear trend in demand
(iv) Lead time is zero

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1.1.2 **Notation**

$\delta_1(t)$ The demand rate(unit per unit time) during the period before deterioration sets in where

$\delta_1(t) = \alpha_1 + \alpha_2 t$

T The inventory cycle length $T \; C \; A$

CThe unit cost of the item

T_1 The time the deterioration sets in

AThe ordering cost per order

i The inventory carrying charge (excluding interest charges, naira per unit time)

λ The rate of deterioration

$D(T_2)$ The number of items that deteriorate during the time T_2

H_C The inventory holding cost per cycle

I_0 The initial inventory

$I(t)$ The inventory level at any time t before deterioration begins

I_d The inventory level at the time deterioration begins

$I_d(t)$ The inventory level at any time t after deterioration sets in

T_d The total demand between T_1 and T

2.0 Mathematical Formulation and Solution

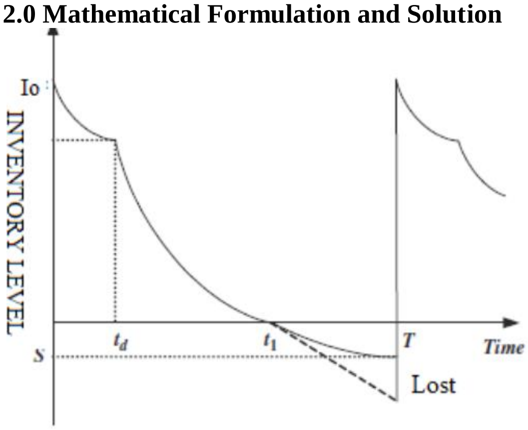

Figure 1: Inventory depletion in a delayed deterioration situation with no shortages.

The Mathematical Model

The depletion of inventory in the intervals $0 \le t \le T_1$ and $T_1 \le t \le T$ before deterioration begins, are described by the following differential equation:

$$\frac{dI(t)}{dt} = Q(t)I(t) = -(\alpha_1 + \alpha_2 t), \qquad 0 \le t \le T_1 \quad (1)$$

Where $Q(t) = Qt$ and $D_1(t) = \alpha_1 + \alpha_2 t$

The solution of Equation (1) using the condition $I(t_1) = 0$ is

$$I(t) = \left[\alpha_1\left(t_1 + \frac{\theta t_1^3}{6}\right) + \alpha_2\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right)\right]e^{\frac{-\theta t_1^2}{2}} - \left[\alpha_1\left(t + \frac{\theta t^3}{6}\right) + \alpha_2\left(\frac{t^2}{2} + \frac{\theta t^4}{8}\right)\right]e^{\frac{-\theta t^2}{2}}, \qquad 0 \le t \le t_1 \qquad (2)$$

(Neglecting the higher power of θ as $0 < \theta \le 1$)

Maximum inventory level for each cycle is obtained by putting the boundary condition $I(0) = W$ in Equation (2). Therefore,

$$I(0) = W = \left[\alpha_1\left(t_1 + \frac{\theta t_1^3}{6}\right) + \alpha_2\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right)\right]e^{\frac{-\theta t_1^2}{2}} \qquad (3)$$

During the shortage interval $[t_1, T]$, the demand at time is partially backlogged defined as $[1 + \delta(T - t)]^{-1}$. Therefore, the differential equation governing the amount of demand backlogged is

$$\frac{dI(t)}{dt} = -\frac{K_0}{1+\delta(T-t)} t_1 < t \leq T \quad (4)$$

With the boundary condition$(t_1) = 0$. The solution of Equation (4) is

$$I(t) = \frac{K_0}{\delta}\ln[1+\delta(T-t)] - \frac{K_0}{\delta}\ln[1+\delta(T-t_1)], \qquad t_1 < t \leq T \quad (5)$$

Maximum amount of demand backlogged per cycle is obtained by putting t = T in Equation (5) therefore,

$$S = -I(t) = \frac{K_0}{\delta}\ln[1+\delta(T-t_1)] \quad (6)$$

Hence, the economic order quantity per cycle is

$$Q = W + S = \alpha_1\left(t_1 + \frac{\theta t_1^3}{6}\right) + \alpha_2\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right) + \frac{K_0}{\delta}\ln[1+\delta(T-t_1)] \quad (7)$$

