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Contribution of Oblateness of the Sun to Radar Sounding Phenomenon According to the General Dynamic Theory of Gravitation

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Abstract

Radar sounding phenomenon within the gravitational field established by the spherical massive Sun to the order of C^{-5}has been resolved using general dynamical theory of gravitation (GDTG). It is however well established, now, that most of the astronomical bodies including the Sun are spheroidal (oblate or prolates) in shape. In this paper, the general dynamical theory of gravitation (GDTG) has been employed to resolve the Radar sounding phenomenon within the gravitational field established by the homogenous spheroidal oblate massive Sun. The results compare favourably with that established by the spherical massive Sun.

Keywords: Radar Sounding, Oblateness of the Sun, Radar Time Delay.

1.0 Introduction

Radar sounding phenomenon within the gravitational field established by the spheroidal massive Sun to the order of C^{-3} has been resolved using General Relativistic Theory (GRT) [1]. The work is based on the solution of Einstein's field equation by Schwarzschild. In that work the matrix tensor was translated from spherical coordinate system to spheroidal coordinate and the time for round trip for a photon moving within the gravitational field established by the homogenous oblate spheroidal massive sun was derived from the world line element. However, in this work, the approach is based on the General Dynamic Theory of Gravitation (GDTG). Here, thetime for round trip for a photon moving within the gravitational field established by the homogenous oblate spheroidal massive sun is derived from the equation of motion for photons moving in the gravitational field of a homogenous spheroidal massive body.Even though this phenomenon has been resolved using the General Dynamic Theory of Gravitation (GDTG) up to order of C^{-5}, the problem is the General Dynamic Theory of Gravitation (GDTG) have developed the field equations and equations of motion for solving the above mentioned physical phenomenon by considering the massive sun, planetary bodies and other stars as homogenous spherical bodies [2]. But it is well known that the only reason for these restrictions is mathematical convenience and simplicity. The fact of nature is that the sun which is G2 star in the milky-way galaxy is oblate [1] in shape.
Consequently spheroidal geometry will have substantial effects in the gravitational fields of astronomical bodies. This work is an attempt to use GDTG to derive the equation of Radar sounding phenomenon within the gravitational field established by the spheroidal massive Sun to the order of C^{-3} by writing the equation of motion for photons moving in the gravitational field of a homogenous spherical massive body in spheroidal coordinate and then solving for the time for a round trip.

2.0 Spheroidal Geometry and Field Equations

The oblate spheriodal coordinate of space (η, ξ, ϕ) are defined in terms of Cartesian coordinates (x, y, z) as [3]:

$$\left.\begin{aligned} x &= a(1-\eta^2)^{\frac{1}{2}}(1+\xi^2)^{\frac{1}{2}}cos\phi \\ y &= a(1-\eta^2)^{\frac{1}{2}}(1+\xi^2)^{\frac{1}{2}}sin\phi \\ z &= a\eta\xi \end{aligned}\right\} \tag{1}$$

Where a is a constant parameter of a particular oblate body and

$$-1 \le \eta \le 1, 0 \le \xi \le \infty, 0 \le \phi \le 2\pi$$

The General Dynamic Theory of Gravitation derived and showed that the equation of motion for photons moving in the gravitational field of a homogenous spherical massive body is given as [2].

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$$\frac{du(r,t)}{dt} - \frac{1}{c^2}\left\{\left[1 + \frac{1}{c^2}\Phi_g(r,t)\right]^{-1}\frac{d}{dt}\Phi_g(r,t)\right\}u(r,t) = -\nabla\Phi_g(r,t) \qquad (2)$$

Equation (2) may be written alternatively as

$$a(r,t) - \frac{1}{c^2}\left\{\left[1 + \frac{1}{c^2}\Phi_g(r,t)\right]^{-1}\frac{d}{dt}\Phi_g(r,t)\right\}u(r,t) = -\nabla\Phi_g(r,t) \qquad (3)$$

