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Analytical Study of the Effects of Lateral Extent and Point Coordinate on Pressure Behavior in a Vertical Well in a Bounded Reservoir System

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Abstract

Reservoir boundaries are encountered when the flowing pressure departs from the straight line portion of the commonly known semi – log plot. And this reflects the lateral extent of the reservoir.Information on the boundaries detection and the effects of the lateral extent and coordinate on pressure behavior is an important and integral aspect of well test analysis. In this paper, numerical mathematical models were derived for the vertical well in a bounded reservoir. Thelateral extent and point coordinate effects on the pressure distributions were analyzed. The pressure responses were simulated and some calculations were performed using the derived models by varying the coordinate parameters.

The results show that dimensionless pressure decreases with increasing lateral extent. Pressure behavior is unaffected by the lateral point coordinates throughout all the flow regimes.Situating the well far away from the boundaries delays the attainment of pseudo – steady state.Infinite – acting flow period subsists for a long time for as long as flow is close to the well bore. It also shows that theinfinite – acting flow period is unaffected by the lateral extent of the reservoir. However, reservoir depletion during late time reduces with increasing lateral extent.

Key words: Bounded Reservoir, Dimensionless Pressure, Dimensionless Pressure Derivative,Lateral Extent, Lateral Point Coordinate, Log – Log Plot, Reservoir Boundaries, Pressure Distribution and Reservoir Thickness

Nomenclature

p_D = Dimensionless pressure.
h_D = Dimensionless reservoir thickness.
x_D = Point co-ordinate along the X – direction.
z_D = Point co-ordinate along the Z – direction.
S = Source function.
τ = Time.
t_D = Dimensionless time
x_{ed} = Reservoir lateral extent.
k = Reservoir permeability
ct = Total compressibility
μ = Viscosity
ø = Porosity

1.0 Introduction

Oil well test analysis is a branch of reservoir engineering. Information obtained from flow and pressure transient tests about in situ reservoir conditions are important to determining the productive capacity of a reservoir. Pressure transient analysis also yields estimates of the average reservoir pressure. The reservoir engineer must have sufficient information about the condition and characteristics of reservoir/well to adequately analyze reservoir performance and to forecast future production under various modes of operation. The production engineer must know the condition of production and injection wells to persuade the best possible performance from the reservoir. Pressures are the most valuable and useful data in reservoir engineering. Directly or indirectly, they enter into all phases of reservoir engineering calculations. Therefore accurate determination of reservoir parameters is very important.

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2.0 Problem Definition

Information on the boundaries detection and the effects of the lateral extent and coordinate on pressure behavior of a well is an important and integral aspect of well test analysis. Therefore, the need to simulate the reservoir to derive representative models for pressure behavior description becomes very germane.

3.0 Technical Objective

The objectives of this research include:

1. To simulate the effects of lateral point coordinate on the pressure behavior in a vertical well system in a bounded reservoir usingnumerical mathematical modelsderived from Source and Green functions.
2. To simulate the effects of lateral extent on the pressure behavior in a vertical well systemin a bounded reservoirusingnumerical mathematical modelsderived from Source and Green functions.
3. To vary the coordinate parameters for proper simulation of the different possible scenarios in the reservoir.
4. To determine the various flow regimes.
5. To analyze the effect of lateral extent and point coordinate on the pressure behavior in a bounded reservoir.

4.0 Literature Review

Advances in well testing have resulted in several research works over the past decades.Several researchers have successfully worked on flow test in wells and pressure build up [1-5]. Theuse of transient testing in reservoir management is reported in [6-10]. Oloro and Adewole [8] also analyzed in their work usingSource and Green’s Function the Factors that Affect Pressure Distribution of Horizontal Wells in a Layered Reservoir with Simultaneous Gas – cap and Bottom Water Drive. Owolabi et al.[9] presented the use of Source and Green’s Function in Pressure Distribution in a Layered Reservoir with Gas – cap and Bottom Water. Extensive researches have been reported by several researchers on transient pressure analysis of horizontal Wells [11-13]. Interestingly, the applicability of Source and Green’s Function in Pressure Distribution in vertical well testing is yet to be reported. And this has been implemented.

5.0 Basic Theory of Transient Well Testing [14]

The well test concept is basically sending a signal to the reservoir, then receiving its response from the formation at the end the permeability can be obtained from it decline. Response which is received at the wellbore is used to evaluate the near wellbore properties. If response from a boundary is reached, so the distance is possible to estimate from the time delay. The whole process of well test requires specific set-up. The standard well test set-up consists of surface rates, wellhead pressure (WHP), bottom hole pressure (BHP) including bottom hole temperature (BHT), and acquisition interpretation. Transient well testing applies the inverse solution of indirect measurement where input and output are known from the test, while the system is going to predict or estimate from interpretation. The system means well and reservoir characteristics, output represents pressure responses, and input shows a change of rate. As part of field data, well test data contributes in production analysis model (i.e. well test models, material balance models, and decline curve analysis). Those data are collected become reservoir information that furthermore designed to be a predictive model. This model allows engineers to simulate production forecast and run various scenarios with different production strategies. Finally, economic study and decision making for the field development is conducted by considering many aspects.

6.0 Types of Transient Tests[14]

Certain types of tests are dedicated to specific stage of reservoir discovery, development, and production. In exploration and appraisal wells, Drillstem tests (DSTs) and wireline formation test are normally run. During primary, secondary, and enhanced recovery stages, the conventional transient well tests (i.e. drawdown, buildup, interference, and pulse tests) are run. Step-rate, injectivity, falloff, interference, and pulse tests are executed during secondary and enhanced recovery stages. Some tests are implemented throughout the life of reservoir, such as multilayer and vertical permeability tests.

