Journal of the Nigerian Association of Mathematical Physics
Volume34,(March, 2016), pp 85 – 98
© J. of NAMP

On Modeling and Estimating Volatility in Financial Assets Using ARCH and GARCH Models

[1]F. Ewere and [2]E. O. Iduh

[1,2]Department of Mathematics, University of Benin, Benin City, Nigeria.

Abstract

This paper focuses on the estimation of volatility using Autoregressive Conditional Heteroscedastic (ARCH) and Generalized ARCH (GARCH) models. Volatility clustering which is an important feature of most financial assets is examined. The closing prices of stocks from the period January 2nd to December 31st 2012 of four major companies: First Bank of Nigeria PLC (FBN), Guarantee Trust Bank (GTB), Unilever Nigeria PLC (UNIL) and Nestle Foods PLC (NEST) was used in this study. Using FBN as the proxy endogenous variable, we found that the ARCH term and the closing prices of stock returns from UNIL are major factors responsible for the volatility in the stock returns of FBN for the period under consideration.

Keywords: Volatility, Volatility Clustering, ARCH model, GARCH model, endogenous variable, exogenous variable

1.0 Introduction

Data in which the variance of the error terms are not equal, in which error term may reasonably be expected to be larger for some point or ranges of the data than others are said to suffer from heteroscedaticity. ARCH and Generalized ARCH (GARCH) models treat heteroscedasticity as a variance to be modeled and these models have been widely used in financial time series analysis. Volatility refers to the spread of all likely outcomes of an uncertain variable. Statistically, volatility is often measured as the sample standard deviation. Sometimes, variance σ^2 is used also as a volatility measure. An important feature of any series of financial asset returns that provides a motivation for the ARCH class of models is known as "volatility clustering" or "volatility pooling". Volatility clustering describes the tendency of large changes in asset price (of either sign) to follow large changes and small changes (of either sign) to follow small changes. That is to say, the current level of volatility tends to be positively correlated with its level during the immediately preceding periods.

ARCH model was first introduced in [1] and it has been used in asset pricing to develop volatility test. A rigorous study of the behavior of speculative prices was first conducted in [2]. There was then a period of long silence until Mandelbrot [3-5] revived the interest in the time series properties of asset prices with his theory that 'random variables with an infinite population variance are indispensable for a workable description of price changes' (cf [4], p. 421). Prior to the introduction of ARCH model, researchers were very much aware of change in variable but used only informal procedure to take account of this. It has been argued in [6] that "it is both logically inconsistent and statistically inefficient to use volatility measure that are based on the assumption of constant volatility over some period when the resulting series moves through time".

2.0 Methodology

A basic question here is: How could volatility clustering be parameterized (modeled)? One approach is to use an ARCH model. To understand how the model works, a definition of the conditional variance of a random variable u_t is required. The conditional variance u_t may be denoted as σ_t^2 and is written as

$$\sigma_t^2 = \text{var}\left(u_t \middle| u_{t-1}, u_{t-2.}\ldots \right) = \mathrm{E}\left[\left(u_t - \mathrm{E}(u_t)^2 \right) \middle| u_{t-1}, u_{t-2}\ldots \right] \qquad (1)$$

Corresponding author: F. Ewere, E-mail:friday.ewere@uniben.edu, Tel.: +2348058504893

It is usually assumed that $\mathrm{E}(u_t) = 0$, so that

$$\sigma_t^2 = \text{var}\left(u_t | u_{t-1}, u_{t-2.} \ldots \right) = \mathrm{E}\left[u_t^2 | u_{t-1}, u_{t-2} \ldots \right] \quad (2)$$

Equation (2) states that the conditional variance of a zero mean normally distributed random variable u_t is equal to the conditional expected value of the square of u_t. Under the ARCH model, the 'autocorrelation in volatility is modeled by allowing the conditional variance of the error term σ_t^2 to depend on the immediately previous value of the squared error and we write

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} \quad (3)$$

The model described above is known as ARCH(1) since the conditional variance depends on only one lagged squared error. Observe that equation (3) is only a partial model, since nothing has been mentioned about the conditional mean. Under ARCH, the conditional mean equation (which describes how the dependent variable y_t, varies over time) could take almost any form that the researcher wishes. One example of a full model would be

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + u_t, \; u_t \sim N(0, \sigma_t^2) \quad (4)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} \quad (5)$$

The model given by Equations (4) and (5) would easily be extended to the general case where the error variance depends on the q lags of squared errors, which would be known as an ARCH(q) model and written as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} + \alpha_2 u^2_{t-2} + \ldots + \alpha_q u^2_{t-q} \quad (6)$$