To write equation (3) as an equation of motion for photons moving in the gravitational field of a homogenous spheroidal oblate massive body (sun), we carry out the transformation of equation (1) spherical coordinate system and subsequently into the spheroidal coordinates so that our first term in the L.H.S of equation (3) becomes

$$a(\eta,\xi,\phi) = a_\eta\hat{\eta} + a_\xi\hat{\xi} + a_\phi\hat{\phi}$$

$$= \hat{\eta}a\left[\frac{(1-\eta^2)}{(\eta^2+\xi^2)}\right]^{1/2}\left\{\frac{(\eta^2+\xi^2)}{(1-\eta^2)}\ddot{\eta} + \frac{2\xi}{(1-\eta^2)}\dot{\eta}\dot{\xi} + \frac{2(1-\xi^2)^{1/2}}{(1-\eta^2)^2}\dot{\eta}^2 - \frac{\eta}{(1+\xi^2)}\dot{\xi}^2 + \eta(1+\xi^2)\dot{\phi}^2\right\}$$
$$+ \hat{\xi}a\left[\frac{(1-\xi^2)}{(\eta^2+\xi^2)}\right]^{1/2}\left\{\frac{(\eta^2+\xi^2)}{(1-\xi^2)}\ddot{\xi} + \frac{2\eta}{(1+\xi^2)}\dot{\eta}\dot{\xi} + \frac{\xi(1+\eta^2)}{(1-\xi^2)^2}\dot{\xi}^2 - \frac{\xi}{(1-\eta^2)}\dot{\eta}^2 - \xi(1-\eta^2)\dot{\phi}^2\right\}$$
$$+ \hat{\phi}a(1-\eta^2)^{1/2}(1+\xi^2)^{1/2}\left\{\ddot{\phi} - \frac{2\eta}{(1-\eta^2)}\dot{\eta}\dot{\xi} + \frac{2\xi}{(1+\xi^2)}\dot{\phi}\dot{\xi}\right\} \qquad (4)$$

And the instantaneous velocity of a moving particle in an oblate spheroidal coordinate [4] as

$$U(\eta,\xi,\phi) = U_\eta\hat{\eta} + U\hat{\xi} + U_\phi\hat{\phi}$$
$$= \hat{\eta}a\frac{(\eta^2+\xi^2)^{1/2}}{(1-\eta^2)}\dot{\eta} + \hat{\xi}a\frac{(\eta^2+\xi^2)^{1/2}}{(1-\xi^2)^{1/2}}\dot{\xi} + \hat{\phi}a(1-\eta^2)^{1/2}(1+\xi^2)^{1/2}\dot{\phi} \qquad (5)$$

The universal gravitational scalar potential [4] exterior to the spheroidal oblate massive sun has been obtained as [6]

$$\Phi_g^+(\eta,\xi,\phi) = \{B_o^+Q_o(-i\xi)P_o(\eta) + B_2^1Q_2(-i\xi)P_2(\eta)\} \qquad (6)$$

Where P_o,P_2 and Q_o,Q_2 are corresponding pair of independent Legendre functions, B_o and B_2 are constants. The del-operator in terms of the spheroidal-oblate coordinates is also given as [6]

$$\nabla(\eta,\xi,\phi) = \left\{\hat{\eta}\frac{(1-\eta^2)^{1/2}}{a(\eta^2+\xi^2)^{1/2}}\frac{\partial}{\partial\eta} + \hat{\xi}\frac{(1-\xi^2)^{1/2}}{a(\eta^2+\xi^2)^{1/2}}\frac{\partial}{\partial\xi} + \hat{\phi}\frac{1}{a(1-\eta^2)^{1/2}(1+\xi^2)^{1/2}}\frac{\partial}{\partial\phi}\right\} \qquad (7)$$

Using equation (6) and (7), the negative gradient of the universal scalar potential is computed as

$$-\nabla\Phi_g^+(\eta,\xi,\phi) = -\frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_0^+Q_0(-i\xi)\frac{\partial}{\partial\eta}P_0(\eta) - \frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+Q_2(-i\xi)\frac{\partial}{\partial\eta}P_2(\eta)$$
$$-\frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_o^+P_o(\eta)\frac{\partial}{\partial\xi}Q_o(-i\xi)$$
$$-\frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+P_2(\eta)\frac{\partial}{\partial\xi}Q_2(-i\xi) \qquad (8)$$

