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Quality Factor of Practically Designed Electrically Small Antennas

Ikechiamaka F. N[1], Kwaha B.J[2] and Chagok N.M.D[3]

[1]Department of Physics, Federal University Dutsin-Ma, Katsina State, Nigeria
[2,3]Department of Physics, University of Jos, Plateau State, Nigeria.

Abstract

Quality factor (Q) is one essential parameter that can give a crude assessment of electrically small antenna performance due to its relationship with fractional bandwidth as well as gain and efficiency of the antenna. Quality factor of four electrically small antennas were predicted with the use of existing expressions in literature. Q was calculated for Monopole (Whip) antenna of radius enclosing the antenna,$a = 0.05m$, Microstrip antenna of $a = 0.07m$, Planer Inverted–F antenna of $a = 0.034m$ and $a = 0.023m$ for air and FR4 substrates respectively. The plots of the results of the predicted Q show that Q diverges as the size a of the antenna goes to zero but are in tandem as a increases. The paper recommends that at this time when miniaturized antennas for portable wireless applications are in high demand, there is need for more research to be carried out by antenna design engineers and other scholars in this field to enable the derivation of expression for prediction of Q that will be near exactfor effective small antenna design.

Key words: Quality factor, electrically small antenna, Fundamental limit

1.0 Introduction

The challenge of antenna miniaturization with regards to fundamental limits of electrically small has become an interesting field of research in recent times. The increasing demand for portable and compact wireless systems (equipment) such as mobile phones, global positioning systems (GPS), Radio Frequency Identification Devices (RFIDs) and others has consequently increased the need for smaller antennas for their effective design. More so, the growth in the number of applications in mobile devices requires the design of efficient smaller and multiband antennas for their operations.
An antenna is considered to be electrically small when its physical size is much smaller than a wavelength at the operational frequency such that it satisfies the condition [1]

$$\frac{a}{\lambda} = \frac{1}{2\pi} \qquad (1)$$

where a is the radius of the sphere enclosing the antenna and λ is the wavelength. Antennas are characterized by a number of parameters such as bandwidth (BW), radiation efficiency (η), gain (G), quality factor (Q) and so on. Though Q may not be rated as important as the first three listed parameters in evaluating the antenna performance in wireless systems, it specifies overall antenna performance and limitations of its size on gain. High Q implies high storage of reactive energy in the near field, large current, large ohmic losses and narrow bandwidth [2]. Q is a more fundamental quantity defined in terms of antenna fields and its relationship with gain and radiation efficiency has been derived [3]. Due to its importance, a number of researchers have used different techniques to derive expression for the calculation of Q. This paper considers the calculation of Q for some designed small antennas based on some already derived formulae and those calculated from measured bandwidth of some simulated electrically small antennas. The results will be compared to deduce their correlation with each other and consequently suggest the expression that is mostly suitably for effective electrically small antenna design.

2.0 Limits on Quality Factor (Q) of an Antenna

*Q*uality factor is a parameter that describes how much power that transform as losses in the system. A high *Q* indicates a lower rate of energy loss relative to the stored energy represented mathematically as [4]

$$Q = \omega \times \frac{(total\ energy\ stored)}{average\ power\ loss\ in\ the\ load} = \frac{2\omega W}{P_{rad}} \qquad (2)$$

Corresponding author: Ikechiamaka F. N, E-mail: fosuagwu@fudutsinma.edu.ng, Tel.: +2348065631514

where ω is the radian frequency, p_{rad} is the radiated power and W is the time – averaged non propagating stored electric or magnetic energy.It is the ratio of the power stored in the reactive field to the radiated power. It is used to describe antenna as a resonator and quantifies the potential bandwidth of an antenna. Q relates to bandwidth thus [1];

$$Q = \frac{f_0}{f_2 - f_1} = \frac{f_0}{Bandwidth} \quad (3)$$

where $f_0$is the center(resonant) frequency , f_1 and f_2 are the frequencies when the center frequency has dropped 3 dB from the maximum value.Higher value implies a sharp resonance and narrow bandwidth [5]. The approximate bandwidth for an RLC (resistance, inductance and capacitance) circuit type in terms of Q is [6]

$$\text{BW} = \frac{S-1}{Q\sqrt{S}} \quad (4)$$

where S is equivalent to S: 1 VSWR and BW is the fractional bandwidth.