The GARCH model which was developed independently in [7] and [8] allows the conditional variance to be dependent upon previous own lags, so that the conditional variance equation in the simplest case is now

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} + \beta \sigma^2_{t-1} \quad (7)$$

Equation (7) is a GARCH (1, 1) model where σ_t^2 is known as the conditional variance since it is a one period ahead estimate for the variance calculated based on any past information thought relevant. The GARCH (1,1) model can be extended to a GARCH(p, q) formulation where the current conditional variance is parameterized to depend upon q lags of the squared mean and p lags of the conditional variance and we write

$$\sigma_t^2 = \alpha_o + \alpha_1 u^2_{t-1} + \alpha_2 u^2_{t-2} + \ldots + \alpha_q u^2_{t-q} + \beta_1 \sigma^2_{t-1} + \beta_2 \sigma^2_{t-2} + \ldots \beta_p \sigma^2_{t-p} \quad (8)$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u^2_{t-i} + \sum_{j=1}^{p} \beta_j \sigma^2_{t-j} \quad (9)$$

2.1 Estimation of ARCH and GARCH Models

The steps involved in estimating ARCH or GARCH models are:

(i) Specify the appropriate equation for the mean and the variance

(ii) Specify the Log-Likelihood Function (LLF) to maximize under a normality assumption for the disturbances.

(iii) Maximize the function and generate parameter values and construct their errors.

3.0 Datasets and Analysis

The data consists of closing prices of stock returns (Monday to Friday trading periods) of FBN, GTB, UNIL and NEST for the period January 2nd to December 31st 2012 available at the website of Central Securities Clearing System Nigeria Plc, see [9]. In order to make the data continuous, periods where data was not available either due to public holidays or other events were assumed to be the same price as the last period closing price before the break. The datasets were found to be non–stationary when tested for unit root at the levels using Augmented-Dickey Fuller (ADF) method available in the software EViews. The data was further tested for unit root at first difference using Augmented Dickey-Fuller method and the was found to be stationary at first difference. If it is stationary at first difference, then it will also be stationary at second, third and other difference. The model is estimated using this stationary data of FBN, GTB, UNIL and NEST which we denote as DFBN, DGTB, DUNIL and DNEST respectively. The total number of observations is 261.

3.1 Garch (1,1) Model Specification Using Eviews

In this section, we specify the appropriate model to be used for the data estimation. This model is divided into two: the mean equation and the variance equation. Using the stationary data: DFBN, DGTB, DUNIL and DNEST, we have the following:

The Mean Equation:

$$DFBN = C_1 + C_2 lag_1 DFBN + C_3 DGTB + e \qquad (10)$$

The residuals (e) are extracted from the model and must be tested for ARCH effect. If there is an ARCH effect and clustering volatility on the residuals after computing equation (10), it is then suggestive that the residual or error term is conditionally heteroscedastic and can be represented by ARCH and GARCH model.

The Variance Equation:

$$H_t = C_4 + C_5 H_{t-1} + C_6 e^2{}_{t-1} + C_7 DUNIL + C_8 DNEST \qquad (10)$$

The residual (e) obtained from equation (10) is used in deriving equation (11)

H_t is the variance of the residual (error term) obtained from equation (10). It is also known as current's day variance or volatility of FBN. H_{t-1} is the previous day's residual variance or volatility of FBN stock returns. It is known as the GARCH term. $e^2{}_{t-1}$ is the previous period's squared residual derived from equation (10) and it is also known as previous day's stock return information about volatility. It is the ARCH term.

Equation (11) is a GARCH (1, 1) model as it has one ARCH ($e^2{}_{t-1}$) and one GARCH term (H_{t-1}). The mean and variance equations given by (10) and (11) are estimated simultaneously using [10].

4.0 Results and Discussion

4.1 Arch Testing

Hypothesis Formulation:

Null: There is no ARCH effect

Alternative: There is ARCH effect

The decision rule is to reject the null hypothesis if the p-value of the observed R squared is less than 5% and to accept the alternative that there is ARCH effect on the residuals.

