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SPECTRAL BASED COMPUTATIONAL METHODS FOR SOLUTIONS OF FOURTH VARIABLE COEFFICIENTS PARABOLIC PARTIAL DIFFERENTIAL EQUATION

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Abstract

Spectral based computational methods for solutions of variable coefficients partial differential equation were developed at step numbers j = 2; resulting into a Trapezoidal rule spectral based computational schemes. The accuracy, consistency, stability and convergence properties of these methods were determined. The methods were implemented on some sampled problems that involve constant coefficient parabolic partial differential equations; and evaluated by comparing them with some existing difference methods. The results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros. This work will provide better numerical solutions to a class of dynamical problems having time dependent boundary conditions. Higher ordered parabolic partial differential equations with defined theoretical solutions to given boundary conditions can be solved directly using this method.

Keywords: Trigonometric Functions, Taylor Series Expansion, Time, Finite& Space Difference

1.0 Introduction

Spectral methods had been described as the prevailing numerical tools for large scale calculations [1].This method describes the spatial discretization techniques that rely on the expansion of the flow solutions to function that has global support and; the first to be used in the practical flow simulation [2]. Spectral methods for the solutions of partial differential equations were firstly proposed numerically by some meteorologists as contained in [1]. That report reiterated the expense of computing non-linear terms which imposed some difficulties not until when the transform methods which forms the backbone of many large scale spectral computations were developed. Better approaches were proposed which explained the spectral collocation techniques such that the fundamental unknowns are the solution values at some selected points, and the series expansion is used solely for the purpose of approximating derivatives [3]. The roles of spectral methods in meteorology were detailed for areas of heat transfer, boundary layers, reacting flows, compressible flows, and magneto-hydrodynamics; spectral methods have proved to be a viable alternative to the traditional finite difference methods and finite – element methods. Many fields of application; and theoretical development in the numerical analysis of spectral methods had been of historical references; amongst which are; fluid – dynamics applications using multi- grid techniques and compressible – flow application as referred in [1],

Authors in [4] and [[5] laid emphasis on the detailed description of many spectral algorithms, aims at giving the fundamental ideas, focusing on the popular Chebyshev-collocation and Fourier-Galerkin methods and presents an exhaustive discussion of the theoretical aspects of these numerical methods. A continuous hybrid method using the method of line approaches with Symmetric, Legendre or Chebyshev polynomial as the bases functions differently for solving parabolic partial differential equations as proposed by [6].

In the course of this research work; it was noted that most of these cited researchers focused on the use of Orthogonal, Legendre or Chebyshev polynomials and developed their models using weighted residual methods but ignored the use of global spectral functions involving both sinusoidal and cosinuzoidal identities as the basis function which could be adopted in the sequel of developing some spectral based computational methods for the solutions of second order parabolic partial differential equations.

This study focus on the general second order linear non-homogenous partial differential equation of the form

$a(x,t)U_{tt}+2b(x,t)U_{xt}+c(x,t)U_{xx}=d(x,t,u,U,U_x,U_t)$ (1.01)

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[as reported in[7] and [8] where a, b, c, d are constants, as either characteristics or canonical; especially for a second order equation in which the derivatives of second order all occur linearly; with coefficients only depending on the independent variables.
The characteristic equation of (1.01) is

$$\frac{dy}{dt} = \frac{b \pm \sqrt{(b^2 - ac)}}{2a};$$

$equation\ (1.01)\ is\ Parabolic;\ if\ b^2 - ac = 0; \rightarrow one\ characteristic;;$
$equation\ (1.01)\ is\ called\ Hyperbolic\ ;\ if\ b^2 - ac > 0; \rightarrow two\ characteristic;;$
$equation\ (1.01)\ is\ called\ Elliptic\ ;\ if\ b^2 - ac < 0; \rightarrow two\ characteristic;$
Equation (1.01) can be written in canonical form as follows;
$U_{xx} - U_t = 0\ is\ Parabolic; (one\ dimensional\ heat\ equation)$
$U_{xx} - U_{tt} = 0\ is\ Hyperbolic; (one\ dimensional\ wave\ equation)$
$U_{xx} + U_{tt} = 0\ is\ Elliptic; (two -\ dimensional\ Laplace\ equation)$
[as reported in [7]

2.0 Methodology

In this work, the spectral based theoretical solution $U_{(x_{m+i},t_{n+j})} = aCos(x_{m+i},t_{n+j}) + bSin(x_{m+i},t_{n+j}),\ i,j = 0,1,2,3.$ is partly differentiated at constant time t but varying on the space (x), and substituted into a general partial differential equation of the form;
$a(x,t)U_{tt} + 2b(x,t)U_{xt} + c(x,t)U_{xx} = d(x,t,u,U,U_x,U_t)$ leading to a general parabolic partial differential equation of the form $c(x,t)U_{xx} = d(x,t,u,U,U_x,U_t)$ and; by adopting the spectral based theoretical solution $U_{(x_{m+i},t_{n+j})} = aCos(x_{m+i},t_{n+j}) + bSin(x_{m+i},t_{n+j}),\ i,j = 0,1,2,3.$ to aforementioned general parabolic equation and further simplifications were performed resulting into development of a class of spectral based computational methods for solution of partial differential equations at step numbers j = 2, we obtained three distinct spectral based computational schemes as stated below;