The inventory holding cost per cycle is

$$HC = C_1\int_0^{t_1} I(t)dt$$

$$C_1\int_0^{t_1}\left[\alpha_1\left(t_1 + \frac{\theta t_1^3}{6}\right) + \alpha_2\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{8}\right)\right]e^{\frac{-\theta t_1^2}{2}}dt - \left[\alpha_1\left(t + \frac{\theta t^3}{6}\right) + \alpha_2\left(\frac{t^2}{2} + \frac{\theta t^4}{8}\right)\right]e^{\frac{-\theta t^2}{2}}dt \quad (8)$$

$$= C_1\left[\alpha_1\left(\frac{t_1^2}{2} + \frac{\theta t_1^4}{12}\right) + \alpha_2\left(\frac{t_1^3}{3} + \frac{\theta t_1^5}{15}\right)\right]$$

The differential equation representing the status of the inventory when the deterioration sets in

$$\frac{dI_d(t)}{dt} = -(\delta_2 + \lambda I_d(t)), \qquad T_1 \leq t \leq T \ (2)$$

Their solutions $I(t)$ from equation (1) is given by $\frac{dI(t)}{dt} = -\delta_1(t) = -(\alpha_1 + \alpha_2 t), \qquad 0 \leq t \leq T_1$

And $I_d(t)$ are obtained from equation (2) as follows:

$$\int dI(t) = -\int(\alpha_1 + \alpha_2 t)dt \Rightarrow I(t) = -\alpha_1 t - \frac{\alpha_2 t^2}{2} + \beta_1 \ (3)$$

Where β_1 is an arbitrary constant?Applying the initial condition at $t=0$, $I(t) = I_0$, we have from equation (3) which yields when substituted into (3)

$$I(t) = -\alpha_1 t - \frac{\alpha_2 t^2}{2} + I_0 \ (4)$$

Also at $t= T_1$, $I(t) = I_d$, we get from equation (4)

$$I_d = -\alpha_1 T_1 - \frac{\alpha_2 T_1^2}{2} + I_0 = -\left(\alpha_1 + \frac{\alpha_2 T_1}{2}\right)T_1 + I_0 \quad (5)$$

So that from equation (5) we have

$$I_0 = I_d + \left(\alpha_1 + \frac{\alpha_2 T_1}{2}\right)T_1 \ (6)$$

Substituting equation (6) into equation (4), gives:

$$I(t) = I_d + (T_1 - t)\alpha_1 + (T_1^2 - t^2)\frac{\alpha_2}{2} \quad (7)$$

The integrating factor for equation (2) is given by $I(t, I_d) = e^{\int \lambda dt} = e^{\lambda t}$, multiplying (2) through by the integrating factor and integrating yields:

$\int \frac{d}{dt}(I_d(t)e^{\lambda t}) = -\delta_2\int e^{\lambda t}dt$, which gives on simplification:

$$I_d(t) = -\frac{\delta_2}{\lambda} + \beta_2 e^{-\lambda t} \quad (8)$$

Where β_2 is an arbitrary constant? We apply the condition at $t = T_1$, $I_d(t) = I_d$ which yields: $I_d = -\frac{\delta_2}{\lambda} + \beta_2 e^{-\lambda T_1}$

$$\Rightarrow \beta_2 = \left(I_d + \frac{\delta_2}{\lambda}\right)e^{\lambda T_1} \quad (9)$$

Substituting β_2 into equation (8) to get

$$I_d(t) = \frac{\delta_2}{\lambda}(e^{(T_1-t)\lambda} - 1) + I_d e^{(T_1-t)\lambda} \quad (10)$$

Now at $t = T$, $I_d(t) = 0$, equation (10) becomes

$$I_d = \frac{-\delta_2}{\lambda}(1 - e^{(T-T_1)\lambda}) \quad (11)$$

Substituting(11) into equation (10) gives:

$$I_d(t) = \frac{\delta_2}{\lambda}(e^{(T-t)\lambda} - 1) \quad (12)$$

Substituting I_d from equation (11) into (7), yields

$$I(t) = \frac{-\delta_2}{\lambda}(1 - e^{(T_1-T)\lambda}) + (T_1 - t)\beta_1 + (T_1^2 - t^2)\frac{\alpha_2}{2} \quad (13)$$

The product of the demand rate at the beginning of deterioration and the time period when the item deteriorates gives the total demand for the inventory, T_d between T_1 and T. Hence, $T_d = \delta_2 T_2$

And the number of items that deteriorate during the interval $T_1 \le t \le T$ is given by:

$$d(T_2) = I_d - \delta_2 T_2 \quad (14)$$

Substituting I_d from equation (11)into equation (14) yields

$$d(T_2) = -\frac{\delta_2}{\lambda}(1 - e^{(T_1-T)\lambda}) - \delta_2 T_2 = -\frac{\delta_2}{\lambda}(1 - e^{(T-T_1)\lambda} + (T - T_1)\lambda) \quad (15)$$

The total Inventory Cost: This is made up of the sum of the inventory carrying cost, ordering Cost, backorder cost and the cost due to deterioration of materials. The costs are as given below.