7.0 Flow Regimes Categories[14]

At different times, fluid flows in the reservoir with different ways generally based on the shape and size of the reservoir. Flow behavior classification is studied in terms of pressure rate of change with respect to time. Three main flow regimes will be described in this sub-chapter; they are steady-state flow, pseudo steady state flow, and transient state flow.

8.0 Steady State Flow[14]

In steady state flow, there is no pressure change anywhere with time (Equation (1)). It occurs during the late time region when the reservoir has gas cap or aquifer support. This condition is also called constant pressure boundary which pressure maintenance might apply in the producing formation.

$$\frac{\partial p}{\partial t} = 0 \qquad (1)$$

9.0 Pseudo Steady State Flow[14]

This flow regime also occurs in late-time region, but it forms when there is no flow in the reservoir outer boundaries. No flow boundaries can be caused by the effect of nearby producing or presence of sealing faults. It is a closed system or acts as a tank where a constant rate production results constant pressure drop for each unit of time (Equation (2)).This flow is also called semi-steady state or depletion state.

$$\frac{\partial p}{\partial t} = constant \qquad (2)$$

10.0 Transient State Flow[14]

When the pressure/rate changes with time due to well geometry and the reservoir properties (i.e. permeability and heterogeneity), it indicates that transient (unsteady state) flow occurs (Equation (3)). It is observed before boundary effects are reached or also called infinite acting time period. Higher compressibility of the fluid leads the more pronounced the unsteady state effect of the reservoir fluid.

$$\frac{\partial p}{\partial t} = f(x, y, t) \qquad (3)$$

11.0 Radius of Investigation [14]

Quantitatively and qualitatively, radius of investigation has great significance both in planning and analyzing a well test. It describes the distance (from the tested wellbore) of the transient pressure into the formation if there is an unstable pressure caused by production or closure of a well. It will show that this distance has a correlation with physical properties of the rock and fluid and also depends on the length of time of well testing.

There is a time *t* when the pressure disturbance reaches the distance *ri*(radius of investigation). The relationship between *t* and $r_i$is given by:

$$r_{i=} \left(\frac{kt}{948\emptyset\mu c_t}\right)^{0.5} \qquad (4)$$

From equation above, $r_i$describes a distance at which the pressure disturbance (increase or decrease) simply influences due to production or injection of fluid at constant rate.

The concept of radius of investigation is a guide to plan a well test. For a certain radius of investigation needed, duration of the test is possible to estimate using this concept. Therefore, optimum and effective time will be used which affect the cost of well testing. Effective and optimum cost for well testing is very important since it is considerably expensive, especially for offshore wells.

12.0 Drawdown [14]

Drawdown test is ideally performed when the pressure is equalized throughout the formation. This condition can be reached by shutting-in the well prior to drawdown test or after having several days of workover job. Performing a drawdown test at new wells becomes a good recommendation because the reservoir still has a uniform pressure.

Basically, this test measures bottom hole pressure during a period of constant production rate. The equipment is first set into the well, and then begins a constant flow rate. The consideration for having this test is simply when there are some uncertainties in buildup interpretations. Therefore, analysis from drawdown test can be used for comparative analysis.

13.0 Buildup [14]

A constant production rate *q* for a period of time *t* is usually conducted prior to buildup test. Producing a well at constant rate represents the drawdown part of the well history. Buildup test is started right after t_p(which is representing the duration of production) with zero production by shutting-in the well at the wellhead. Measurement of bottom hole pressure is normally performed since the beginning of drawdown part until the end of buildup test.

A method to analyze the pressure response of buildup test is using Horner method. It is a semilog plot of shut-in pressure p_{ws}versushorner time $(tp+\Delta t)/\Delta t$. This plot creates a straight-line which represents the transient flow during the middle-time of the test. Middle-time region indicates that the pressure transient has spread away from the wellbore into the formation. Slope of the straight-line *m* is a tool to predict reservoir permeability.

14.0 Methodology

This section presents the derived models and their applications, representing a vertical well in a bounded reservoir for lateral extent and point coordinate effects on the pressure distributions analysis.

In this research,the following assumptions about the well and reservoir being modeled were madein developing the bounded reservoir model.The reservoir is homogeneous, rectangular in shape, anisotropic and the boundaries in a given axis are felt by the transient pressure at the same time

In developing the mathematical equation representing the above described model, the Source and Green's function expression and the Newman's product rule were employedto determine the dimensionless pressure analysis.

Newman's product rule is expressed as:

$$S(x_{D,}z_{D,}t_D) = S(x_D t_D).S(z_D t_D)\ldots\ldots \quad (5)$$

And the dimensionless pressure response is expressed as:

$$p_D = 2\pi h_D \int_0^{td} S(x_D\tau).S(z_D\tau)d\tau \ldots\ldots \quad (6)$$

a. Dimensionless Pressure Models

The analytical models for the transient pressure response of a vertical well in a bounded reservoir are expressed below

b. Early Time Flow Period

This is an infinite-acting period when the transient pressure has not felt any boundaries. The required source function expressions for this flow period as read from the source function tableare as follows:

$$S(x_, t) = \frac{1}{2\sqrt{\eta_x \pi t}} e^{[-\frac{(x-x_w)^2}{4\eta_x t}]}\ldots\ldots \quad (7)$$

Source is an infinite plane source in an infinite reservoir

Source function number: I(X)

$$S(z_, t) = \frac{1}{2\sqrt{\eta_z \pi t}} e^{[-\frac{(z-z_w)^2}{4\eta_z t}]}\ldots\ldots \quad (8)$$