The plot of the residual for clustering volatility for the period of January 2nd to December 31st (261 days) 2012 for FBN is given in Figure 1

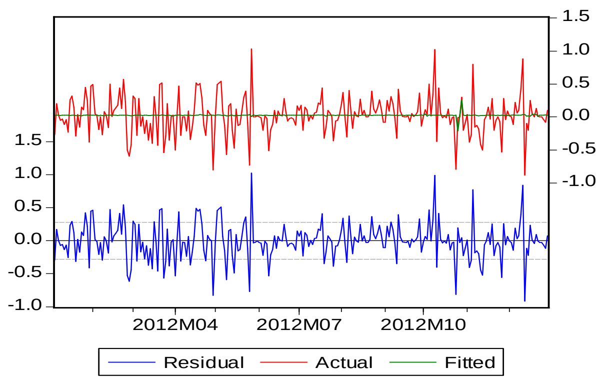


Figure 1: Plot of the residual for clustering volatility for FBN

Figure 1 shows that a period of low or small volatility is followed by another period of period of low or small volatility and period of high or large volatility is followed by another period of high or large volatility. We can therefore conclude that the residuals of FBN stock return estimated using equation (10) has a clustering volatility for the period of January to December 2012. Next, the test for ARCH effect on the residuals is conducted. The result of this test is given in Table 1:

Table 1: Test for ARCH effect on the residuals

ARCH Test:

F-statistic	14.24023	Prob. F(1,256)	0.000200
Obs*R-squared	13.59524	Prob. Chi-Square(1)	0.000227

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/06/13 Time: 06:05
Sample (adjusted): 1/05/2012 12/31/2012
Included observations: 258 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.058617	0.010201	5.746398	0.0000
RESID^2(-1)	0.229651	0.060857	3.773623	0.0002

R-squared	0.052695	Mean dependent var	0.076185
Adjusted R-squared	0.048994	S.D. dependent var	0.149497
S.E. of regression	0.145789	Akaike info criterion	-1.005595
Sum squared resid	5.441114	Schwarz criterion	-0.978052
Log likelihood	131.7217	F-statistic	14.24023
Durbin-Watson stat	1.985881	Prob(F-statistic)	0.000200

Here, the value of the observed R squared is13.59524. The P-value is 0.000227. Since the P-value for this model is less than 5%, we reject the null hypothesis and conclude that the residuals of FBN stock return for the period of January to December 2012 has ARCH effects. Now that the residuals of equation (10) has a clustering volatility and ARCH effects, we now evaluate the GARCH(1, 1) model using the residuals of equations (10) and (11)

4.2 Estimating the GARCH (1, 1) Model: Variance Equation

The GARCH model is estimated using three underlying distributions. They are:
(i) Normal Gaussian Distribution (ii) Student's t with fixed degrees of freedom and
(iii) Generalised Error Distribution (GED)

In the GARCH (1,1) model, under the variance equation (11), there will be one ARCH term e^2_{t-1} represented as RESID(-1)^2 and one GARCH term H_{t-1} represented as GARCH(-1) in EViews computation. The coefficients of either the ARCH or GARCH term or the exogenous variables are said to be significant or contribute to the volatility of FBN stock return if and only if the value of the p-value corresponding to these coefficients are less than 5%; otherwise they are not significant.
Now, using the Normal Gaussian distribution, the result of the GARCH(1,1) model is given in Table 2

Table 2: GARCH(1,1) model using the Normal Gaussian distribution

Dependent Variable: DFBN
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 02/26/14 Time: 09:17
Sample (adjusted): 1/04/2012 12/31/2012
Included observations: 259 after adjustments
Convergence achieved after 40 iterations
Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1) + C(8)*DNEST + C(9)*DUNIL

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.661265	0.309233	2.138406	0.0325
C	-0.156772	0.076639	-2.045580	0.0408
LAG1DFBN	-0.008619	0.070741	-0.121833	0.9030
DGTB	0.018361	0.024975	0.735168	0.4622
	Variance Equation			
C	0.048678	0.008547	5.695689	0.0000
RESID(-1)^2	0.409938	0.110782	3.700415	0.0002
GARCH(-1)	-0.009233	0.112582	-0.082014	0.9346
DNEST	-0.000460	0.000419	-1.096945	0.2727
DUNIL	0.009261	0.008359	1.107903	0.2679
R-squared	-0.011128	Mean dependent var		0.025676
Adjusted R-squared	-0.043485	S.D. dependent var		0.277403
S.E. of regression	0.283370	Akaike info criterion		0.227207
Sum squared resid	20.07470	Schwarz criterion		0.350803
Log likelihood	-20.42327	Durbin-Watson stat		1.952723

Under Normal Gaussian distribution, the standard deviation or volatility {@SQRT(GARCH)} of First Bank is 0.661265 and it is very significant because the p-value is less than 5%. For the one period lag of the first difference stationary data of closing prices of stock returns of FBN (LAG1DFBN), its coefficient is negatively related to FBN and not significant as its p-value is more than 5%.This means that previous price return of FBN does not affect the current stock return of FBN. The coefficient of DGTB is positively related to the current price of FBN stock return but not significant. The value of the coefficient of the ARCH term RESID (-1) ^2 is 0.409938 and the corresponding p-value is 0.0002. The coefficient of the ARCH term is positive and significant meaning that the previous day's FBN stock return information (that is ARCH) can influence today's stock return volatility.