Methods for step number j = 2

(i) $U_{(m,n+2)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+2)} + 2f_{(m,n+1)} + f_{(m,n)}]$ ***Scheme* 1**

Equation (i) is a spectral Trapezoidal Rule to the method with step number two [16]

(ii) $U_{(m+2,,n)} = 2U_{(m+1,n)} - U_{(m,n)} + \frac{h^2}{2^2}(g_{(n+2,t)} + 2g_{(n+1,t)} + g_{n,t})$ ***Scheme* 2**

(iii) $U_{(m,n+2)} = U_{m,n} + k[f_{x,(n+1)} + f_{x,n}] + \frac{h^2}{4}(g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t})$ ***Scheme* 3**

Equate (i) and (iii) to obtain a generalized spectral based method for cases in step number two as

(a) $f_{m,n+2} - f_{m,n} = \frac{h^2}{2k}(g_{m+2,t} + 2g_{m+1,t} + g_{m,t})$

Equation (1.01) was re-defined based on the behavior of its constant coefficients as reiterated by [9] as follows; taking partial derivatives of equation (1.01) with respect to x at time t constant then $a = b = 0, and\ c \neq 0;\ then;\ b^2 - ac = 0\ will\ be\ satisfied.$ Equation (1.01) becomes a general linear second order parabolic partial differential equation of the form

$$c(x,t)U_{xx} = d(x,t,u,U,U_x,U_t) \quad (1.1)$$

$where;\ U_t = U_t(x,t,u), \quad U_{xx} = U_{xx}(x,t,u),$
$with\ initial\ condition\ 0 < x < L;\ \forall\ t \geq 0$
$and\ boundary\ conditions\ , U(0,t) = U(L,t) = 0$
$the\ region\ xt - plane\ in\ which\ the\ solution\ is\ sought\ is\ described\ by\ inequality$
$0 \leq x \leq L;\ and\ t \geq 0\ and,\ c = f(x)\ \forall x \epsilon (q,r), q, r, L\ are\ set\ of\ integers,$
$c\ is\ a\ singular\ coefficient;$ [[9], [10], [11] and [12]]

Various authors like [6], [8] , [12] and [15] , to mention but a few; reconsidered equation (1.1) at c = f(x) = 1; as x = 1; to suit the development of their schemes before generalizing the resulting difference methods. Thus, equation (1.1) can be written as a diffusion equation of the form

$$U_t = U_{xx} \quad (1.2)$$

$where; \quad U_t = U_t(x,t,u), \quad U_{xx} = U_{xx}(x,t,u),$
$with\ initial\ condition\ 0 < x < L;\ \forall\ t \geq 0$
$and\ boundary\ conditions, U(0,t) = U(L,t) = 0,$
$the\ region\ xt - plane\ in\ which\ the\ solution\ is\ sought\ is\ described\ by\ inequality$
$0 \leq x \leq L;\ and\ t \geq 0\ and,\ p = f(x)\ \forall x \epsilon (q,r);\ q, r, L\ are\ set\ of\ integers,$

$p\ is\ a\ variable\ coefficient.$ The grid points (x_m, t_n) where $x_m = x_0 + mh, t_n = t_0 + nk,\ h = \frac{r-q}{m},\ r, q, m, n \in I$; a set of integers, k and h are constants step size; t is the time, and x is the distance coordinate as emphasized in [15], [10], [18].

3.0 Derivation of the methods

Equation (1.1) is a sample linear non homogenous one dimensional parabolic partial differential equation and how one solve it numerically has to begin with outlining some of its basic properties in the case of one space dimension and to more than one space dimensions qualitatively and, which can be generally apply to parabolic partial differential equation. Equation (1.1) is first degree in time (t) and thereby requires one initial condition and since it is second degree in distance(x) and requires two boundary conditions. For details see [13], [14] [15], [16], [19]. In one space dimension the boundary conditions are typically applied at the two endpoints. Hence, the parabolic partial differential equation is a mixture of an ODE - IVP and a two point ODE-BVP and, generally requires numerically implicit methods.

Assuming the theoretical solution of equation (1.1) is in form of the basis spectra function

$$U_{(x_{m+i}, t_{n+j})} = aCos(x_{m+i}, t_{n+j}) + bSin(x_{m+i}, t_{n+j}), \dots\ i, j = 0,1,2,3 \dots \quad (1..3)$$

and time (t) and distance (x) as independent variables.

By considering time (t) - as an independent variable and, assume distance(x) to be stable then the equation (1.3) becomes

$$U_{(x_m, t_{n+j})} = aCos(x_m, t_{n+j}) + bSin(x_m, t_{n+j}), \quad (1.4)$$

$$At\quad j = 0;\quad U_{(x_m, t_n)} = aCos(x_m, t_n) + bSin(x_m, t_n) \quad (1.5)$$

$$At\quad j = 1;\quad U_{(x_m, t_{n+1})} = aCos(x_m, t_{n+1}) + bSin(x_m, t_{n+1}) \quad (1.6)$$

$$At\quad j = 2;\quad U_{(x_m, t_{n+2})} = aCos(x_m, t_{n+2}) + bSin(x_m, t_{n+2}), \quad (1.7)$$