Source is an infinite plane source in an infinite reservoir

Source function number: I(Z)

In dimensionless form;

$$S(x_{D,}t_D) = \frac{1}{2\sqrt{\pi t_D}}\sqrt{\frac{k}{k_x}} e^{[-\frac{(x_D-x_{wD})^2}{4t_D}]}\ldots\ldots \quad (9)$$

$$S(z_{D,}t_D) = \frac{1}{2\sqrt{\pi t_D}}\sqrt{\frac{k}{k_z}} e^{[-\frac{(z_D-z_{wD})}{4t_D}]}\ldots\ldots \quad (10)$$

Using the Newman's product method, the model for the dimensionless pressure response of a vertical well during the early radial flow period is given as;

Substituting Equations (8) and (9) into Equation (6) gives:

$$p_D = -\frac{1}{2}\frac{k}{\sqrt{k_x k_z}}\int_0^{t_{De}} \frac{e^{-[(x_D-x_{wD})^2+(z_D-z_{wD})^2]}\Big/_{4\tau}}{\tau} d\tau\ldots\ldots \quad (11)$$

c. First Transition Time Flow Model

In the first transition time flow model, the boundaries in the x - direction have been seen while the boundaries in the z – direction have not been seen by the well. The required source functions as read from the source function table are as follows:

$$S(x_, t) = \frac{1}{x_e}\left[1 + 2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\eta_x t}{x_e^2}\right)\cos n\pi\frac{x_w}{x_e}\cos n\pi\frac{x}{x_e}\right]\ldots\ldots \quad (12)$$

Source is an infinite plane source in an infinite slab reservoir

Source function number: VII (X)

$$S(z_, t) = \frac{1}{2\sqrt{\eta_z \pi t}} e^{[-\frac{(z-z_w)}{4\eta_z t}]}\ldots\ldots \quad (13)$$

Source is an infinite plane source in an infinite reservoir.

Source function number: I(Z)

In dimensionless form;

$$S(x_{D,}t_D) = \frac{1}{x_{eD}}\sqrt{\frac{k}{k_x}}\left[1 + 2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2 t_D}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\ldots\ldots \quad (14)$$

$$S(z_{D,}t_D) = \frac{1}{2\sqrt{\pi t_D}}\sqrt{\frac{k}{k_z}} e^{[-\frac{(z_D-z_{wD})}{4t_D}]}\ldots\ldots \quad (15)$$

Using the Newman's product method, the model for the dimensionless pressure response of a vertical well during the intermediate flow period is given as;

Substituting Equations (13) and (14) into Equation (6) gives:

$$p_D = \frac{k}{\sqrt{k_x k_z}}\frac{\pi h_D}{x_{eD}}\int_{t_{De}}^{t_{D1}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\tau}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\times\frac{1}{\sqrt{\tau}}e^{[-\frac{(z_D-z_{wD})^2}{4\tau}]}\,d\tau. \quad (16)$$

d. Second Transition Time Flow Model

In the second transition time flow model, the boundaries in the z - direction have been seen while the boundaries in the x – direction have not been seen by the well. The required source functions as read from the source function table are as follows:

$$S(x,t) = \frac{1}{2\sqrt{\eta_x\pi t}}e^{[-\frac{(x-x_w)}{4\eta_x t}]}\ldots \quad (17)$$

Source is an infinite plane source in an infinite reservoir.
Source function number: I(X)

$$S(z,t) = \frac{1}{z_e}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\eta_z t}{z_e^2}\right)\cos n\pi\frac{z_w}{z_e}\cos n\pi\frac{z}{z_e}\right]\ldots \quad (18)$$

Source is an infinite plane source in an infinite slab reservoir.
Source function number: VII (Z)
In dimensionless form;

$$S(x_D,t_D) = \frac{1}{2\sqrt{\pi t_D}}\sqrt{\frac{k}{k_x}}e^{[-\frac{(x_D-x_{wD})}{4t_D}]}\ldots \quad (19)$$

$$S(z_D,t_D) = \frac{1}{h_D}\sqrt{\frac{k}{k_z}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2 t_D}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]\ldots \quad (20)$$

Using the Newman's product method, the model for the dimensionless pressure response of a vertical well during the intermediate flow period is given as;
Substituting Equations (18) and (19) into Equation (6)

$$p_D = \frac{k}{\sqrt{k_x k_z}}\pi\int_{t_{D1}}^{t_{D2}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\tau}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]\times\frac{1}{\sqrt{\tau}}e^{[-\frac{(x_D-x_{wD})^2}{4\tau}]}\,d\tau\ldots \quad (21)$$

e. Late Time Flow Model

In this case, the boundaries in both the X and z directions have been felt. The required source function expressions as read from the source function table are as follows:

$$S(x,t) = \frac{1}{x_e}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\eta_x t}{x_e^2}\right)\cos n\pi\frac{x_w}{x_e}\cos n\pi\frac{x}{x_e}\right]\ldots \quad (22)$$

Source is an infinite plane source in an infinite slab reservoir
Source function number: VII (X)

$$S(z,t) = \frac{1}{z_e}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\eta_z t}{z_e^2}\right)\cos n\pi\frac{z_w}{z_e}\cos n\pi\frac{z}{z_e}\right]\ldots \quad (23)$$

Source is an infinite plane source in an infinite slab reservoir
Source function number: VII (Z)
In dimensionless form;

$$S(x_D,t_D) = \frac{1}{x_{eD}}\sqrt{\frac{k}{k_x}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2 t_D}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\ldots \quad (24)$$

$$S(z_D,t_D) = \frac{1}{h_D}\sqrt{\frac{k}{k_z}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2 t_D}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]\ldots \quad (25)$$