For this distribution, the coefficient of GARCH is -0.009233 and the corresponding p-valueis 0.9346. The coefficient of GARCH is negative and not significant. It means that previous day's FBN stock return volatility cannot influence today's FBN stock volatility. Also, the coefficients of the exogenous variables (DUNIL and DNEST) are not significant. This means that volatility in the stock return of UNIL and NEST does not influence the volatility in stock return of FBN. Hence, the volatility of FBN stock is influenced by its own internal shock, that is, within information or market events that takes place in the company and not the external shock of other companies like UNIL and NEST. Again, the model is considered using another distribution- Student's t with fixed degrees of freedom. Table 3 gives the result of the GARCH (1,1) model using this distribution.

Table 3: GARCH (1,1) model using Student's t with fixed degrees of freedom.

Dependent Variable: DFBN
Method: ML - ARCH (Marquardt) - Student's t distribution
Date: 02/26/14 Time: 09:19
Sample (adjusted): 1/04/2012 12/31/2012
Included observations: 259 after adjustments
Convergence achieved after 60 iterations
Variance backcast: ON
t-distribution degree of freedom parameter fixed at 10
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1) + C(8)*DNEST
+ C(9)*DUNIL

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.435075	0.116150	3.745816	0.0002
C	-0.089515	0.026261	-3.408596	0.0007
LAG1DFBN	-0.017258	0.067044	-0.257418	0.7969
DGTB	0.016471	0.022231	0.740915	0.4587
	Variance Equation			
C	0.043895	0.007264	6.042512	0.0000
RESID(-1)^2	0.508436	0.119296	4.261958	0.0000
GARCH(-1)	-0.044424	0.076165	-0.583253	0.5597
DNEST	-0.000658	0.000398	-1.652820	0.0984
DUNIL	0.021457	0.006235	3.441296	0.0006
R-squared	-0.003116	Mean dependent var		0.025676
Adjusted R-squared	-0.035216	S.D. dependent var		0.277403
S.E. of regression	0.282245	Akaike info criterion		0.168709
Sum squared resid	19.91563	Schwarz criterion		0.292306
Log likelihood	-12.84787	Durbin-Watson stat		1.953601

Table 3 shows that the standard deviation which is the volatility of FBN given as @SQRT (GARCH) is 0.435075 and is significant. Also, LAG1DFBN is negatively related to FBN stock return. The coefficient of DGTB is positively related to the volatility of FBN but not significant. The ARCH term here is also significant and has a major impact on the volatility of FBN stock return. The GARCH term is negatively related to the volatility and is not significant. Also, DNEST is negatively related to the volatility of FBN but not significant. The exogenous variable DUNIL is positively related to the volatility of FBN and is very significant meaning it has affected FBN stock return. We also estimate the GARCH(1,1) model using the Generalized Error Distribution (GED) with fixed parameter. Table 4 provides the result.

Table 4: GARCH(1,1) model using the Generalized Error Distribution (GED)

Dependent Variable: DFBN
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)
Date: 02/26/14 Time: 09:20
Sample (adjusted): 1/04/2012 12/31/2012
Included observations: 259 after adjustments
Failure to improve Likelihood after 11 iterations
Variance backcast: ON
GARCH = C(5) + C(6)*RESID(-1)^2 + C(7)*GARCH(-1) + C(8)*DNEST
+ C(9)*DUNIL

	Coefficient	Std. Error	z-Statistic	Prob.
@SQRT(GARCH)	0.376925	0.147414	2.556922	0.0106
C	-0.080144	0.034651	-2.312889	0.0207
LAG1DFBN	0.030237	0.043405	0.696637	0.4860
DGTB	0.016091	0.004946	3.253485	0.0011
	Variance Equation			
C	0.056908	0.014841	3.834602	0.0001
RESID(-1)^2	0.567534	0.241664	2.348448	0.0189
GARCH(-1)	-0.055911	0.074346	-0.752036	0.4520
DNEST	-0.000647	0.000862	-0.749711	0.4534
DUNIL	0.027580	0.009838	2.803373	0.0051
GED PARAMETER	0.812895	0.108873	7.466431	0.0000
R-squared	-0.001965	Mean dependent var		0.025676
Adjusted R-squared	-0.038181	S.D. dependent var		0.277403
S.E. of regression	0.282649	Akaike info criterion		0.065607
Sum squared resid	19.89278	Schwarz criterion		0.202937
Log likelihood	1.503831	Durbin-Watson stat		2.036379

From Table 4, the coefficient of the standard deviation or volatility is positive and significant. The endogenous variable LAG1DFBN is positively related to the volatility of current FBN stock return but is not significant. Also, DGTB is positively related to FBN stock return and is significant. It means that current price in GTB stock affects the current price in First Bank stock return. The ARCH term is also found to be significant here while the GARCH term is not significant. Also, the exogenous variable DNEST is negatively related to the volatility of FBN stock return and is not significant while DUNIL is positively related and significant.
The results of estimating the model using the three distributions clearly indicates that volatility in FBN is largely influence by its own shock (the ARCH term) and the exogenous variable DUNIL.