(a) The inventory ordering cost per order is given as A

(b) The inventory holding cost per period is computed as:

$$H_C = iC\int_0^{T_1} I(t)dt + iC\int_{T_1}^{T} I_d(t)dt = \left(e^{(T-T_1)\lambda} + \frac{1}{T_1\lambda}\left(e^{(T-T_1)\lambda} - 1\right) - 1 + \left[\frac{\alpha_1}{2\delta_2} + \frac{\alpha_2 T_1}{3\delta_2}\right]T_1\lambda\right)\frac{iC\delta_2 T_1}{\lambda} \quad (16)$$

Cost due to deterioration of materialsis computed as:

$$C(d(T_2)) = C\left(-\frac{\delta_2}{\lambda}(1 - e^{(T-T_1)\lambda} + (T - T_1)\lambda)\right) \quad (17)$$

The total inventory cost per unit is given as:

$TI_C(T) = \frac{1}{T}$ (Inventory ordering cost + Cost of deteriorated items + Inventory holdingcost)/length of the ordering cycle

$$= \frac{1}{T}\left(A + Cd(T_2) + H_C\right)$$

$$= \frac{A}{T} - \frac{C\delta_2}{T\lambda} + \frac{C\delta_2 e^{(T-T_1)\lambda}}{\lambda T} - \frac{C\delta_2\lambda(T - T_1)}{T} + \left(\frac{e^{(T-T_1)\lambda}}{T} + \frac{1}{T_1 T\lambda}\left(1 - e^{(T-T_1)\lambda}\right)\right.$$

$$\left. - \frac{1}{T} + \left[\frac{\alpha_1}{2\delta_2} + \frac{\alpha_2 T_1}{3\delta_2}\right]\frac{T_1\lambda}{T}\right)\frac{iC\delta_2 T_1}{\lambda}$$

To determine the value of T which minimizes the total inventory cost per unit time, we evaluate

$\frac{dTI_C(T)}{dT} = 0$ Simplifying to get:

$$-A+\frac{C\delta_2}{\lambda}\left(1+(T\lambda-1)e^{(T-T_1)\lambda}\right)-C\delta_2\lambda T_1+\left\{(T\lambda-1)e^{(T-T_1)\lambda}+\frac{1}{T_1\lambda}\left(-1-(T\lambda-1)e^{(T-T_1)\lambda}\right)+1\right.$$
$$\left.-\left[\frac{\alpha_1}{2\delta_2}+\frac{\alpha_2 T_1}{3\delta_2}\right]T_1\lambda\right\}\frac{iC\delta_2 T_1}{\lambda}=0 \quad (19)$$

The corresponding Economic Order Quantity (EOQ) can be determined from the following:

$$EOQ=\delta_1T_1+\delta_2T_2+d(T_2)=\delta_1T_1+\delta_2T_2-\frac{\delta_2}{\lambda}(1-e^{(T-T_1)\lambda})-\delta_2T_2$$
$$=\delta_1T_1-\frac{\delta_2}{\lambda}(1-e^{(T-T_1)\lambda}) \quad (20)$$

3.0 Numerical Examples

The solutions of sevendifferent numerical examples representing the application of the model are given in the table 1 using the model Maple (2017) mathematical software gives the corresponding Optimal cycle length (T), the minimum total inventory cost (TC) and Economic Order Quantity (EOQ) as given in Table 1:

Table 1

S/N	A (NAIRA)	C	δ_1 (Units)	δ_2 (Units)	i	T_1	λ	α_1	α_2	T	$TI_C(T)$	*EOQ (Units)*
1	130	120	132	150	0.13	0.0247	0.4	60	80	0.0575 21 days	5168.45	08
2	99	200	282	31	0.12	0.0521	0.44	100	200	0.1041 37 days	1919.71	16
3	80	50	320	120	0.14	0.0603	0.25	120	250	0.1808 65 days	1272.47	34
4	290	112	575	200	0.11	0.0657	0.17	400	250	0.1863 67 days	4426.71	62
5	300	115	780	210	0.11	0.0548	0.18	500	400	0.1808 65 days	5406.40	70
6	120	110	760	140	0.12	0.0246	0.3	50	70	0.0669 24 days	8441.57	25
7	400	118	130	220	0.11	0.0648	0.9	60	80	0.2062 74 days	4567.66	42

6. Conclusion

In this paper, we present an EOQ model for delayed deteriorating items with the assumption of linear time dependent demand and partial backlogging before deterioration and a constant demand rate after deterioration. The model determines the cycle length for the inventory and also the amount to be ordered per cycle length. Five different numerical examples on the application of the model are also given.

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