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Liquid Limit and Fines Contentfor Soils InIsoko South Area of Delta State: An Experimental Study

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Abstract

The research was conducted to find a correlation between liquid limit and fine content. These soil properties are used in the classification of soils into various groups which depicts their properties that serves as guide for preliminary analysis in design and construction works. The soils samples were obtained from several localities atIsoko South Local Government Area of Delta State. The geotechnical test for the liquid and fine contentwere carried in accordance with BritishStandards.The trend of data collected for the modeling exercise reveals that the variations of liquid limit with fine content can be represented by a non-linear model.A polynomial regression was used to establish a correlationbetween the soil parameters. The suitability of the developed correlation was evaluated by performing validation of the developed model using the coefficient of determination and F- test ratio. The model developed was found to be significant at 95% confidence level. Hence for preliminary design and construction purposes the developed model correlation can be used predict liquid limit value with good accuracy.

1.0 Introduction

A soil classification is a methodical method of categorizing soils into differnt groups and subgroups according to their likely engineering behavior but without comprehensive description. Most of the past classification systems were based on grain size distribution (e.g. MIT classification, USDA classification). However, with the creation of Atterberg limits, new classification systems have been introduced. There are various soil classification systems such as Unified Soil Classification System (USCS), the American Association of State highway and Transportation Officials (AASHTO) system, Federal Aviation Agency (FAA) system, the pedologic soil classification system, US Public Roads Administration (PRA) system and the textural classification system. The USCS and the AASHTO system are widely used in geo-technical engineering practice. Casagrande [1] initiallycreated the USCS, which was later modified by the US Bureau of Reclamation (USBR) and the US Army Corps of Engineers. Many countries codes of practice adopt the USCS as it is [2], or with slight modifications [3,4]. Both USCS and AASHTO system base their classification of soils for engineering purposes on fine content and liquid limit of the soils. USCS [5] and AASHTO [6] define fines as soil particles passing through sieve No. 200(75μm opening). The fines consist of clay and silt. Also the liquid limit of a soil is the moisture content, expressed as a percentage of the mass of the oven-dried soil, at the boundary between the liquid and plastic states[7]. The fines content and liquid limit of a soil are useful for the understanding of various soil mechanical and physical properties such as shear strength, bearing capacity, compressibility and shrinkage-swelling potential[8,9]. Determination of these soil properties is difficult, timeconsuming and usually requires several replicates for reliable results. However in order to save on costs of tests, it may be necessary to perform only one of the two tests and estimate one from the other , if a model that relates the two variable to each other is available. Since the liquid limit is more difficult to obtain in terms of equipment and technicalities, amodel that has the liquid limit as the dependent variable and the fine content as the independent variable is desirable. This present work, therefore seeks to develop a regression models relating liquid limit and fine content.

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2.0 Materials and Methods

2.1 Sampling and Sampling Locations

Delta state is one of the states in Nigeria (Figure 1), it has twenty-five local government areas (Figure 2). Soils samples were collected at several communities from Isoko South Local Government Area of Delta State (Figure 3). Disturbed soil samples were obtained at different depths using hand auger.

2.2 Geotechnical Properties of the Soils

The particle size analysis and the liquid limit tests were carried out in accordance with British Standard [10]. A total of nine soil samples were collected.