4.3 Model Selection

A crucial question here is: Which of these distributions best fit the model? A GARCH model will be most appropriate when:
(i) there is no serial correlation
(ii) the residuals are normally distributed and (iii) there is no ARCH effect.
When these conditions are met by any of the distributions listed in section 4.2, then the model is said to be the best. Alternatively, we can choose the best model by inspecting the value of the Akaike Information Criterion (AIC) and Schwarz Criterion (SC) of the distributions. The distribution with the highest value of AIC and SC is selected as the best. Therefore, the following assumptions or hypotheses must be fulfilled:

(a) Null: There is no serial correlation in the residuals or error term
Alternative: There is serial correlation in the residuals

To carry out this test, correlogram square residuals (Q statistics) test can be performed using at least 20 lags. The rule of the thumb is to accept the null hypothesis if most of the p-values within the number of lags used are more than 5%, otherwise, reject the null hypothesis

(b) Null: Residuals are normally distributed
Alternative: Residuals are not normally distributed

Jacque-Bera statistics is used to conduct this test. The decision rule here is to accept the null hypothesis if the p-value of the Jacque-Bera statistics is more than 5% otherwise reject the null hypothesis

(c) Null: There is no ARCH effect
Alternative: There is ARCH effect

ARCH test would be used to perform this. The decision rule here is to accept the null hypothesis if the p-value of the Observed R squared is more than 5% otherwise reject the null hypothesis. Here, all null hypotheses are desirable to achieve 'best model' status. The estimation of the model under the three distributions is thus:

Under the Normal-Gaussian distribution, results of the tests for 'best model' are given in Tables 5, 6 and Figure 2.

Table 5: Testing for serial correlation under the Normal-Gaussian Distribution

Date: 07/16/13 Time: 02:24

Sample: 1/04/2012 12/31/2012

Included observations: 259

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
.|. |	.|. |	1	-0.010	-0.010	0.0242	0.877
.|. |	.|. |	2	0.030	0.030	0.2627	0.877
.|. |	.|. |	3	0.020	0.021	0.3702	0.946
.|. |	.|. |	4	-0.014	-0.014	0.4214	0.981
.|. |	.|. |	5	-0.046	-0.048	0.9961	0.963
*|. |	*|. |	6	-0.101	-0.102	3.7340	0.713
.|. |	.|. |	7	0.025	0.026	3.8995	0.791
.|. |	.|. |	8	-0.051	-0.042	4.5891	0.800
.|* |	.|* |	9	0.158	0.161	11.301	0.256
.|. |	.|. |	10	-0.018	-0.020	11.391	0.328
.|** |	.|** |	11	0.220	0.214	24.612	0.010
.|. |	.|. |	12	0.008	-0.011	24.628	0.017
.|. |	.|. |	13	-0.028	-0.028	24.844	0.024
.|. |	.|. |	14	0.008	-0.007	24.862	0.036
.|. |	.|. |	15	-0.017	0.022	24.941	0.051
.|. |	.|. |	16	-0.007	-0.002	24.954	0.071
.|. |	.|. |	17	-0.028	0.033	25.173	0.091
.|. |	.|. |	18	0.060	0.023	26.173	0.096
.|* |	.|* |	19	0.083	0.116	28.107	0.081
.|** |	.|* |	20	0.228	0.178	42.759	0.002

From Table 5, most of the p-values within the number of lags chosen are more than 5%. Therefore we accept the null hypothesis and conclude that there is no serial correlation. Next, we determine if the residuals are normally distributed.