The minimum quality factor, Q, of an omnidirectional antenna and its volume was given by Chu for a linearly polarized antenna as [7]

$$\text{Q} = \frac{1+2(ka)}{(Ka)^3+[1+(Ka)^2]} \quad (5)$$

where$k = 2\pi/\lambda$ is the free space wave number and a is the radius of imaginary sphere enclosing the maximum dimension of the antenna. The expression in equation (5) is the approximate value of the lower limit on radiation Q of an electrically small antenna. Equation (5) was expanded by Mclean using field theory to derive the minimum attainable Qexpression for a linearly polarized antenna given as [8]

$$\text{Q} = \frac{1}{(Ka)^3} + \frac{1}{Ka} \quad (6)$$

Collin and Rothschild approached the derivation using circuit theory. By subtracting the energy associated with radiation from the total energy they obtained Q for the lowest spherical mode which is same as equation (6) [2]. Hansen and Collin extended the work of Collin and Rothschild by calculating the total energy stored in the sphere through integrating the inward complex pointing vector over the sphere surface; they obtained the expression of Q as [9]

$$Q = \frac{1}{\sqrt{2}ka} + \frac{3}{2(ka)^3} \quad (7)$$

Thal and Gustafsson also worked independently and arrived at different formulae for Q. Thal's equations for Q are [2]

$$Q = \frac{1.5}{(ka)^3} ka \to 0 \quad \text{(for TM mode)} \quad (8)$$

$$Q = \frac{1}{(ka)^3} ka \to 0 \quad \text{(for TE and TM mode)} \quad (9)$$

$$Q = \frac{3}{(ka)^3} ka \to 0 \quad \text{(for TE mode)} \quad (10)$$

On the other hand, Gustafsson's formula for Q is [3]

$$Q = \frac{G}{\eta}\frac{1}{2(ka)^3} = \frac{1.5}{(ka)^3} \quad (11)$$

where G is the gain and η is the efficiency of the antenna.

3.0 Materials and Method

Four electrically small antennas considered in this work include; Monopole (whip) antenna, Microstrip antenna, Planer Inverted-F Antenna (PIFA) with air substrate and PIFA with FR4 substrate. These antennas were designed based on transmission line model and simulated using High Frequency Structural Simulator. First, calculation of Q based on already derived expression was made, and then Q was also calculated using the measured bandwidth of the simulated antennas.

Calculation of Q for Practical Antennas

Q is calculated based on the length of the designed antenna ground plane L_g. The value of a used in obtaining Q is equal to $\frac{1}{2}L_g$. All calculations are made for frequency of $900MHz$.

For Monopole, the length of recent handsets which is about $100mm$ is adopted as length of antenna ground plane [10].

For $L_g = 100mm$, $a = \frac{1}{2}L_g = 50mm = 0.05m$ and

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 6\pi = 18.85m^{-1}$$

Substituting a and k into equation (5) gives Q_{Chu} for Monopole as

$$Q_C = \frac{1 + 2(18.85 \times 0.05)^2}{(18.85 \times 0.05)^3[1 + (18.85 \times 0.05)^2]} = 1.76$$

Q_{Mclean} obtained from equation 6 gives

$$Q_M = \frac{1}{18.85 \times 0.05} + \frac{1}{(18.85 \times 0.05)^3} = 2.26$$

Q_{HC} from Hansen and Collin expression (equation (7)) is

$$Q_{HC} = \frac{1}{\sqrt{2}(18.85 \times 0.05)} + \frac{3}{2(18.85 \times 0.05)^3} = 2.54$$

Q from Thal's TM, TE, TE and TM mode expressions in equations (8), (9) and (10) yield

$$Q_{Ttm} = \frac{1.5}{(18.85 \times 0.05)^3} = 1.79$$

$$Q_{Ttem} = \frac{1}{(18.85 \times 0.05)^3} = 1.20$$

$$Q_{Tte} = \frac{3}{(18.85 \times 0.05)^3} = 3.58$$

For planer antennas such as Microstrip and Planer Inverted –F Antennas, the length of ground plane (L_g) is given as [11]

$$L_g = 6h + l \qquad (12)$$

where h is the substrate thickness and l is the antenna patch length. The length of patch for Microstrip antenna is given as [1]

$$l = \frac{c}{2f_0\sqrt{\varepsilon_e}} - 2h \qquad (13)$$

where $\varepsilon_e \approx \frac{\varepsilon_r+1}{2}$ is the effective dielectric constant of the substrate, ε_r is the intrinsic dielectric constant of the substrate and c is the speed of light. The patch length for Microstrip antenna is calculated by substituting the following data into equation (17); $\varepsilon_r = 2.2$ Roger Duroid Substrate, $f_0 = 900MHz$, $c = 3.0 \times 10^8 m/s$ and $h = 2mm = 0.002m$.