Subtract equation (1.5) from equation (1.6) and simplify to obtain equation

$$U_{(x_m, t_{n+1})} - U_{(x_m, t_n)} = a(Cos(x_m, t_{n+1}) - Cos(x_m, t_n)) + b(Sin(x_m, t_{n+1}) - Sin(x_m, t_n)) \quad (1.8).$$

Simplifying equation (1.8) to have equation

$$U_{(x_m, t_{n+1})} - U_{(x_m, t_n)} = -2\sin\left(x_m, \frac{k}{2}\right)\left[asin[x_m, \frac{(2t_n + k)}{2}\right] - b\cos\left(x_m, \frac{(2t_n + k)}{2}\right] \quad (1.9)$$

By setting $(x_m,\ t_n) = (0, 0)$ as a starting point; then equation (1.9) becomes

$$U_{(m,n+1)} - U_{(m,n)} = -2sin\frac{h}{2}\left[asin\frac{h}{2} - bcos\frac{h}{2}\right] \quad (1.10)$$

To proceed with step j = 2; subtract equation (1.6) from equation (1.7) and simplify to obtain

$$U_{(x_m, t_{n+2})} - U_{(x_m, t_{n+1})} = a\left(Cos(x_m, t_{n+2}) - Cos(x_m, t_{n+1})\right) - b(Sin(x_m, t_{n+2}) - Sin(x_m, t_{n+1})) \quad (1.11)$$

Simplify equation (1.11) to have equation (1.12)

$$U_{(m,n+2)} - U_{(m,n+1)} = a[-2sin[x_m, \frac{(2t_n + 3k)}{2}]sin[x_m, \frac{k}{2}] + b[2Sin[x_m, \frac{k}{2}]cos[x_m, \frac{2t_n + 3k)}{2}].$$

$$U_{(m,n+2)} - U_{(m,n+1)} = -2\sin\left[x_m, \frac{k}{2}\right]\left[asin[x_m, \frac{(2t_n + 3k)}{2}\right] - bCos\left[x_m, \frac{2t_n + 3k}{2}\right] \quad (1.12)$$

By adopting similar procedure taking in equation (1.10) in (1.12) and simplifying into

$$U_{(m,n+2)} - U_{(m,n+1)} = -2sin\frac{k}{2}\left[asin\frac{3k}{2} - bcos\frac{3k}{2}\right] \quad (1.13)$$

Subtract equation (1.11) from equation (1.13) and simplify to obtain

$$U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)} = -2sin\frac{k}{2}\left[\left[a[sin\frac{3k}{2} - sin\frac{k}{2}]\right] + b\left[cos\frac{3k}{2} - cos\frac{k}{2}\right]\right] \quad (1.14)$$

Simplify equation (1.14) to obtain equation

$$U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)} = -4sin^2\left(\frac{k}{2}\right)\left[\left[a[Cos2k]\right] + b[Sin2k]\right] \quad (1.15).$$

Define f_t as the first derivative of equation (1.4) with respect to time t in equation (1.1) as follows to obtain;

$$U' = f(x, t, u) = f_t \quad (1.16)$$

$$\text{Then, } f_{(x_m, t_{n+j})} = -aSin(x_m, t_{n+j}) + bCos(x_m, t_{n+j}) \quad . \quad (1.17)$$

$$At\ j = 0;\ f_{(x_m, t_n)} = -aSin(x_m, t_n) + bCos(x_m, t_n) \quad (1.18)$$

$$At\ j = 1;\ f_{(x_m, t_{n+1})} = -aSin(x_m, t_{n+1}) + bCos(x_m, t_{n+1}), \quad (1.19)$$

$$At\ j = 2,\ f_{(x_m, t_{n+2})} = -aSin(x_m, t_{n+2}) + bCos(x_m, t_{n+2}) \quad (1.20)$$

Add equations (1.17) and (1.18) to obtain

$$f_{(m,n+1)} + f_{(m,n)} = \left[-[aSin(x_m, t_{n+1}) + aSin(x_m, t_n)] + b[Cos(x_m, t_{n+1}) + Cos(x_m, t_n)]\right] \quad (1.21)$$

Simplify equation (1.21) and obtain

$$f_{(m,n+1)} + f_{(m,n)} = -2cos\left(x_m, \frac{k}{2}\right)\left[asin\left(x_m, \frac{k}{2}\right) - bcos\left(x_m, \frac{k}{2}\right)\right] \quad (1.22)$$

Adopting equilibrium or quasi – equilibrium phenomena; in which the solution in one space depends on the solution everywhere else. Then equating equations (1.10) and (1.22) and simplify to have equation

$$\frac{U_{(m,n+1)} - U_{(m,n)}}{f_{(m,n+1)} + f_{(m,n)}} = \frac{-2\sin[x_m, \frac{k}{2}]\left[\sin[x_m, \frac{k}{2}] - bcos[x_m, \frac{k}{2}]\right] \dots}{-2\cos[x_m, \frac{k}{2}]\left[asin[x_m, \frac{k}{2}] - bcos[x_m, \frac{k}{2}]\right] \dots} = tan[x_m, \frac{k}{2}] \approx tan\frac{k}{2} \quad (1.23)$$