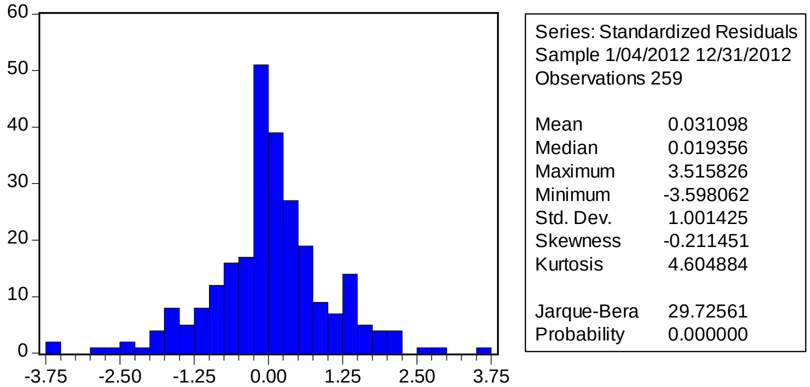


Figure 2: Normality Test

The p-value of the Jarque-Bera test is less than 5% and we reject the null hypothesis and conclude that the residuals are not normally distributed under the Normal-Gaussian distribution. We now carry out the ARCH test.

Table 6: ARCH test using the Normal-Gaussian distribution.

ARCH Test:

F-statistic	0.023615	Prob. F(1,256)	0.877991
Obs*R-squared	0.023797	Prob. Chi-Square(1)	0.877403

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 07/16/13 Time: 03:56
Sample (adjusted): 1/05/2012 12/31/2012
Included observations: 258 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.008741	0.133984	7.528834	0.0000
WGT_RESID^2(-1)	-0.009607	0.062520	-0.153670	0.8780

R-squared	0.000092	Mean dependent var	0.999102
Adjusted R-squared	-0.003814	S.D. dependent var	1.898097
S.E. of regression	1.901713	Akaike info criterion	4.131109
Sum squared resid	925.8275	Schwarz criterion	4.158652
Log likelihood	-530.9131	F-statistic	0.023615
Durbin-Watson stat	1.998069	Prob(F-statistic)	0.877991

The observed value of the p-value is 0.87740 which is greater than 5%. Hence, we do not reject the null hypothesis and conclude that there is no ARCH effect.

The tests using the Normal-Gaussian distribution shows that the residuals are not serially correlated and there are no ARCH effects which is an indication that the model is good. However, the residuals are not normally distributed. We repeat same tests but using the Student's t with fixed degrees of freedom distribution and the results are given in Tables 7, 8 and Figure 3.

Table 7:Testing for Serial Correlation
Date: 07/16/13 Time: 05:53
Sample: 1/04/2012 12/31/2012
Included observations: 259

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|. |</td><td>.|. |</td><td>1</td><td>-0.033</td><td>-0.033</td><td>0.2850</td><td>0.593</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>0.013</td><td>0.012</td><td>0.3293</td><td>0.848</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>0.020</td><td>0.021</td><td>0.4384</td><td>0.932</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>-0.013</td><td>-0.012</td><td>0.4842</td><td>0.975</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>-0.048</td><td>-0.050</td><td>1.1059</td><td>0.954</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>6</td><td>-0.096</td><td>-0.099</td><td>3.5484</td><td>0.738</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>7</td><td>0.038</td><td>0.033</td><td>3.9347</td><td>0.787</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>-0.052</td><td>-0.045</td><td>4.6506</td><td>0.794</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>9</td><td>0.148</td><td>0.149</td><td>10.590</td><td>0.305</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>-0.018</td><td>-0.016</td><td>10.682</td><td>0.383</td></tr>
<tr><td>.|** |</td><td>.|** |</td><td>11</td><td>0.229</td><td>0.228</td><td>24.991</td><td>0.009</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>12</td><td>0.019</td><td>0.015</td><td>25.086</td><td>0.014</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>13</td><td>-0.027</td><td>-0.016</td><td>25.289</td><td>0.021</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.019</td><td>0.004</td><td>25.384</td><td>0.031</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>15</td><td>-0.017</td><td>0.019</td><td>25.460</td><td>0.044</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>-0.007</td><td>-0.002</td><td>25.474</td><td>0.062</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>-0.030</td><td>0.030</td><td>25.722</td><td>0.080</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>18</td><td>0.051</td><td>0.015</td><td>26.456</td><td>0.090</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>19</td><td>0.080</td><td>0.115</td><td>28.264</td><td>0.078</td></tr>
<tr><td>.|** |</td><td>.|* |</td><td>20</td><td>0.207</td><td>0.169</td><td>40.425</td><td>0.004</td></tr>
</table>

Majority of the p-values here are more than 5% meaning that the residuals are not serially correlated. We now test for the normality of the residuals.