Substituting equation (4), (5), (6) and (8) into equation (3), the RHS of equation (3) turns out to be;

$$-\frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_0^+Q_0(-i\xi)\frac{\partial}{\partial\eta}P_0(\eta) - \frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+Q_2(-i\xi)\frac{\partial}{\partial\eta}P_2(\eta) - \frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_o^+P_o(\eta)\frac{\partial}{\partial\xi}Q_o(-i\xi)$$
$$-\frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+P_2(\eta)\frac{\partial}{\partial\xi}Q_2(-i\xi)$$

And the LHS of equation (3) becomes

$$a\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}\{1\} + a\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{1/2}\{2\} + a(1-\xi^2)^{1/2}(1+\xi^2)^{1/2}\{3\}$$
$$-\frac{1}{c^2}\left\{\left[1 + \frac{1}{c^2}\Phi_g^+(\eta,\xi,\phi)\right]^{-1}\frac{d}{dt}\Phi_g^+(\eta,\xi,\phi)\right\}u(\eta,\xi,\phi)$$

Where:

$$\{1\} = \left\{\frac{(\eta^2+\xi^2)}{(1-\eta^2)}\ddot{\eta} + \frac{2\xi}{(1-\eta^2)}\dot{\eta}\dot{\xi} + \frac{2(1-\xi^2)^{1/2}}{(1-\eta^2)^2}\dot{\eta}^2 - \frac{\eta}{(1+\xi^2)}\dot{\xi}^2 + \eta(1+\xi^2)\dot{\phi}^2\right\}$$

$$\{2\} = \left\{\frac{(\eta^2+\xi^2)}{(1-\xi^2)}\ddot{\xi} + \frac{2\eta}{(1+\xi^2)}\dot{\eta}\dot{\xi} + \frac{\xi(1+\eta^2)}{(1-\xi^2)^2}\dot{\xi}^2 - \frac{\xi}{(1-\eta^2)}\dot{\eta}^2 - \xi(1-\eta^2)\dot{\phi}^2\right\}$$
$$\{3\} = \left\{\ddot{\phi} - \frac{2\eta}{(1-\eta^2)}\dot{\eta}\dot{\xi} + \frac{2\xi}{(1+\xi^2)}\dot{\phi}\dot{\xi}\right\}$$

Equating the LHS and the RHS of equation (3) from the foregoing we have

$$\begin{aligned} a\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}\{1\} + a\left(\frac{1+\xi^2}{\eta^2+\xi^2}\right)^{1/2}\{2\} + a(1-\xi^2)^{1/2}(1+\xi^2)^{1/2}\{3\} \\ -\frac{1}{c^2}\left\{\left[1+\frac{1}{c^2}\Phi_g^+(\eta,\xi,\phi)\right]^{-1}\frac{d}{dt}\Phi_g^+(\eta,\xi,\phi)\right\}u(\eta,\xi,\phi) \\ = -\frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_0^+Q_0(-i\xi)\frac{\partial}{\partial\eta}P_0(\eta) - \frac{1}{a}\left(\frac{1-\eta^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+Q_2(-i\xi)\frac{\partial}{\partial\eta}P_2(\eta) \\ -\frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_o^+P_o(\eta)\frac{\partial}{\partial\xi}Q_o(-i\xi) \\ -\frac{1}{a}\left(\frac{1-\xi^2}{\eta^2+\xi^2}\right)^{1/2}B_2^+P_2(\eta)\frac{\partial}{\partial\xi}Q_2(-i\xi) \end{aligned} \qquad (9)$$

Equation (9) is the equation of motion for photon moving in the gravitational field established by the oblate spheroidal massive sun according to General Dynamic Theory of gravitation (GDTG)

3.0 Application

Consider an observer at r_1 corresponding to $\xi_1$sending radar signal or pulses in a radial direction towards a small body at position r_2 corresponding to ξ_2 within the gravitational field established by the homogenous oblate spheroidal massive sun of mass M such that $\xi_1 > \xi_2$ as shown in Figure 1.