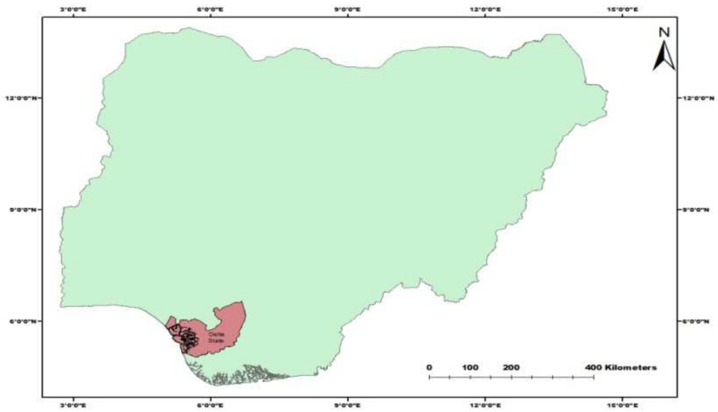

Figure 1: Map of Nigeria Showing Delta State

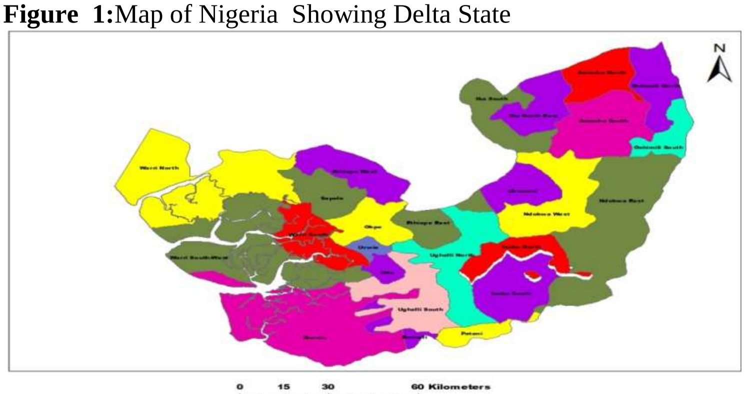

Figure 2: Map of Delta State Showing the Local Government Areas

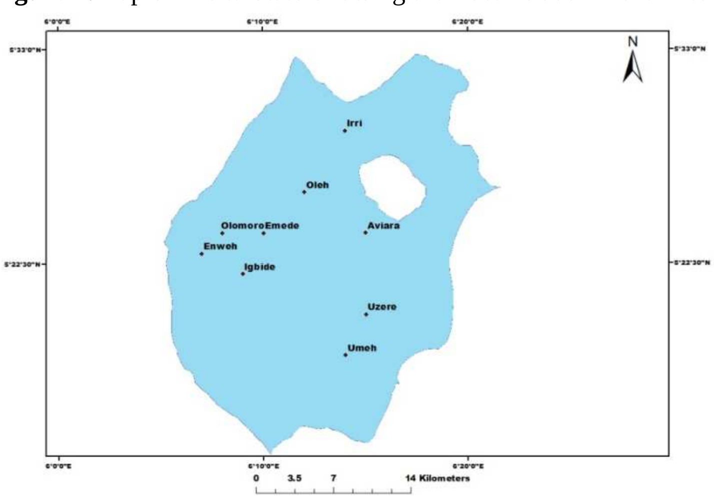

Figure 3: Map of Isoko South Local Government Area Showing Soils Sample Points

2.3 Polynomial Regression Modeling

The polynomial regression is a form of linear regression in which the relationship between the independent variable x and the dependent variable y is modeled as an nth degree polynomial. Polynomial regression fits a nonlinear model to the data and has been used to describe nonlinear phenomena such as the growth rate of tissue[11], the distribution of carbon isotopes in lake sediments[12] and the progression of disease epidemics[13].

The premise of polynomial regression is that a data set of n paired (x,y) members: (x_1, y_1), (x_2,y_2), (x_3, y_3)….. (x_n,y_n) can be processed using a least-square methods to create a predictive polynomial equation[14]. Thus,

$$y = a_0 + a_1 x + \ldots + a_k x^k \tag{1}$$

The essence of the method is to reduce the residual R at each data point given by

$$R^2 = \sum_{i=1}^{n} \left[y_i - \left(a_0 + a_1 x_i + \ldots + a_{k\,i}^{\;k} \right) \right]^2 \tag{2}$$