By adopting Taylor series expansion for $tan\frac{k}{2}\ as\ follows$

$$tan\frac{k}{2} = k\left\{\frac{1}{2} + \frac{k^2}{24} + \frac{k^4}{240} + \cdots \dots \dots \dots ..\right\} \quad (1.24)$$

to a sufficiently small value of k; as k approaches zero then, $tan\frac{k}{2} \approx \frac{k}{2}$ and equation (1.23) becomes

$$\frac{U_{(m,n+1)} - U_{(m,n)}}{f_{(m,n+1)} + f_{(m,n)}} = \frac{k}{2}$$

Simplifying

$$U_{(m,n+1)} - U_{(m,n)} = \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] \quad (1.25)$$

$$U_{(m,n+1)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] \quad (1.26)$$

Equation (1.26) is similar to Trapezodial Rule which is one – step implicit method. [Lambert,1973]

Add equations (1.19) and (1.20) to obtain

$$f_{(m,n+2)} + f_{(m,n+1)} = a\left[-2sin[x_m, \frac{2t_n+3k}{2}]cos[x_m, \frac{k}{2}]\right] + b\left[2cos[x_m, \frac{k}{2}]cos[x_m, \frac{2t_n+3k}{2}]\right] \quad (1.27)$$

Simplifying equation (1.27) to get

$$f_{(m,n+2)} + f_{(m,n+1)} = -2cos[x_m, \frac{k}{2}]\left[asin[x_m, \frac{2t_n+3k}{2}]\right] - \left[bcos[x_m, \frac{2t_n+3k}{2}]\right] \quad (1.28)$$

Divide equation (1.12) by equation (1.28); we have;

$$\frac{U_{(m,n+2)} - U_{(m,n+1)}}{f_{(m,n+2)} + f_{(m,n+1)}} = \frac{-2\sin\left[x_m, \frac{k}{2}\right]\left[asin\left[x_m, \frac{(2t_n+3k)}{2}\right] - bCos[x_m, \frac{2t_n+3k}{2}]\right]}{-2cos[x_m, \frac{k}{2}]\left[asin[x_m, \frac{2t_n+3k}{2}]\right] - \left[bcos[x_m, \frac{2t_n+3k}{2}]\right]} = \tan\left(x_m, \frac{k}{2}\right) \quad (1.29)$$

Adopting Taylor series to expand $tan[x_m, \frac{k}{2}] \approx tan\frac{k}{2}\ as\ follows$

$$tan\frac{k}{2} = h\left\{\frac{1}{2} + \frac{h^2}{24} + \frac{h^4}{240} + \cdots \dots \dots \dots ..\right\} \quad (1.30)$$

to a sufficiently small value of k;

as k approaches zero $tan\frac{k}{2} \approx \frac{k}{2}$; adopting this assumption in equation (1.29) and simplify to obtain

$$U_{(m,n+2)} - U_{(m,n+1)} = \frac{k}{2}[f_{(m,n+2)} + f_{(m,n+1)}$$

$$U_{(m,n+2)} = U_{(m,n+1)} + \frac{k}{2}\left[f_{(m,n+2)} + f_{(m,n+1)}\right] \quad (1.31)$$

$$U_{(m,n+2)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+1)} + f_{(m,n)}] + \frac{k}{2}\left[f_{(m,n+2)} + f_{(m,n+1)}\right]. \quad (1.32)$$

$$U_{(m,n+2)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+2)} + 2f_{(m,n+1)} + f_{(m,n)}] \quad (1.33)$$

reconsidering equation (1.1) and theoretical solution (1.3) for the second derivative in terms of time-space independent variables

$$U'' = f'(t,x,u) = f_t + ff_x \quad (1.33.1)$$

$$g_{n,t} = U''_{n,t} = -(aCosx_n, t_n + bSinx_n, t_n) \quad (1.34)$$

$$g_{(n+1),t} = U''_{(n+1),t} = -(aCosx_{n+1}, t_n + bSinx_{n+1}, t_n), \quad (1.35)$$

$$g_{(n+2),t} = U''_{(n+2),t} = -\left(aCosx_{n+2}, t_n + bSinx_{n+2,}t_n\right), \quad (1.36)$$

Add equation (1.34) and equation (1.35) to have;

$$g_{(n+1),t} + g_{n,t} = -[(aCosx_{n+1}, t_n + bSinx_{n+1}, t_n) + \left(aCosx_n, t_n + bSinx_{n,}t_n,\right)]$$

$$g_{(n+1),t} + g_{n,t} = -[a(Cosx_{n+1}, t_n + Cosx_n, t_n) + b\left(Sinx_{n+1,}t_n + Sinx_n, t_n,\right)]$$

$$g_{(n+1),t} + g_{n,t} = -2\cos\left(\frac{h}{2}, t_n\right)\left[acos\left(\frac{2x_n+h}{2}, t_n\right) + bsin\left(\frac{2x_n+h}{2}, t_n\right)\right]. \quad (1.37)$$

Add equation (1.35) and equation (1.36) to obtain;

$$g_{(n+2),t} + g_{(n+1),t} = -2\cos\left(\frac{h}{2}, t_n\right)\left[acos\left(\frac{2x_n+3h}{2}, t_n\right) + bsin\left(\frac{2x_n+3h}{2}, t_n\right)\right] \quad (1.38)$$

Add equation (1.37) and equation (1.38) to obtain

$$g_{(n+2),t} + 2g_{(n+1),t} + g_{n,t} = 4cos^2\left(\frac{h}{2}, t_n\right)[\text{acos}((2x_n + 2h), t_n) + bsin((2x_n + 2h), t_n)] \quad (1.39)$$