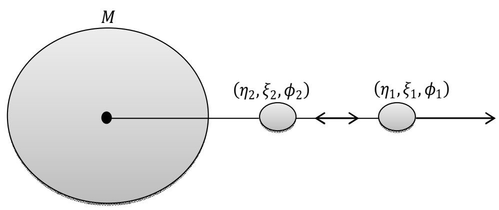

Figure1: Radar Sounding Experiment

We are interested in computing the total time needed for the radar pulses to travel from $\xi_1 to \xi_2$ and back to ξ_1 in a radial direction. For radial motion of radar signals in equatorial plane

$$\eta = 0\ and d\eta = d\phi = 0$$

And also using the facts that

$$P_o(\eta) = 1$$
$$P_2(\eta) = \frac{1}{2}(3\eta^2 - 1)$$

As Legendre functions in equation (9), it follows that

$$\begin{aligned} \ddot{\xi} + \frac{1}{\xi(1-\xi^2)}\dot{\xi}^2 - \frac{a}{c^2}\dot{\xi}\frac{(1-\xi^2)^{\frac{1}{2}}}{(1+\xi^2)^2}\left\{\left[1+\frac{1}{c^2}\left(B_o^+Q_o(-i\xi) - \frac{1}{2}B_2^1Q_2(-i\xi)\right)\right]^{-1}\frac{d}{dt}\left[B_o^+Q_o(-i\xi) - \frac{1}{2}B_2^1Q_2(-i\xi)\right]\right\} \\ = -\frac{1}{a}\frac{(1-\xi^2)^{\frac{3}{2}}}{\xi^2(1+\xi^2)^{\frac{1}{2}}}B_o^+\frac{\partial}{\partial\xi}Q_o(-i\xi) + \frac{1}{2a}\frac{(1-\xi^2)^{\frac{3}{2}}}{\xi^2(1+\xi^2)^{\frac{1}{2}}}B_2^+\frac{\partial}{\partial\xi}Q_2(-i\xi) \end{aligned} \qquad (10)$$

Or alternatively, equation (10) may be written as

$$\ddot{\xi} + \frac{1}{\xi(1-\xi^2)}\dot{\xi}^2 - \frac{a}{c^2}\dot{\xi}\frac{(1-\xi^2)^{\frac{1}{2}}}{(1+\xi^2)^2}Q_3(\xi) = -\frac{1}{a}\frac{(1-\xi^2)^{\frac{3}{2}}}{\xi^2(1+\xi^2)^{\frac{1}{2}}}Q_4(\xi) + \frac{1}{2a}\frac{(1-\xi^2)^{\frac{3}{2}}}{\xi^2(1+\xi^2)^{\frac{1}{2}}}Q_5(\xi) \qquad (11)$$

Where:

$$Q_3(\xi) = \left\{\left[1+\frac{1}{c^2}\left(B_o^+Q_o(-i\xi) - \frac{1}{2}B_2^1Q_2(-i\xi)\right)\right]^{-1}\frac{d}{dt}\left[B_o^+Q_o(-i\xi) - \frac{1}{2}B_2^1Q_2(-i\xi)\right]\right\}$$

$$Q_4(\xi) = \left[B_0^+ \frac{\partial}{\partial \xi} Q_o(-i\xi)\right]$$

$$Q_5(\xi) = \left\{B_2^+ \frac{\partial}{\partial \xi} Q_2(-i\xi)\right\}$$

By transformation
$\dot{\xi}(\xi) = w(\xi);\ w^2 = (\xi)$
Equation (11) integrates as

$$\dot{\xi}^2 = \left\{\frac{a}{c^2}\xi(1+\xi^2) - \xi\left[\frac{1+\xi^2}{(1+\xi)^2}\right]\right\}^2 \tag{12}$$

But

$$U^2\xi = \frac{a^2\xi^2}{1+\xi^2}\dot{\xi}^2[1] \tag{13}$$

Substituting equation (12) into equation (13) we have

$$\frac{d\xi}{dt} = \pm a\xi(1+\xi^2)^{-\frac{1}{2}}\left\{\frac{a}{c^2}\xi(1+\xi^2) - \xi\left[\frac{1+\xi^2}{(1+\xi)^2}\right]\right\} \tag{14}$$

Therefore the coordinate time of the radar signal is obtained from equation (14) as

$$dt = \frac{(1+\xi^2)^{\frac{1}{2}}}{a\xi}\left\{\frac{a}{c^2}\xi(1+\xi^2) - \xi\left[\frac{1+\xi^2}{(1+\xi)^2}\right]\right\}^{-1} d\xi \tag{15}$$