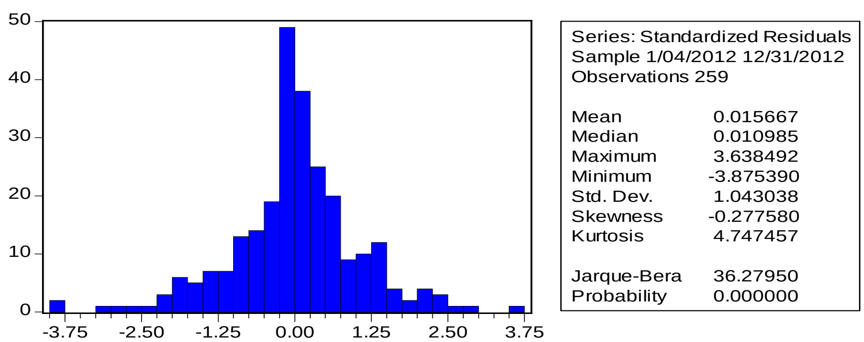


Figure 3: Normality of the residuals
The p-value of the Jacque-Bera statistics test is less than 5% and the conclusion is that the residuals are not normally distributed. The ARCH test is also conducted.

Table 8: Arch test using the Student's t with fixed degrees of freedom

ARCH Test:

F-statistic	0.278959	Prob. F(1,256)		0.597842
Obs*R-squared	0.280833	Prob. Chi-Square(1)		0.596156

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 07/16/13 Time: 06:20
Sample (adjusted): 1/05/2012 12/31/2012
Included observations: 258 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.118765	0.147597	7.579851	0.0000
WGT_RESID^2(-1)	-0.033004	0.062488	-0.528166	0.5978
R-squared	0.001088	Mean dependent var		1.082871
Adjusted R-squared	-0.002813	S.D. dependent var		2.101545
S.E. of regression	2.104500	Akaike info criterion		4.333754
Sum squared resid	1133.803	Schwarz criterion		4.361297
Log likelihood	-557.0543	F-statistic		0.278959
Durbin-Watson stat	1.997748	Prob(F-statistic)		0.597842

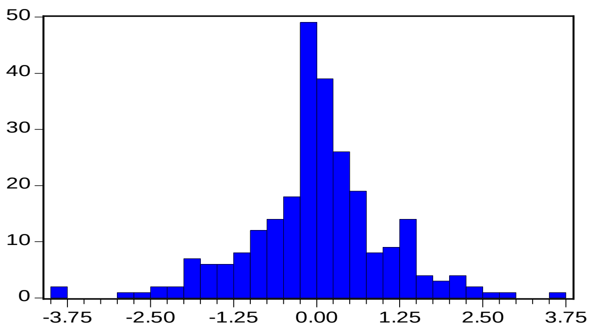
For this test, the p-value of the observed R square here is 0.596156. We do not reject the null hypothesis, meaning that there is no ARCH effect.

Like the case of the Normal Gaussian distribution, the residuals of the GARCH (1,1) model under the Student's t with fixed degrees of freedom distribution are not serially correlated and there are no ARCH effects which is desirable for the model. But, the residuals are not normally distributed. Again, we estimate the model using the Generalized Error Distribution (GED). The results are given in Tables 9, 10 and Figure 4.

Table 9: **Testing for serial correlation**
Date: 07/16/13 Time: 06:40
Sample: 1/04/2012 12/31/2012
Included observations: 259

<table>
<tr><th>Autocorrelation</th><th>Partial Correlation</th><th></th><th>AC</th><th>PAC</th><th>Q-Stat</th><th>Prob</th></tr>
<tr><td>.|. |</td><td>.|. |</td><td>1</td><td>-0.027</td><td>-0.027</td><td>0.1967</td><td>0.657</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>2</td><td>0.023</td><td>0.022</td><td>0.3325</td><td>0.847</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>3</td><td>0.019</td><td>0.020</td><td>0.4256</td><td>0.935</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>4</td><td>-0.014</td><td>-0.013</td><td>0.4765</td><td>0.976</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>5</td><td>-0.046</td><td>-0.048</td><td>1.0387</td><td>0.959</td></tr>
<tr><td>*|. |</td><td>*|. |</td><td>6</td><td>-0.096</td><td>-0.099</td><td>3.5202</td><td>0.741</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>7</td><td>0.033</td><td>0.030</td><td>3.8090</td><td>0.801</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>8</td><td>-0.053</td><td>-0.046</td><td>4.5678</td><td>0.803</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>9</td><td>0.155</td><td>0.156</td><td>11.056</td><td>0.272</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>10</td><td>-0.018</td><td>-0.016</td><td>11.149</td><td>0.346</td></tr>
<tr><td>.|** |</td><td>.|** |</td><td>11</td><td>0.226</td><td>0.222</td><td>25.046</td><td>0.009</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>12</td><td>0.014</td><td>0.006</td><td>25.097</td><td>0.014</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>13</td><td>-0.027</td><td>-0.022</td><td>25.301</td><td>0.021</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>14</td><td>0.016</td><td>0.002</td><td>25.373</td><td>0.031</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>15</td><td>-0.016</td><td>0.022</td><td>25.440</td><td>0.044</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>16</td><td>-0.007</td><td>-0.003</td><td>25.454</td><td>0.062</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>17</td><td>-0.029</td><td>0.031</td><td>25.686</td><td>0.080</td></tr>
<tr><td>.|. |</td><td>.|. |</td><td>18</td><td>0.058</td><td>0.021</td><td>26.616</td><td>0.086</td></tr>
<tr><td>.|* |</td><td>.|* |</td><td>19</td><td>0.079</td><td>0.114</td><td>28.365</td><td>0.077</td></tr>
<tr><td>.|** |</td><td>.|* |</td><td>20</td><td>0.216</td><td>0.173</td><td>41.564</td><td>0.003</td></tr>
</table>