Divide equation (1.39) by equation (1.15) and have;

$$\frac{U_{(m,n+2)}-2U_{(m,n+1)}+U_{(m,n)}}{g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}} = \frac{4sin^2\left(\frac{h}{2},t_n\right)[a\cos\ 2h+bsin\, 2h].}{4cos^2\left(\frac{h}{2},t_n\right)[a\cos\left((2x_n+2h),t_n\right)+bsin\left((2x_n+2h),t_n\right)].}$$

$$= tan^2\left(\frac{h}{2}\ ,t_n\right) = tan^2\frac{h}{2} \quad (1.40)$$

Adopting a Taylor's series expansion of $tan^2\frac{h}{2}$ given as

$$tan^2\frac{h}{2} = \left(tan\frac{h}{2}\right)^2 = h^2\left(\frac{1}{2}+\frac{h^2}{24}+\frac{h^4}{240}+\cdots.+\right)^2 \quad (1.40.1)$$

Put equation (1.40.1) in (1.40) to obtain equation (1.41)

$$U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)} = h^2\left(\frac{1}{2}+\frac{h^2}{24}+\frac{h^4}{240}\right)^2\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right) \quad (1.41)$$

For sufficiently small values of h, equation (1.41) becomes

$$U_{(m,n+2)} - 2U_{(m,n+1)} + U_{(m,n)} = \frac{h^2}{2^2}\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right) \quad (1.42)$$

$$U_{(m,n+2)} = 2U_{(m,n+1)} - U_{(m,n)} + \frac{h^2}{2^2}\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right) \quad (1.42.1)$$

Substitute equation (1.26) for $U_{(m,n+1)}$ in equation (1.42) and adopt the condition for equilibrium and quasi-equilibrium to obtain

$$U_{(m,n+2)} = 2\left(U_{m,n}+\frac{k}{2}[f_{x,(n+1)}+f_{x,n}]\right) - U_{(m,n)} + \frac{h^2}{4}\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right) (1.43)$$

$$U_{(m,n+2)} = U_{m,n} + k[f_{x,(n+1)}+f_{x,n}] + \frac{h^2}{4}\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right) \quad (1.44)$$

Equation (1.44) is the spectra computational based two - steps scheme.

Equating the (1.44) and (1.33) to obtain a generalized spectral based method for step number two

$$\frac{k}{2}[f_{(m,n+2)}+2f_{(m,n+1)}+f_{(m,n)}] = k[f_{x,(n+1)}+f_{x,n}] + \frac{h^2}{4}\left(g_{(n+2),t}+2g_{(n+1),t}+g_{n,t}\right)$$

$$\frac{k}{2}[f_{m,n+2}+2f_{m,n+1}+f_{m,n}] - k[f_{m,n+1}+f_{m,n}] = \frac{h^2}{4}\left(g_{m+2,t}+2g_{m+1,t}+g_{m,t}\right)$$

$$\frac{k}{2}[f_{m,n+2}-f_{m,n}] = \frac{h^2}{4}\left(g_{m+2,t}+2g_{m+1,t}+g_{m,t}\right)$$

$$f_{m,n+2}-f_{m,n} = \frac{h^2}{2k}\left(g_{m+2,t}+2g_{m+1,t}+g_{m,t}\right) \quad (1.45)$$

4.0 Properties of the Scheme in equation (1.33)

Definition 1: In the spirits of [16], [17] as emphasized in [18] that a linear multistep method of the type in equation (1.33) is said to be consistence if and only if it satisfies the following conditions

$(i)\quad the\ order\ p\geq 1\ \ (ii)\sum_{j=0}^{k}\alpha_j = 0\quad (iii)\ \rho(r)=\rho'(r)=0\quad (iv)\quad \rho^n(1)=n!\,\partial(1);$

with the principal root r ≤ 1, $|r_s| : s = 1,2,,,k\ \ \forall\ |r_s| < |r_1|$, [Lambert 1991]

Consistency Test on Scheme in equation (1.33)

The order and error constant of the scheme in equation (1.33) are estimated as follows;

$$U_{(m,n+2)} = U_{(m,n)} + \frac{k}{2}[f_{(m,n+2)}+2f_{(m,n+1)}+f_{(m,n)}$$

$$U_{(m,n+2)} - U_{(m,n)} = \frac{k}{2}[f_{(m,n+2)}+2f_{(m,n+1)}+f_{(m,n)}]$$

$$U_{m,n+2} = \frac{(2h)^0}{0!}y_n + \frac{(2h)^1}{1!}y_n^1 + \frac{(2h)^2}{2!}y_n^2 + \frac{(2h)^3}{3!}y_n^3 + \frac{(2h)^4}{4!}y_n^4 + \frac{(2h)^5}{5!}y_n^5 + ..$$

$$U_{m,n} = y_n$$

$$\frac{1}{2}f_{m,n+2,} = \frac{1}{2}[\frac{(2h)^0}{0!}y_n^1 + \frac{(2h)^1}{1!}y_n^2 + \frac{(2h)^2}{2!}y_n^3 + \frac{(2h)^3}{3!}y_n^4 + \frac{(2h)^4}{4!}y_n^5 + \frac{(2h)^5}{5!}y_n^6 + \cdots]$$

$$\frac{1}{2}(2)f_{m,n+1,} = [\frac{(h)^0}{0!}y_n^1 + \frac{(h)^1}{1!}y_n^2 + \frac{(h)^2}{2!}y_n^3 + \frac{(h)^3}{3!}y_n^4 + \frac{(h)^4}{4!}y_n^5 + \frac{(h)^5}{5!}y_n^6 + \cdots]$$

$$\frac{1}{2}f_{m,n+1,,} = \frac{1}{2}y_n^1$$

By collecting the coefficients (C_is) of equal powers of y_n and simplifying; we obtain