Hence the total time needed for the round trip of the radar signal for $\xi_1 to \xi_2$ and back to ξ_1 is given as

$$dt = 2\int_{\xi_1}^{\xi_2} \frac{(1+\xi^2)^{\frac{1}{2}}}{a\xi}\left\{\frac{a}{c^2}\xi(1+\xi^2) - \xi\left[\frac{1+\xi^2}{(1+\xi)^2}\right]\right\}^{-1} d\xi \tag{16}$$

The relationship between the proper time, $D\tau$ and the coordinate time, Dt is given by [5]

$$D\tau = \left[1 - \frac{2}{c^2} + (\eta, \xi, \phi)\right]^{-\frac{1}{2}} Dt \tag{17}$$

Substituting equation (16) into equation (17), expanding and integrating then considering the first few terms we have

$$D\tau = \frac{2a}{c^2}(\xi_1 - \xi_2)^{-4}\left(\frac{1}{\xi_1^2} - \frac{1}{\xi_2^2}\right)^{-12}\left(\frac{1}{\xi_2^3} - \frac{1}{\xi_2^3}\right) \tag{18}$$

Equation (18) is the total time to the order of c^{-2} taken for the round trip of radar signal within the gravitational field established by the homogenous spherical oblate sun. The expression for the radial distance in the equatorial plane from equation (1) in terms of ξ is given by [3] as

$$r = a(1+\xi^2)^{1/2} \tag{19}$$

Hence

$$\left.\begin{aligned}\xi_1 &= \left(\frac{r_1^2}{a^2} - 1\right)^{1/2} \\ \xi_2 &= \left(\frac{r_2^2}{a^2} - 1\right)^{1/2}\end{aligned}\right\} \tag{20}$$

Substituting equation (20) into equation (18) we have

$$D\tau_g = \frac{1}{a^2c^2}(r_1^2 - r_2^2) - \frac{14}{a^3}(r_1^2 - r_2^2) \tag{21}$$

Equation (21) is the total time for the round trip of the radar pulses in terms of a measurable distance (r) to the order of c^{-2} within the gravitational field of the spheroidal oblate sun according to General Dynamical Theory of Gravitation.

The total time for the round trip of the radar pulses within the gravitational field of the spheroidal oblate sun according to Newton's Dynamical Theory of Gravitation is given as [6].

$$D\tau_n = \frac{1}{ac}\left\{r_1 - r_2 - \frac{1}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right)\frac{a^2}{c^2}[Q(r_1) - Q(r_2)]\right\} \tag{22}$$

Subtracting equation (22) from equation (21) we have

$$D\tau = \frac{1}{ac}\left[r_1 - r_2 - \frac{1}{2}\left(\frac{1}{r_1} + \frac{1}{r_2}\right) - \frac{1}{ac}\left[(r_1^2 - r_2^2) - \frac{14}{a^2}(r_1^2 - r_2^2)\right] + \frac{a}{c^3}[Q(r_1) - Q(r_2)]\right] \tag{23}$$

Equation (23) is the time delay to the order of c^{-3} which the radar signal will experience in the gravitational field of the homogenous spheroidal oblate sun according to General Dynamical Theory of Gravitation (GDTG).

4.0 Summary and Conclusion

In this paper we formulated and derived equations of motion for the radar pulses (photon) in the gravitational field of an oblate spherical sun, as equation (4), (5) and (9). Then we solved them for the radial motion in the equatorial plane to obtain the instantaneous speed of the radar pulses as equation (14). Equation (14) is applied to compute the time taken by the radar pulses to move from an observer (ξ_1) to reflecting target (ξ_2) and back to the observer within the gravitational field of the sun considered to be an oblate spheroidal body and given as equation (18) and the time delay is obtained as equation (23).

Equation (4)-(9) open the way for solutions of the equations of motion for photons (radar pulses) in all directions $(\hat{\eta}, \hat{\xi}, \hat{\phi})$ in the gravitational field of the spheroidal sun. Also our expression i.e. equation (23) contains the oblate spheroidal correction to the corresponding total time delay to the order of c^{-3} for homogenous spherical (r, θ, ϕ) given as

$$D\tau_o = \frac{2}{c}\left\{(r_1 - r_2) + -\frac{k}{c^2}\ln\left(\frac{r_1}{r_2}\right)\right\}$$

Therefore the contribution of the oblateness of the radar sounding problem according to the General Dynamical Theory of Gravitation (GDTG) may be computed theoretical via equation (23) to the order c^{-3} for experimental verification.

5.0 References

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