Using the Newman's product method, the model for the dimensionless pressure response of a vertical well during the late time flow period is given as;
Substituting Equations (23) and (24) into Equation (6) gives:

$$p_D = \frac{k}{\sqrt{k_x k_z}}\frac{2\pi}{x_{eD}}\int_{t_{D2}}^{t_{D3}}\left[1+2\sum_{i=1}^{n} exp\left(\frac{-n^2\pi^2\tau}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\times$$
$$\left[1+2\sum_{i=1}^{n} exp\left(\frac{-n^2\pi^2\tau}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]d\tau\ldots \quad (26)$$

The sum of all the dimensionless pressure equations (Equations (10), (15), (20) and (25) for all the flow periods is expressed in Equation (26).

$$p_D = -\pi \frac{k}{\sqrt{k_x k_z}}\left[\left\{\frac{1}{2}\int_0^{t_{De}} \frac{e^{-[(x_D-x_{wD})^2+(z_D-z_{wD})^2]}/_{4\tau}}{\tau}\right\} - \left[\frac{h_D}{x_{eD}}\int_{t_{De}}^{t_{D1}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\tau}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\times\frac{1}{\sqrt{\tau}}e^{[-\frac{(z_D-z_{wD})^2}{4\tau}]}\right] -$$

$$\left[\int_{t_{D1}}^{t_{D2}}\left[1+2\sum_{i=1}^{n} exp\left(-\frac{n^2\pi^2\tau}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right]\times\frac{1}{\sqrt{\tau}}e^{[-\frac{(x_D-x_{wD})^2}{4\tau}]}\right] -$$

$$\langle\frac{2}{x_{eD}}\int_{t_{D2}}^{t_{D3}}\left[1+2\sum_{i=1}^{n} exp\left(\frac{-n^2\pi^2\tau}{x_{eD}^2}\right)\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\times$$

$$1+2\sum_{i=1}^{n} exp\left(\frac{-n^2\pi^2\tau}{h_D^2}\right)\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\rangle \quad (27)$$

f. Solutions to Flow Equations

g. Dimensionless Pressure:

h. Early Time Flow

Using the Gauss-Laguerre quadrature, the early radial flow equation (Equation (11)) can be expressed as:

$$p_D = -\frac{1}{2}\frac{k}{\sqrt{k_x k_z}}E_i\left[-\frac{(x_D-x_{wD})^2+(z_D-z_{wD})^2}{4t_D}\right] \quad (28)$$

i. First Transition Time Flow

Using the Gauss - Legendre quadrature, the first transition time flow equation (Equation (15)) can be expressed as:

$$p_D = \frac{\pi h_D}{x_{eD}}\frac{k}{\sqrt{k_x k_z}}\left(\frac{b-a}{2}\right)\sum_{i=1}^{n} w_i f\left[1+2exp\langle\frac{-n^2\pi^2}{x_{eD}^2}\left(\frac{z_i(b-a)+b+a}{2}\right)\rangle . cosn\pi\frac{x_{wD}}{x_{eD}}cosn\pi\frac{x_D}{x_{eD}}\right]$$
$$\times\frac{exp\left[-\frac{(z_D-z_{wD})^2}{4(\frac{z_i(b-a)+b+a}{2})}\right]}{\sqrt{(\frac{z_i(b-a)+b+a}{2})}} \quad (29)$$

j. Second Transition Time Flow

Using the Gauss - Legendre quadrature, the second transition time flow equation (Equation (29)) can be expressed as:

$$p_D = \frac{\pi k}{\sqrt{k_x k_z}}\left(\frac{b-a}{2}\right)\sum_{i=1}^{n} w_i f\left[1+2exp\langle\frac{-n^2\pi^2}{h_D^2}\left(\frac{z_i(b-a)+b+a}{2}\right)\rangle . cosn\pi\frac{z_{wD}}{H_D}cosn\pi\frac{z_D}{H_D}\right]$$
$$\times\frac{exp\left[-\frac{(x_D-x_{wD})^2}{4(\frac{z_i(b-a)+b+a}{2})}\right]}{\sqrt{(\frac{z_i(b-a)+b+a}{2})}} \quad (30)$$

k. Late Time Flow

Using the Gauss – Legendre quadrature, the late time flow can be expressed as:

$$p_D = \frac{2\pi}{x_{eD}}\frac{k}{\sqrt{k_x k_z}}\left(\frac{b-a}{2}\right)\sum_{i=1}^{n} w_i f\left\{\begin{bmatrix}1+2exp\langle\frac{-n^2\pi^2}{x_{eD}^2}\left(\frac{z_i(b-a)+b+a}{2}\right)\rangle\\ \cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\end{bmatrix}\right.$$
$$\times\begin{bmatrix}1+2exp\langle\frac{-n^2\pi^2}{h_D^2}\left(\frac{z_i(b-a)+b+a}{2}\right)\rangle\\ \cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\end{bmatrix} \quad (31)$$

l. Dimensionless Pressure Derivative:

m. Early Time Flow

The dimensionless pressure derivative at early – time flow can be expressed as:

$$p_D^{'} = \frac{1}{2}\frac{k}{\sqrt{k_x k_z}}exp\left[-\frac{(x_D-x_{wD})^2+(z_D-z_{wD})^2}{4t_D}\right] \quad (32)$$

n. First Transition Time Flow

The dimensionless pressure derivative at first transition - time flow can be expressed as:

$$p_D^{'} = \frac{t_D\pi h_D}{x_{eD}}\frac{k}{\sqrt{k_x k_z}}\sum_{i=1}^{n}\left[1+2exp\langle\frac{-n^2\pi^2 t_D}{x_{eD}^2}\rangle . cosn\pi\frac{x_{wD}}{x_{eD}}cosn\pi\frac{x_D}{x_{eD}}\right]$$
$$\times\frac{exp\left[-\frac{(z_D-z_{wD})^2}{4t_D}\right]}{\sqrt{t_D}} \quad (33)$$

o. Second Transition Time Flow

The dimensionless pressure derivative at second transition – time flow can be expressed as:

$$p_D^{'} = t_D\pi\frac{k}{\sqrt{k_xk_z}}\sum_{i=1}^{n}\left[1+2exp\left\langle\frac{-n^2\pi^2t_D}{h_D^2}\right\rangle.cosn\pi\frac{z_{wD}}{H_D}cosn\pi\frac{z_D}{H_D}\right] \times\frac{exp\left[-\frac{(x_D-x_{wD})^2}{4t_D}\right]}{\sqrt{t_D}} \ldots \quad (34)$$

p. Late Time Flow

The dimensionless pressure derivative at late – time flow can be expressed as:

$$p_D^{'} = \frac{t_D2\pi}{x_{eD}}\frac{k}{\sqrt{k_xk_z}}\sum_{i=1}^{n}\left\{\left[1+2exp\left\langle\frac{-n^2\pi^2t_D}{x_{eD}^2}\right\rangle\cos n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\right]\right. \times\left[1+2exp\left\langle\frac{-n^2\pi^2t_D}{h_D^2}\right\rangle\cos n\pi\frac{z_{wD}}{h_D}\cos n\pi\frac{z_D}{h_D}\right] \ldots \quad (35)$$

q. Flow Time Equations and Solutions

These equations are presented here for estimating the various flow regimes based on the concept of Odeh and Babu[13].

r. Early Time Flow

The duration of this flow period may be approximated by the minimum of the two following terms

$$t_{e1} = \frac{1800d_z^{2^2}\emptyset\mu c_t}{k_v} \ldots \quad (36)$$

And

$$t_{e1} = \frac{125L^2\emptyset\mu c_t}{k_v} \ldots \quad (37)$$

Where dz is the shortest distance between the well and the z – boundary, ft.

Dimensionless time is expressed as:

$$t_D = \frac{0.00264kt}{\emptyset\mu c_tr_w^2} \ldots \quad (38)$$

Substituting separately Equations (36) and (37) into Equation (38) gives:

$$t_{De} = \frac{4.752(h-z_w)^2}{k_vr_w^2} \ldots \quad (39)$$

And

$$t_{De} = 0.33\frac{k}{k_v} \ldots \quad (40)$$

Therefore, the duration of the early time radial flow period may be approximated by the minimum of the following two terms

$$t_{De} = \frac{4.752(h-z_w)^2}{k_vr_w^2} \text{ And } t_{De} = 0.33\frac{k}{k_v} \ldots \quad (41)$$

s. First Transition Flow Time

The time for this flow starts at

$$t_{e1} = \frac{1800\emptyset D_z^2\mu c_t}{k_v} \ldots \quad (42)$$

And ends at

$$t_{e2} = \frac{160L^2\emptyset\mu c_t}{k_x} \ldots \quad (43)$$

Where Dz is the longest distance between the well and the z – boundary, ft.

Substituting separately Equations (42) and (43) into Equation (38) gives:

$$t_{D1} = \frac{4.752(h-z)^2}{k_vr_w^2} \ldots \quad (44)$$

And

$$t_{D2} = 0.4224\frac{k}{k_x} \ldots \quad (45)$$

Therefore, the first transition time flow starts at

$$t_{D1} = \frac{4.752(h-z)^2}{k_vr_w^2} \text{ And ends at } t_{D2} = 0.4224\frac{k}{k_x} \ldots \quad (46)$$

t. Second Transition Flow Time

The time for this flow starts at

$$t_{e3} = \frac{1480r_w^2\emptyset\mu c_t}{k_x} \ldots \quad (47)$$

And ends at

$$t_{e3} = \frac{2000\emptyset\mu c_t\left(d_x+\frac{r_w}{4}\right)^2}{k_x} \qquad (48)$$

Where dx is the shortest distance between the well and the z – boundary, ft.
Substituting separately Equations (47) and (48) into Equation (38) gives:

$$t_{D3} = 3.9072\,\frac{k}{k_x} \qquad (49)$$

And

$$t_{D4} = \frac{5.28k\left((x_e-x)+\frac{r_w}{4}\right)^2}{k_x} \qquad (50)$$

Therefore, the second transition time flow starts at

$$t_{D3} = 3.9072\,\frac{k}{k_x} \text{ And ends at } t_{D4} = \frac{5.28k\left((x_e-x)+\frac{r_w}{4}\right)^2}{k_x} \qquad (51)$$

v. Late Time Flow

This flow period ends at a maximum of

$$t_{e4} = \frac{4800\emptyset\mu c_t D_x^2}{k_x} \qquad (52)$$

And

$$t_{e4} = \frac{1800\emptyset D_z^2\mu c_t}{k_z} \qquad (53)$$

Where Dx is the longest distance between the well and the X – boundary.
Substituting Equations (52) and (53) into Equation (38) gives:

$$t_{D5} = \frac{12.67k(x_e-x_w)^2}{k_x r_w^2} \qquad (54)$$

And

$$t_{D5} = \frac{4.752(h-z)^2}{k_z r_w^2} \qquad (55)$$

Therefore, the late time flow period ends at a maximum of

$$t_{D5} = \frac{5.0688(x_e-x_w)^2}{L^2} \text{ And } t_{D5} = \frac{1.9008(h-z_w)^2}{L^2} \qquad (56)$$

15.0 Results and Discussion

This section presents the results of the analysis of the effects of lateral extent and point coordinate on the pressure behaviour in a vertical well in a bounded reservoir system.