$C_0 = 1-1 = 0 \qquad C_1 = 2-\frac{1}{2}-1-\frac{1}{2} = 0 \qquad C_2 = 2-1-1 = 0$

$C_3 = \frac{4}{3}-1-1 = \frac{-2}{3} = -0.66666667 \neq 0$ The order of the method is 2

$Error\ constant\ of\ Scheme(1) = -0.66666667$

The first and second characteristic polynomials of scheme (1) are $\rho(r)\ and\ \sigma(r)$ respectively

$\rho(r) = (r)^2 - 1$ and $\partial(r) = \frac{1}{2}(r^2+2r+1)$

$for\ r = 1,\qquad (1)^2 - 1 = \rho(1) = 0$(zero stability)

$\sum_{j=0}^{k} j\alpha_j = \sum_{j=0}^{k} \beta_j$ $\quad j\alpha_j = 2\alpha_2 = 2 = \beta_j = \frac{1}{2}(1+2+1) = 2$

$\rho'(r) = 2r,\quad \rho''(r) = 2,$

$\partial(r) = \frac{1}{4}(r^2 + 2r + 1)\quad \rho''(1) = 2 = 2!\,\partial(1)$

Definition 2

A linear multistep method is convergent if and only if it is consistent and zero – stable. [14]. The Scheme in equation (1.33) is consistence, zero stable and convergent.

To affirm the stability criteria of the scheme equation (1.33); using Crank –Nicolson with matrix difference method (as reported in [12]

$U_{m,n+2} - U_{m,n} = \frac{k}{2}\left(f_{m,n+2,} + 2f_{m,n+1,} + f_{m,n}\right)$

$U(x,t+2k) - U(x,t) = \frac{k}{2}[U(x,t+2\text{k}) + 2\text{U(x,t+k)} + U(x,t)]$

$\frac{2}{k}[U(x,t+2k) - U(x,t)] = [U(x,t+2\text{k}) + 2\text{U(x,t+k)} + U(x,t)]$

$Let\ S = \frac{2}{k}$; then, $S[U(x,t+2k) - U(x,t)] = [U(x,t+2\text{k}) + 2\text{U(x,t+k)} + U(x,t)]$

$[SU(x,t+2k) - SU(x,t) = [U(x,t+2\text{k}) + 2\text{U(x,t+k)} + U(x,t)]$

$U(x,t+2\text{k}) + 2\text{U(x,t+k)} + U(x,t) + SU(x,t) = SU(x,t+2k)$

$U(x,t+2\text{k}) + 2\text{U(x,t+k)} + (S+1)U(x,t) = SU(x,t+2k),$

$assuming\ r = S+1, therefore,$

$U(x,t+2\text{k}) + 2\text{U(x,t+k)} + rU(x,t) = SU(x,t+2k)$ (**)

On the t level, U is known but on t+2k, U is unknown. Therefore, the unknown parameter can be introduced as $U_i = U(x,\ ik) and\ known\ quantity\ to\ be\ b_i = SU(ih, t+2k)$

Equation (**) can be transform into tri - diagonal matrix difference of the form

$$\begin{pmatrix} 1, & 2 & \dots & & \dots & r \\ 0 & 1 & 2 & & r-1 & \\ \dots & \dots & \dots & & \dots & \dots \\ \dots & \dots & \dots & & \dots & \dots \\ 0 & 0 & 0 & & 2 & 1 \end{pmatrix} \begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_{n-2} \\ U_{n-1} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{n-2} \\ b_{n-1} \end{pmatrix}$$

$AU_i = b_i$, A is upper triangular matrix

$therefore,\quad A^{-1}b_i = U_i\ i = 1,2,3\dots,21$

k = 0.1; $\quad S = \frac{2}{k} = \frac{2}{0.1} = 20$

$r = S + 1 = 20 + 1 = 21$

These result into 21 x 21 upper triangular matrix A as shown below

$$\mathbf{A} = \begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 & 21 \\
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 & 20 \\
0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 & 19 \\
0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 \\
0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & `4 & 5 & 6 & 7 & 8 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 & 5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 & 4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

Inverse of matrix A is given as

$$
A^{-1} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

Table 1: Showing values of b_i and Estimate of unknown Parameter U_i

Values of b_i	Estimate of unknown Parameters U_i
0	-1.14E-86
1.16E-85	-2.16E-86
2.21E-85	-2.97E-86
3.04E-85	-3.50E-86
3.57E-85	-3.68E-86
3.76E-85	-3.50E-86
3.57E-85	-2.97E-86
3.04E-85	-2.16E-86
2.21E-85	-1.14E-86
1.16E-85	-1.71E-100
4.60E-101	1.14E-86
-1.16E-85	2.16E-86
-2.21E-85	2.97E-86
-3.04E-85	3.50E-86
-3.57E-85	3.68E-86
-7.33E-85	3.50E-86
3.57E-85	2.97E-86
-3.04E-85	2.16E-86
-2.21E-85	1.14E-86
-1.16E-85	-1.16E-85
-9.20E-101	-9.20E-101