As with the previous two distributions, the residuals are found not to be serially correlated because majority of its p-values are more than 5%. The Normality test of this model is now presented.



Series: Standardized Residuals	
Sample 1/04/2012 12/31/2012	
Observations 259	
Mean	0.018935
Median	0.009815
Maximum	3.607359
Minimum	-3.809961
Std. Dev.	1.036657
Skewness	-0.250487
Kurtosis	4.686373
Jarque-Bera	33.39836
Probability	0.000000

Figure 4: Normality Test
Under the GED, the p-value of the Jacque-Bera statistics is less than 5%. Like the previous distributions, the residuals of this model are not normally distributed. The
ARCH test is conducted.

Table 10: Arch test using the Generalized Error Distribution
ARCH Test:

F-statistic	0.192515	Prob. F(1,256)	0.661201
Obs*R-squared	0.193874	Prob. Chi-Square(1)	0.659712

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 07/16/13 Time: 07:07
Sample (adjusted): 1/05/2012 12/31/2012
Included observations: 258 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.099330	0.144863	7.588760	0.0000
WGT_RESID^2(-1)	-0.027422	0.062499	-0.438766	0.6612

R-squared	0.000751	Mean dependent var	1.069867
Adjusted R-squared	-0.003152	S.D. dependent var	2.058524
S.E. of regression	2.061766	Akaike info criterion	4.292725
Sum squared resid	1088.225	Schwarz criterion	4.320267
Log likelihood	-551.7615	F-statistic	0.192515
Durbin-Watson stat	1.997392	Prob(F-statistic)	0.661201

From the ARCH test, the p-value of the observed R squared is 0.659712. The null hypothesis cannot be rejected and hence there are no ARCH effects.
Results from estimating the model using the three distributions given in section 4.2 clearly indicates that the weakness of the model is the non-normality of the residuals. However, many researchers have suggested that non-normality may not be a serious problem as estimators are still consistent. The implication of this therefore is that estimating the model using any of the distributions listed in section 4.2 would be appropriate. Alternatively, estimating the model using the Normal Gaussian distribution could be considered to be the “best” using the AIC and the SC since it has the highest value for both AIC and SC (see Tables 2, 3 and 4).

5.0 Conclusion

In this paper, estimation of volatility in financial assets using ARCH and GARCH models have been presented. The study utilizes the daily stock prices from January 2nd to December 31st 2012 of FBN, GTB, UNIL and NEST. Results from the study shows that the ARCH term, which is the within factor in FBN (management of the bank, number of account holders, tangible and intangible assets and liabilities of the bank, level of information and technology among other factors) and the exogenous variable DUNIL are major factors affecting the volatility in the return of FBN stock for the period under consideration.

6.0 Reference

[1] Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of U.K. inflation. Econometrica, 50, 987-1008.

[2] Bachelier, L (1900). Theorie de la Speculation. Annales de L’Ecole Normale Superieure, 17, 21-86.

[3] Mandelbrot, B. (1963a). The variation of certain speculative prices. Journal of Business, 36, 394-419.

[4] Mandelbrot, B. (1963b). New methods in statistical economics. Journal of Political Economy, 71, 421-440.

[5] Mandelbrot, B. (1967). The variation of some other speculative prices. Journal of Business, 40, 393-413.

[6] Campbell, J. Y., Lo, A. W. and Mackinlay, A. C. (1997). The Econometrics of financial markets, Princeton, New Jersey: Princeton University Press.

[7] Bollerslev, T (1986). Generalized autoregressive conditional heteroscedasticity. Journal of Econometrics, 31, 307-327.

[8] Taylor, S. J. (1986). Modeling financial time series. John Wiley, Chichester, U.K.

[9] www.cscsnigeriaplc.com.

[10] Eviews (2000). Eviews Users guide, Version 5.0. Quantitative Microsoftware, U.S.A