1. Effect Of Lateral Extent (Xe_D) On Pressure Distribution

The effect of the parameter, xe_D, on pressure distribution of a vertical well in a bounded reservoir system was examined by keeping all other parameters constant while varying xe_D.

The reservoir lateral extent being varied for the third set of wellbore and reservoir parameters were taken as $x_{eD} =$ 16.52, x_{eD} = 20.4 and x_{eD} = 30.6. These parameters represent different values of the dimensionless drainage radius.

A plot of the dimensionless pressure and dimensionless pressure derivative results against the dimensionless time is also illustrated in Figure 1 on the log-log axis for the first set of parameters while varying x_{eD}.

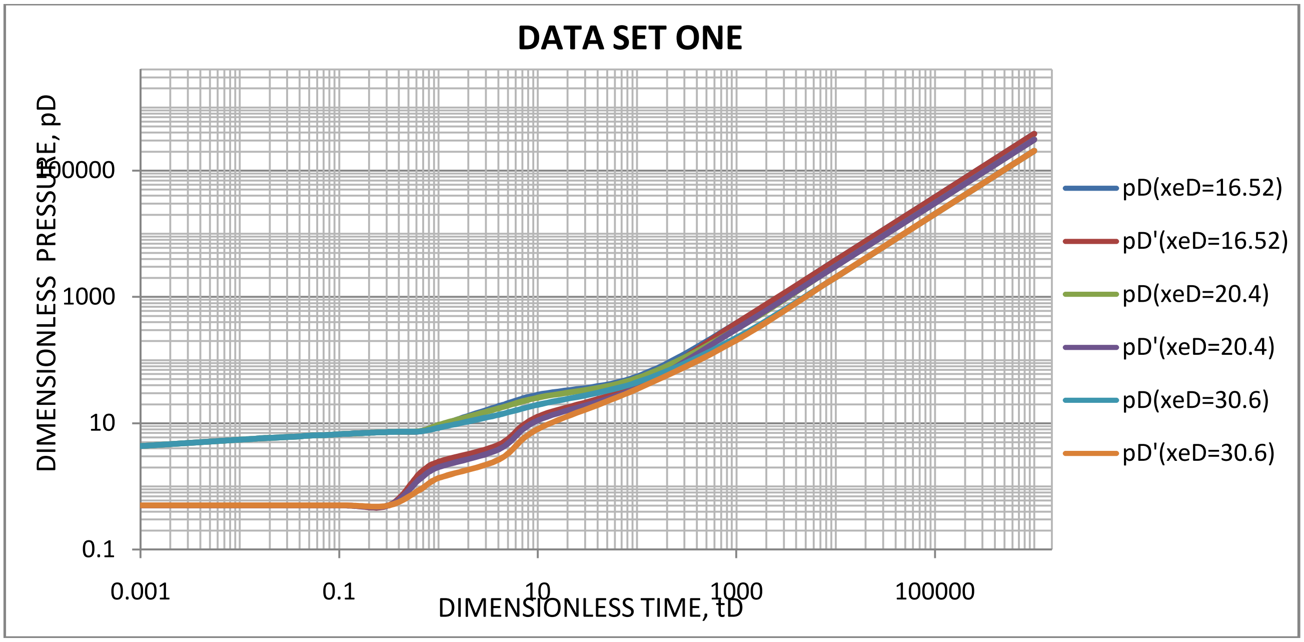

Figure 1: Dimensionless Pressure and Dimensionless Pressure Derivative plots for Lateral Extent Values (Set One)

It can be observed from Figure1 that during the early – time flow, the pressure values remain unchanged for the different values of x_{eD}being investigated. It shows the curves merging between t_D= 0.001 and t_D= 0.318. Since the effect of $x_{eD}$is unnoticed, it thus means that it has no effect on pressure distribution during the infinite – acting flow period. During this period of flow, the dimensionless pressure curves increase infinitely with time with a slope of 1.151. This is usually the convention for a vertical well pressure distribution. During this period flow which is called infinite – acting flow period, the reservoir behaves as though it is infinite in size with fluid flowing radially into the well bore from all directions. Since the influence of the boundary has not been felt at this time, near wellbore characterization is possible and clean oil production guaranteed. Furthermore, it can be observed that the curves seem to separate apart from each other as the reservoir undergoes its first transition between t_D = 0.318 and t_D = 0.439. It is observed during this period, that there is an obvious change in the dimensionless pressure as the value of x_{eD}increases. The curves further separate as the reservoir undergoes another transition, this time, to pseudo – steady state. As can be seen from the curves, the dimensionless pressure distribution tends to reduce as the value of x_{eD}increases during these periods. The transition periods are desired because they delay, to some extent, the attainment of pseudo – steady state, thereby preventing early rapid depletion of the reservoir. As the transition flow recedes over time, pseudo – steady state flow becomes prevailing, indicating the influence of all boundaries on pressure distribution. It can be observed from the dimensionless curves that the pressure distribution increases rapidly at late – time. However, the rate of depletion decreases as x_{eD}increases and vice versa.

The reservoir pressure response is quite noticeable in the dimensionless pressure derivative curves because hidden features in the dimensionless pressure curves are clearly revealed. Figure 1 shows that at early – time, the derivative curves stabilize at constant pressure values of 0.5. The merging of the curves during this period, is an indication that x_{eD}has no effect, whatsoever, on pressure response during the early – time flow. As the infinite – acting flow period gradually disappears, the transition flow becomes dominant with obvious separation of the curves as clearly shown on the pressure derivative curves. This behavior validates the earlier representation on the dimensionless pressure curves as shown in the plot. During the pseudo – steady state flow, it is observed that the dimensionless pressure derivative curves merge characteristically with the dimensionless pressure curves and increase at a constant rate.