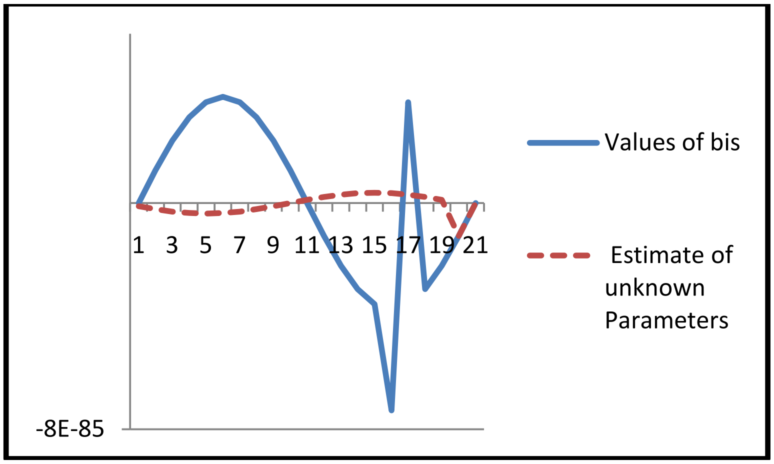

Figure 1: The graph of r against U_i and the graph of r against b_i

The points of interception of the two graphs indicate the point which satisfies the conditions for equilibrium and quasi – equilibrium of the parameters of the method in scheme (1).The spectral based method in scheme 1 is tri-diagonal and diagonal dominant because
$|r| = 1 + \frac{2k}{h^2} > 1$. This shows that the method is stable as referenced in [10].

5.0 Implementation of the scheme on some sampled problems;
Numerical Example:

Consider the fourth order parabolic partial differential equation of the form

$$U_{tt}(x,t) + (1+x)U_{xxxx}(x,t) = \left(x^4 + x^3 - \frac{6}{7!}x^7\right) cost, \qquad 0 < x < 1, \qquad t > 0$$

with the following initial and boundary conditions

$$U(x,0) = \frac{6}{7!}x^7;\ U_t(x,0) = 0;\ U(0,t) = 0;\ U(1,t) = \frac{6}{7!}cost,$$

$$U_{xx}(0,t) = 0;\ \ U_{xx}(1,t) = \frac{6}{20}cost$$

The theoretical solution to this problem is $U(x,t) = \left(\frac{6}{7!}x^7\right) cost,$

[Authored by [17] and [22]]

Table 2: Comparison of the degree of accuracy of Scheme 1

time (t)	X	Exact Result [ER]	Scheme 1	Results authored by [17]	Absolute Error (Scheme 1)	Absolute Error authored by [17]
0.03	0.1	1.18994E-10	1.1899E-10	1.19011E-10	6.73600E-17	1.66976E-14
	0.2	1.52312E-08	1.5231E-08	1.52316E-08	2.33783E-14	3.51848E-13
	0.3	2.60240E-07	2.6024E-07	2.60242E-07	2.69310E-13	2.27091E-12
	0.4	1.94960E-06	1.9496E-06	1.94961E-06	3.40758E-12	1.00513E-11
	0.5	9.29641E-06	9.2964E-06	9.29645E-06	9.01229E-12	3.77047E-11
	0.6	3.33107E-05	3.3311E-05	3.33108E-05	1.44717E-11	1.25717E-10
	0.7	9.79967E-05	9.7997E-05	9.79971E-05	7.97296E-11	3.75281E-10
	0.8	2.49549E-04	0.00024955	2.49550E-04	6.36170E-10	1.00966E-09
	0.9	5.69145E-04	0.00056914	5.69147E-04	7.08982E-10	2.47453E-09
0.04	0.1	1.18952E-10	1.1895E-10	1.19015E-10	2.74700E-16	6.26517E-14
	0.2	1.52373E-08	1.5226E-08	1.52273E-08	1.14088E-11	1.00357E-11
	0.3	2.60240E-07	2.6015E-07	2.60158E-07	9.13752E-11	8.24071E-11
	0.4	1.94892E-06	1.9489E-06	1.94895E-06	5.93135E-12	3.55230E-11
	0.5	9.29316E-06	9.2931E-06	9.29328E-06	1.35392E-11	1.20295E-10
	0.6	3.32991E-05	3.3299E-05	3.32994E-05	7.60302E-11	3.66160E-10
	0.7	9.79624E-05	9.7962E-05	9.79634E-05	8.68386E-11	1.02186E-09
	0.8	2.49461E-04	0.00024946	2.49464E-04	7.86879E-13	2.62666E-09
	0.9	5.68946E-04	0.00056895	5.68946E-04	9.57641E-10	4.30396E-15