$$l = \frac{3.0 \times 10^8}{2 \times 9.0 \times 10^8\sqrt{\frac{3.2}{2}}} - 2 \times 0.002 = 0.128m$$

$$L_g = (6 \times 0.002) + 0.128 = 0.14m$$

$$\therefore a = \frac{L_g}{2} = 0.07m$$

Substituting for $\boldsymbol{k}$ and $\boldsymbol{a}$ into equations (5), (6). (7), (8), (9) and (10), the Q for Microstrip antenna obtained are presented in Table 1;

Table 1: Calculated Values of Q for Microstrip Antenna

Q_C	Q_M	Q_{HC}	Q_{Ttm}	Q_{Ttem}	Q_{Tte}
0.89	1.19	1.19	0.65	0.44	1.31

Length of patch for PIFA is obtained from [12]

$$l_1 = \frac{c}{6f_0\sqrt{\epsilon_e}} \qquad (14)$$

$l_1 = \frac{3\times10^8}{6\times9\times10^8} = 0.056m$ (for air substrate with $\epsilon_r = 1$)

$l_1 = \frac{3\times10^8}{6\times9\times10^8\sqrt{\frac{4.4+1}{2}}} = 0.034m$ (for FR4 substrate with $\epsilon_r = 4.4$)

L_g for PIFA obtained using equation (12) is given as

$L_g = 6 \times 0.002m + 0.056m = 0.068m$ (for air substrate)

$L_g = 6 \times 0.002m + 0.034m = 0.046m$ (for FR4 substrate)

Hence, $a = 0.034m$ and $a = 0.23m$ for air and FR4 substrates respectively. Using these values of a, the set of values of Q obtained for PIFA is presented in Table 2

Table 2: Calculated values of Q for PIFA with different substrates

$\boldsymbol{Q}$	PIFA (air)	PIFA(FR4)
$\boldsymbol{Q_C}$	4.91	14.22
$\boldsymbol{Q_M}$	5.36	14.58
$\boldsymbol{Q_{HC}}$	6.80	20.04
$\boldsymbol{Q_{Ttm}}$	5.70	18.41
$\boldsymbol{Q_{Ttem}}$	3.80	12.27
$\boldsymbol{Q_{Tte}}$	11.40	36.81

The designed Monopole, Microstrip and PIFA (FR4) were also simulated using High Frequency Simulator Software (HFSS). The measured fractional bandwidth (BW_f) and Voltage standing wave ratio (S) were substituted into equation (5) to obtain the simulated result of Q as presented in Table 3.

Table 3: Simulated values of Q

Antenna	BW_f	VSWR (S)dB	Q
Monopole	0.2	2.0	3.50
Microstrip	0.022	2.2	36.78
PIFA (FR4)	0.022	2.0	32.14

The summary of results of Q for Monopole antenna, Microstrip antenna, PIFA with air substrate and PIFA with FR4 substrates is shown in Table 4 and the plot of calculated Q against antenna size is shown in Figure 1.

Table 4: Summary of Calculated Values of Q for Some Designed Antennas Using Different expressions.

Minimum Q	Monopole $a = 0.05m$	Microstrip $a = 0.07m$	PIFA(air) $a = 0.034m$	PIFA(FR4) $a = 0.023m$
Chu	1.76	0.89	4.91	14.22
Mclean	2.26	1.19	5.36	14.58
Hansen and Collin	2.54	1.19	6.80	20.04
Thal (TM mode)	1.79	0.65	5.70	18.41
Thal (TM and TE modes)	1.20	0.44	3.80	12.27
Thal (TE mode)	3.58	1.31	11.40	36.81