The reservoir lateral extent being varied for the second set of wellbore and reservoir parameters were taken as $x_{eD\,=}$ 50.98, x_{eD} = 60.69 and x_{eD} = 67.97. These parameters represent different values of the dimensionless drainage radius.

A plot of the dimensionless pressure and dimensionless pressure derivative results against the dimensionless time is also illustrated in Figure 2 below on the log-log axis for the second set of parameters while varying x_{eD}.

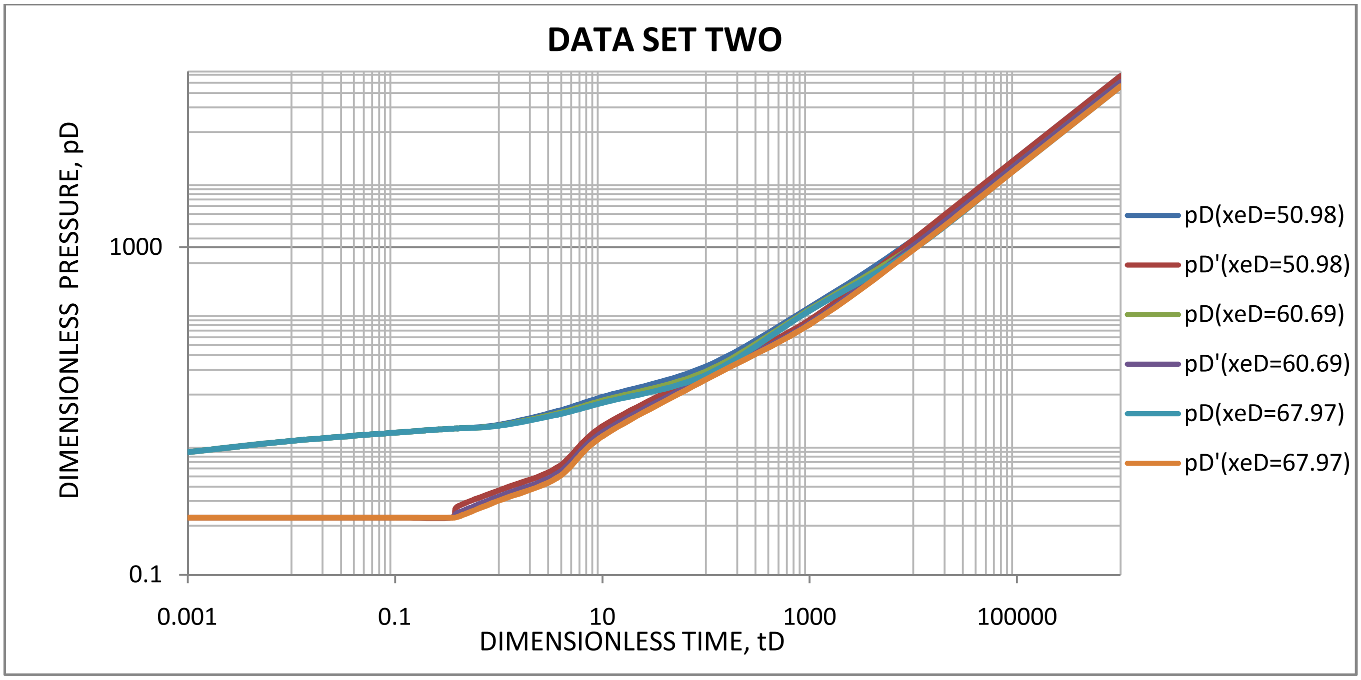

Figure 2: Dimensionless Pressure and Dimensionless Pressure Derivative plots for Lateral Extent Values (Set Two)

It can be observed from Figure2 that during the early – time flow, the pressure values remain unchanged for the different values of x_{eD}being investigated. It shows the curves merging between t_D= 0.001 and t_D= 0.3502. Since the effect of $x_{eD}$is unnoticed, it thus means that it has no effect on pressure distribution during the infinite – acting flow period. During this period of flow, the dimensionless pressure curve increases infinitely with time with a slope of 1.151. This is usually the convention for a vertical well pressure distribution. During this period flow which is called infinite – acting flow period, the reservoir behaves as though it is with infinite in size with fluid flowing radially into the well bore from all directions. Since the influence of the boundary has not been felt at this time, near well bore characterization is possible and clean oil production guaranteed. Furthermore, it can be observed that the curves seem to separate apart from each other as the reservoir undergoes its first transition between t_D = 0.3502 and t_D = 0.398. It is observed during this period, that there is an obvious change in the dimensionless pressure as the value of x_{eD}increases. The curves further separate as the reservoir undergoes another transition, this time, to pseudo – steady state. As can be seen from the curves, the dimensionless pressure distribution tends to reduce as the value of x_{eD}increases during these periods. The transition periods are desired because they delay, to some extent, the attainment of pseudo – steady state, thereby preventing early rapid depletion of the reservoir. As the transition flow recedes over time, pseudo – steady state flow becomes prevailing, indicating the influence of all boundaries on pressure distribution. It can be observed from the dimensionless curves that the pressure distribution increases rapidly at late – time. However, the rate of depletion decreases as x_{eD}increases and vice versa.