Table 3: Results of Scheme 2 on Numerical Example

time (t)	X	Exact Result [ER]	Scheme 2	Results authored by [17]	Absolute Error (Scheme 2)
0.01	0.1	1.19042E-10	1.25543E-10	1.19042E-10	6.50130E-12
	0.2	1.52373E-08	1.53454E-08	1.52373E-08	1.08146E-10
	0.3	2.60344E-07	2.60912E-07	2.60344E-07	5.68352E-10
	0.4	1.95038E-06	1.95224E-06	1.95038E-06	1.86114E-09
	0.5	9.30013E-06	9.30484E-06	9.30013E-06	4.71064E-09
	0.6	3.33240E-05	3.33342E-05	3.33242E-05	1.01550E-08
	0.7	9.80359E-05	9.80553E-05	9.80359E-05	1.93844E-08
	0.8	2.49648E-04	0.000249683	2.49649E-04	3.45537E-08
	0.9	5.69373E-04	0.000569429	5.69373E-04	5.58994E-08
0.02	0.1	1.19024E-10	1.25524E-10	1.19026E-10	6.50047E-12
	0.2	1.52350E-08	1.53431E-08	1.52351E-08	1.08144E-10
	0.3	2.60305E-07	2.60873E-07	2.60305E-07	5.68213E-10
	0.4	1.95009E-06	1.95195E-06	1.95009E-06	1.85828E-09
	0.5	9.29874E-06	9.30344E-06	9.29874E-06	4.70480E-09
	0.6	3.33190E-05	3.33292E-05	3.33319E-05	1.01546E-08
	0.7	9.80212E-05	9.80406E-05	9.80213E-05	1.93750E-08
	0.8	2.49611E-04	0.000249645	2.49611E-04	3.40985E-08
	0.9	5.69287E-04	0.000569343	5.69288E-04	5.64786E-08

Table 4: Results of Scheme 3 on Numerical Example

time (t)	X	Exact Result [ER]	Scheme 3	Results authored by [17]	Absolute Error (Scheme 3)
0.01	0.1	1.19042E-10	1.25543E-10	1.19042E-10	6.50130E-12
	0.2	1.52373E-08	1.53454E-08	1.52373E-08	1.08146E-10
	0.3	2.60344E-07	2.60912E-07	2.60344E-07	5.68352E-10
	0.4	1.95038E-06	1.95224E-06	1.95038E-06	1.86114E-09
	0.5	9.30013E-06	9.30484E-06	9.30013E-06	4.71064E-09
	0.6	3.33240E-05	3.33342E-05	3.33242E-05	1.01550E-08
	0.7	9.80359E-05	9.80553E-05	9.80359E-05	1.93844E-08
	0.8	2.49648E-04	0.000249683	2.49649E-04	3.45537E-08
	0.9	5.69373E-04	0.000569429	5.69373E-04	5.58994E-08
0.02	0.1	1.19024E-10	1.25524E-10	1.19026E-10	6.50047E-12
	0.2	1.52350E-08	1.53431E-08	1.52351E-08	1.08144E-10
	0.3	2.60305E-07	2.60873E-07	2.60305E-07	5.68213E-10
	0.4	1.95009E-06	1.95195E-06	1.95009E-06	1.85828E-09
	0.5	9.29874E-06	9.30344E-06	9.29874E-06	4.70480E-09
	0.6	3.33190E-05	3.33292E-05	3.33319E-05	1.01546E-08
	0.7	9.80212E-05	9.80406E-05	9.80213E-05	1.93750E-08
	0.8	2.49611E-04	0.000249645	2.49611E-04	3.40985E-08
	0.9	5.69287E-04	0.000569343	5.69288E-04	5.64786E-08

6.0 Conclusion

The tables depicted above shows that the results obtained are found to be more rapidly converging as the step lengths h and k approaches zeros. This work will provide better numerical solutions to a class of dynamical problems having time dependent boundary conditions. The method will be alternate to complement the existing ones.

MATLAB CODE; as emphasized by [21]

```
a=0;        %% initial value
b=0.9;        %% a < x < b
c=0.05;        %% a < t < c
h = 0.1;     %% interval along x
m=10;   %% number of points along x
k=0.01;       %% interval long t
n=6;   %% number of points along t

for j = 2:n;
      for i = 2:m;
          x(i)= a + (i-1) * h;
          t(j)= a + (j-1) * k;

          u(i,j) =((6/(factorial(7)))*(x(i))^7)*cos(t(j));

          f(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*sin(t(j));
          g(i,j)= ((6/(factorial(6)))*(x(i))^6)*cos(t(j));
```

```
        g1(i,j)= ((6/(factorial(5)))*(x(i))^5)*cos(t(j));
        g2(i,j)= ((6/(factorial(4)))*(x(i))^4)*cos(t(j));
        g3(i,j)= ((6/(factorial(3)))*(x(i))^3)*cos(t(j));
        g4(i,j)= ((6/(factorial(2)))*(x(i))^2)*cos(t(j));
        f1(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*cos(t(j));
        f2(i,j)  =  ((6/(factorial(7)))*(x(i))^7)*sin(t(j));
        f3(i,j)  =  ((6/(factorial(7)))*(x(i))^7)*cos(t(j));
        f3(i,j)  =  -1*((6/(factorial(7)))*(x(i))^7)*sin(t(j));

        u1(i,j) = u(i,j) + (((k/2)*(f2(i,j)+ 2*f1(i,j)+ f(i,j))))/10^4;

        error(i,j) = abs(u(i,j)-u1(i,j));
    end
end
plot(x(2:m), u(2:m,2),'-', x(2:m), u1(2:m,2),':'); xlabel('x'); ylabel('U(x,t)'); title('Plot of ((6/7!)*x^7)*cos(t) at t = 0.01')
```

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