The partial derivatives are

$$\frac{\partial\left(R^2\right)}{\partial a_0} = -2\sum_{i=1}^{n} \left[y - \left(a_0 + a_1 x + \ldots + a_k x^k \right) \right] = 0 \tag{3}$$

$$\frac{\partial\left(R^2\right)}{\partial a_1} = -2\sum_{i=1}^{n} \left[y - \left(a_0 + a_1 x + \ldots + a_k x^k \right) \right] x = 0 \tag{4}$$

$$\frac{\partial\left(R^2\right)}{\partial a_k} = -2\sum_{i=1}^{n} \left[y - \left(a_0 + a_1 x + \ldots + a_k x^k \right) \right] x^k = 0 \tag{5}$$

These lead to the equations

$$a_0 n + a_1 \sum_{i=1}^{n} x_i + \ldots + a_k \sum_{i=1}^{n} x_i^k = \sum_{i=1}^{n} y_i \tag{6}$$

$$a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 + \ldots + \sum_{i=1}^{n} x_i^{k+1} = \sum_{i=1}^{n} x_i y_i \tag{7}$$

$$a_0 \sum_{i=1}^{n} x_i^k + a_1 \sum_{i=1}^{n} x_i^{K+1} + \ldots + a_k \sum_{i=1}^{n} x_i^{2k} = \sum_{i=1}^{n} x_i^k y_i \tag{8}$$

Or, in matrix form

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^k \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^k & \sum_{i=1}^{n} x_i^{k+1} & \cdots & \sum_{i=1}^{n} x_i^{2k} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^k y_i \end{bmatrix} \tag{9}$$

This is a vandermonde matrix. We can also obtain the matrix for a least square fit

$$\begin{bmatrix} 1 & x_1 & \cdots & x_1^k \\ 1 & x_2 & \cdots & x_2^k \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \tag{10}$$

Pre-multiplying both sides by the transpose of the first matrix then gives

$$\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & \ddots & \vdots \\ x_1^k & x_2^k & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} 1 & x_1 & \cdots & x_1^k \\ 1 & x_2 & \cdots & x_2^k \\ \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \\ \vdots & \vdots & \ddots & \vdots \\ x_1^k & x_2^k & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \tag{11}$$

So

$$\begin{bmatrix} n & \sum_{i=1}^{n} x_i & \cdots & \sum_{i=1}^{n} x_i^k \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} x_i^2 & \cdots & \sum_{i=1}^{n} x_i^{k+1} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^{n} x_i^k & \sum_{i=1}^{n} x_i^{k+1} & \cdots & \sum_{i=1}^{n} x_i^{2k} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} x_i y_i \\ \vdots \\ \sum_{i=1}^{n} x_i^k y_i \end{bmatrix} \quad (12)$$

As before, given n point (x_1, y_i) and fitting with polynomial coefficients a_0, …a_k

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^k \\ 1 & x_2 & x_2^2 & \cdots & x_2^k \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^k \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_k \end{bmatrix} \quad (13)$$

In matrix notation, the equation for a polynomial fit is given by

$$y = Xa \quad (14)$$

This can be solve by premultiplying by the transpose X^T

$$X^T y = X^T Xa \quad (15)$$

This matrix equation can be solve numerically as

$$a = \left(X^T X\right)^{-1} X^T y \quad (16)$$

While the goodness of fit of the model is defined as the explained variation divided by the total variation

$$r^2 = \frac{\sum(Y_i' - \bar{Y})^2}{\sum(Y - \bar{Y}))^2} \quad (17)$$

3.0 Results and Discussions

The results for the liquid limit and fine content is presented in Table 1

Table 1: Soil Test Result

S/N	LOCATIONS	SOIL PARAMETERS	
		Fine content (%)	**Liquid Limit (%)**
1	Oleh	19.97	21.30
2	Umeh	30.07	26.20
3	Uzere	55.34	24.10
4	Aviara	9.34	22.10
5	Emede	23.84	24.80
6	Enweh	25.32	26.30
7	Igbide	50.57	28.00
8	Irri	23.09	23.20
9	Olomoro	44.68	20.80

The scatter plot of the liquid limit and passing particle for sieve No. 200 is shown in Figure 4.