The reservoir pressure response is quite noticeable in the dimensionless pressure derivative curves because hidden features in the dimensionless pressure curves are clearly revealed. Figure 2 shows that at early – time, the derivative curves stabilize at constant pressure values of 0.5. The merging of the curves during this period, is an indication that x_{eD}has no effect, whatsoever, on pressure response during the early – time flow. As the infinite – acting flow period gradually disappears, the transition flow becomes dominant with obvious separation of the curves as clearly shown on the pressure derivative curves. This behavior validates the earlier representation on the dimensionless pressure curves. During the pseudo – steady state flow, it is observed that the dimensionless pressure derivative curves merge with characteristically with the dimensionless pressure curves and increase at a constant rate.

2. Effect Of Lateral Point Coordinates (x_D)

The effect of the parameter, x_D, on pressure distribution of a vertical well in a bounded reservoir system was examined by keeping all other parameters constant while varying x_D.

The reservoir lateral point coordinates being varied for the third set of wellbore and reservoir parameters were taken as $x_{D\,=}$ 19.84, x_D = 28.86 and x_D = 30.66. These parameters represent different points along the lateral coordinate of the reservoir system.

A plot of the dimensionless pressure and dimensionless pressure derivative results against the dimensionless time is also illustrated in Figure 3. below on the log-log axis for the second set of parameters while varying x_D.

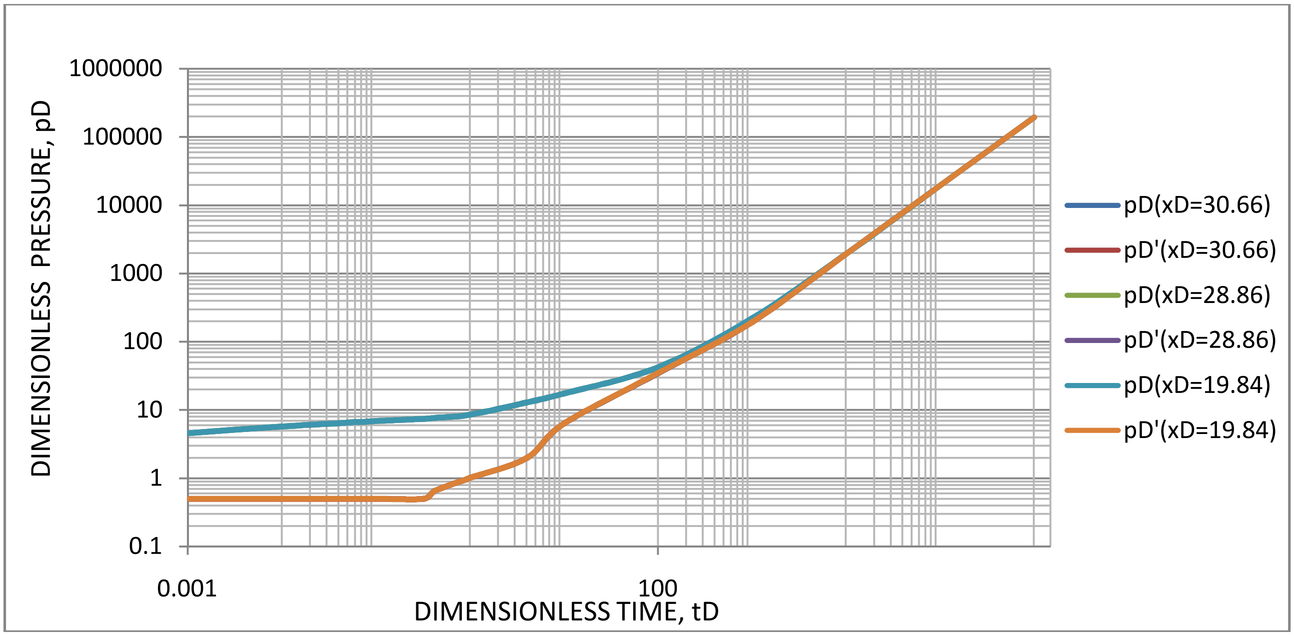

Figure 3: Dimensionless Pressure and Dimensionless Pressure Derivative plots for Lateral Point Coordinate Values

On the dimensionless pressure plot, as seen in Figure 3 above, from early time to late time the curves are seen to rise gradually with no distinct separation between the curves. On the dimensionless pressure derivative plots, at early time, a straight horizontal line trend develops with a value of 0.5, and then gradually rises forming a unit slope straight line at later time. No distinct separation is seen between the curves for the different values of $x_D$from early time to late time.

This means that from the early radial flow period (where the reservoir acts as if it is infinite and fluid flows in all directions to the wellbore) to the point where pseudo - steady state is attained (the remaining oil in the reservoir is being drained by the vertical well), the pressure distribution of the reservoir remains similar for all varied values of x_D.

This is an indication that varying the lateral point coordinate has no significant difference in effect on the pressure response of a vertical well in a bounded reservoir.

16.0 Conclusion

The pressure behavior of a vertical well located in a bounded reservoir has been studied.Pressure responses were generated using mathematical models which were developed and the effect of lateral extent and lateral point coordinate on the pressure behavior has been investigated. From the results of the analysis, the following conclusions were drawn:

1. Infinite – acting flow period is unaffected by the lateral extent of the reservoir. However, reservoir depletion during late time reduces with increasing lateral extent.
2. Pressure behavior is unaffected by the lateral point coordinates throughout all the flow regimes. The direct implication of this is that late and early time productions cannot be differentiated by the effects of the lateral point coordinates in a vertical well located in a bounded reservoir system.
3. Infinite – acting flow period subsists for a long time for as long as flow is close to the well bore.
4. Situating the well far away from the boundaries delays the attainment of pseudo – steady state.

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