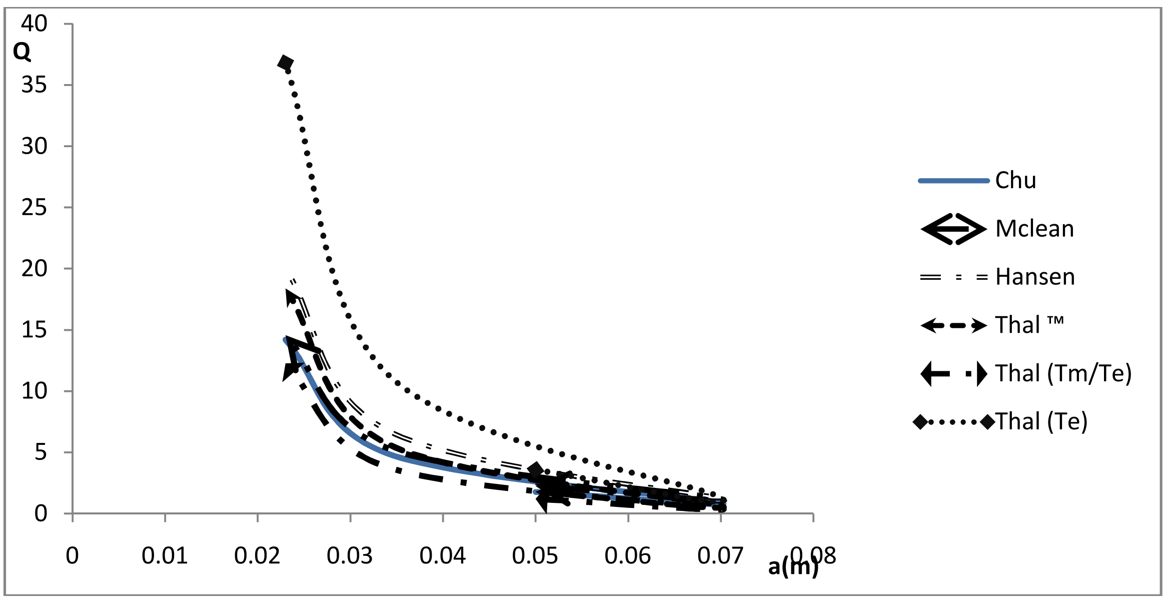

Fig. 1: Plot of Q from different expressions against $a(m)$ for ESAs.

4.0 Discussion of Results

The calculated Q in the Table 4 reveals that at higher values of antenna size (a), such as those of Monopole and Microstrip, Mclean expression and that by Hansen and Collin gave identical results of 2.26 and 2.54 respectively for monopole antenna and 1.19 for Microstrip. But the margin widens as antenna size gets smaller. The plot shows that though the trend in all the formulae is the same, Q differs significantly as a approaches zero.

Suffice it to say that that when the size of the antenna gets smaller, the value for Q obtained in practice (Table 3) is far above most of the calculated values of Q (Table 4). This is revealed by the values of Q for Monopole and PIFA of antenna size $\boldsymbol{a}$ of 0.05m and 0.023m respectively designed and simulated using HFSS. Q obtained using equation (2) for simulated Monopole antenna with operating frequency of 900MHz and bandwidth of 180MHz (impedance bandwidth of 0.2) is 3.5 and that obtained for simulated PIFA of bandwidth 20MHz (impedance bandwidth of 0.022) is 32.14 (Table 3). On the other hand, Q from Thal's TE mode expression are 3.52 and 36.81 for Monopole and PIFA respectively. From these results it can be seen that Thal's (TE) mode expression gave values that are closest to the Q values from the simulated Monopole and PIFA antennas. The difference in values of Q calculated using the derived expressions and that obtained from simulated antennas further proves that the calculated values of Q are the minimum (lower band) Q and not the exact. Table 3 also reveals that bandwidth is inversely proportional to quality factor.

5.0 Conclusion and Recommendation

All the expressions used in the calculation of Q gave the minimum value of Q and by inference the maximum value of fractional bandwidth. However in practice, antennas designed have higher values of Q (low fractional bandwidth) than predicted by all but one of the expressions. It is observed that as antennas get smaller, Q gets larger; therefore, the quality factor of electrically small antenna is limited by its size.

The plot of Q of some practical antennas clearly shows that all the results are in tandem as the value of $\boldsymbol{a}$ increases but diverges significantly with smaller values of $\boldsymbol{a}$. In this 21st century when miniaturization is in high demand, more work needs to be done to derive an equation that will be invariant with size of $\boldsymbol{a}$. As research is ongoing I equally recommend the use of Mclean expression for calculation of minimum Q which is the same as the expression obtained by Collin and Rothschild as this gives minimum values of Q that are between two extrema (especially as $\boldsymbol{a}$ goes to zero) and the use of Thal's (TE mode) expression for predicting the exact value.

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