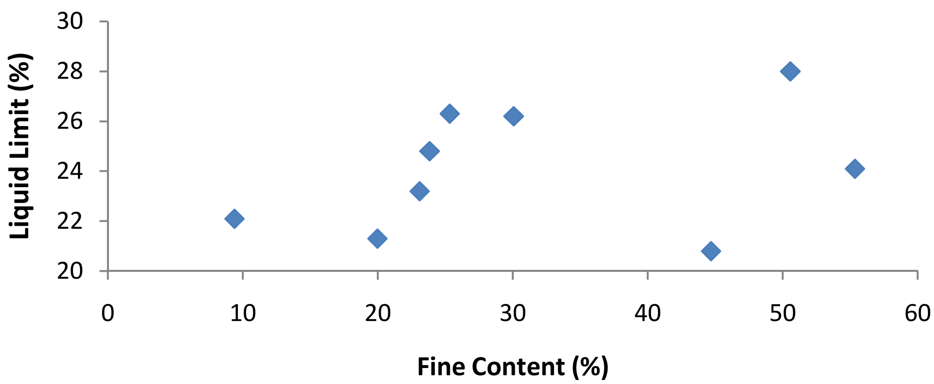

Figure 4: Scatter Plot of Liquid Limit and Fine Content

The plot shows a nonlinear relationship between the dependent and independent variables and a polynomial regressionc was used for modeling. A sixth degree polynomial is given by

$Y = a_0x^6 + a_0x^5 - a_0x^4 + a_0x^3 - a_0x^2 + a_0x + a_0$ (18)

Input the variables in equation (1)

$$\begin{bmatrix} 9 & 2.82E+02 & 1.07E+04 & 4.66E+05 & 2.19E+07 & 1.08E+09 & 5.48E+10 \\ 2.82E+02 & 1.07E+04 & 4.66E+05 & 2.19E+07 & 1.08E+09 & 5.48E+10 & 2.83E+12 \\ 1.07E+04 & 4.66E+05 & 2.19E+07 & 1.08E+09 & 5.48E+10 & 2.83E+12 & 1.48E+14 \\ 4.66E+05 & 2.19E+07 & 1.08E+09 & 5.48E+10 & 2.83E+12 & 1.48E+14 & 7.77E+15 \\ 2.19E+07 & 1.08E+09 & 5.48E+10 & 2.83E+12 & 1.48E+14 & 7.77E+15 & 4.11E+17 \\ 1.08E+09 & 5.48E+10 & 2.83E+12 & 1.48E+14 & 7.77E+15 & 4.11E+17 & 2.19E+19 \\ 5.48E+10 & 2.83E+12 & 1.48E+14 & 7.77E+15 & 4.11E+17 & 2.19E+19 & 1.17E+21 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} 2.17E+02 \\ 6.89E+03 \\ 2.64E+05 \\ 1.15E+07 \\ 5.42E+08 \\ 2.68E+10 \\ 1.36E+12 \end{bmatrix}$$

solving the equation for $a_0, a_1, a_2, a_3, a_4, a_5,$ and a_6, will give

$a_0 = 26.98$; $a_1 = 4.10$; $a_2 = -1.07$; $a_3 = -0.08$; $a_4 = -0.003$; $a_5 = 4.72E-05$ and $a_6 = -2.81E-07$

Hence the fitted model is

$Y = -2.81E-07x^6 + 4.72E-05x^5 - 0.003x^4 + 0.08x^3 - 1.07x^2 + 4.10x + 26.98$ (19)

The graph of the plot of the sixth degree polynomial regression model is shown in Figure 5.

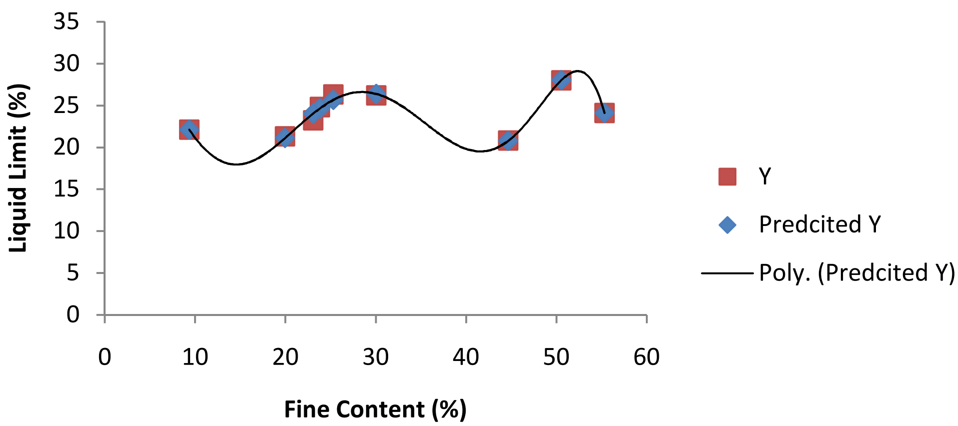

Figure 5: Polynomial Model Plot

The plot shows that the model equation nearly fits the observed values of the liquid limit with minor deviations. The degree of correlation can be obtained using the coefficient of determination (R^2).The results of the predicted values and the errors using the model is given in Table 2.

Table 2: Residual Analysis

S/N	Liquid Limit (Y)	Predicted Y_i'	Residual = Y-Y_i'	(Residual)^2	SS(Regression) = $Y_i' - \bar{Y}$	SS(Regression) (S^2) = $Y_i' - \bar{Y}$	(Y-$\bar{Y}$)^2
1	21.30	21.10	0.20	0.04	-2.99	8.92	7.78
2	26.20	26.37	-0.17	0.03	2.28	5.21	4.46
3	24.10	24.10	0.00	0.00	0.00	0.00	0.00
4	22.10	22.10	0.00	0.00	-1.99	3.94	4.00
5	24.80	24.65	0.15	0.02	0.56	0.31	0.51
6	26.30	25.66	0.64	0.41	1.57	2.47	4.89
7	28.00	28.02	-0.02	0.00	3.93	15.43	15.30
8	23.20	24.03	-0.83	0.69	-0.06	0.00	0.79
9	20.80	20.77	0.03	0.00	-3.32	11.00	10.82
SUM	**216.8**			**1.19**		**47.30**	**48.49**

The goodness of fit is

$r^2 = \frac{47.30}{48.49} = 0.975$

This means that the model can account for about 97.5% of the total variation in the liquid limit leaving 2.5% unexplained. This signifies a good model.The unexplained variation which is the deviations of the observed values from the predicted values can also be used in checking the adequacy of the model using the F-ratio.

Regression (degree of freedom) = 2
Residual (degree of freedom) = 6
Sum of square (regression) = 47.30
Sum of square (residual) = 1.19
Mean square(regression) $= \frac{47.30}{6} = 7.883$
Mean square(residual) $= \frac{0.11549}{2} = 0.0595$
F-ratio $=\frac{7.883}{0.0595} = 13.239$
The $F_{observed}$(13.239) is greater than $F_{critical}$ (5.14). Hence the model parameters are significant at 95% confidence interval. The model is significant as the F value of 13.239 is greater than the critical value of 5.14.

4.0 Conclusion

This present work seeks to develop a regression models relating liquid limit and fine content. These soils properties are used in soil classificationsystem in use in geo-technical engineering practice. They are also informative for the interpretation of several soil mechanical and physical properties such as shear strength, bearing capacity, compressibility and shrinkage-swelling potential. The trend of data collected for the modeling exercise revealed that the variations of liquid limit with fine content can be represented by a non-linear model. A polynomial regression was used to establish a correlation between the soil parameters to save cost and time in performing the experimental procedure. The model was adjudged to be appropriate based on the suitability of the coefficient of determination and F-ratio test.

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