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Hermitian and Skew-Hermitian Rhotrices

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Abstract

In this paper, Rhotrices with complex entries were considered and presented. Special cases of rhotrices with complex entries similar to the Hermitian and Skew-Hermitian matrices were also presented. Some properties of this rhotrices were established together with some examples.

Keywords: Hermitian rhotrix, Skew-Hermitian matrix, Skew-Hermitian rhotrix

1.0 Introduction

The concept of mathematical arrays that are in some way between two-dimensional vectors and 2×2 dimensional matrices were suggested by Atanassov and Shannon in [1]. As an extension to this idea, Ajibade in [2] introduced an object that lies between 2×2 dimensional matrices and 3×3 dimensional matrices called "rhotrix". The initial algebra and analysis of rhotrices were presented in [2]. Ajibade defined a rhotrix R as

$$R = \left\{ \left\langle \begin{matrix} & a & \\ b & c & d \\ & e & \end{matrix} \right\rangle : \mathrm{a, b, c, d, e} \in \mathbb{R} \right\} \quad (1)$$

He also defined the heart of a rhotrix as the perpendicular intersection of the two diagonals of a rhotrix R and is denoted by $h(R)$;

$$R = \left\langle \begin{matrix} & a & \\ b & h(R) & d \\ & e & \end{matrix} \right\rangle$$

Since then many results were presented on Rhotrices [3-9].

Complex rhotrix is a type of rhotrix whose entries comes from the set of complex numbers, any rhotrix with complex entries is called ***Complex Rhotrix***. Example is

$$R_3 = \left\langle \begin{matrix} & 1+i & \\ 2 & 1 & 3-2i \\ & i & \end{matrix} \right\rangle$$

1.1 Conjugate of a Complex Rhotrix

The rhotrix formed by replacing the elements of a rhotrix by their respective conjugate numbers is called the conjugate of that rhotrix. If R_n is an n-dimensional rhotrix, then the conjugate of R_n is denoted by $\overline{R_n}$. That is if $R_n = \langle a_{ij}, c_{lk} \rangle$, then $\overline{R_n} = \langle \overline{a_{ij}}, \overline{c_{lk}} \rangle$

Theorem 1.1

If R_n and P_n are two complex rhotrices and their conjugate rhotrices are $\overline{R_n}$ and $\overline{P_n}$ respectively, then

$$(i)\overline{\overline{R_n}} = R_n (ii)\overline{R_n + P_n} = \overline{R_n} + \overline{P_n} (iii)\overline{kR_n} = \overline{k}\,\overline{R_n} (iv)\overline{R_n P_n} = \overline{R_n}\,\overline{P_n}$$

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Proof

(i) let $R_n = \langle a_{ij}, c_{lk} \rangle$ (2)

then

$$\overline{R_n} = \langle \overline{a_{ij}}, \overline{c_{lk}} \rangle$$

where $\overline{a_{ij}}$ and $\overline{c_{lk}}$ are the complex conjugate of a_{ij} and c_{lk} respectively.

Also

$\overline{\overline{R_n}} = \langle a_{ij}, c_{lk} \rangle$ (3)

Comparing (2) and (3), we have $\overline{\overline{R_n}} = R_n$

(ii) let $R_n = \langle a_{ij}, c_{lk} \rangle$ and $P_n = \langle b_{ij}, d_{lk} \rangle$, then

$\overline{R_n} = \langle \overline{a_{ij}}, \overline{c_{lk}} \rangle$and $\overline{P_n} = \langle \overline{b_{ij}}, \overline{d_{lk}} \rangle$

The elements of $\overline{R_n + P_n} = \overline{\langle a_{ij} + b_{ij}, c_{lk} + d_{lk} \rangle} = \langle \overline{a_{ij}}, \overline{c_{lk}} \rangle + \langle \overline{b_{ij}}, \overline{d_{lk}} \rangle = \overline{R_n} + \overline{P_n}$

(iii)Let$R_n = \langle a_{ij}, c_{lk} \rangle$ be an n-dimensional rhotrix, and let k be any complex number

$$\langle \overline{kR_n} \rangle = \overline{\langle ka_{ij}, kc_{lk} \rangle} = \bar{k}\,\overline{R_n}$$

(iv) let $R_n = \langle a_{ij}, c_{lk} \rangle$ and $P_n = \langle b_{ij}, d_{lk} \rangle$ be two n-dimensional rhotrices, then

$$\overline{R_n P_n} = \langle \sum_{i_2 j_1 = 1}^{t} \overline{(a_{i_1 j_1} b_{i_2 j_2})}, \sum_{l_2 k_1 = 1}^{t-1} \overline{(c_{l_1 k_1} d_{l_2 k_2})} \rangle$$
$$= \langle \sum_{i_2 j_{1=1}}^{t} \overline{(a_{i_1 j_1} b_{i_2 j_2})}, \sum_{l_2 k_1 = 1}^{t-1} \overline{(c_{l_1 k_1} d_{l_2 k_2})} \rangle$$
$$= \overline{R_n}\,\overline{P_n}$$

Hence $\overline{R_n P_n} = \overline{R_n}\,\overline{P_n}$

1.2 Conjugate Transpose of a Complex Rhotrix

If R_n is an n-dimensional complex rhotrix, then to find the conjugate transpose ofR_n, we first calculate the complex conjugate of each entry ofR_n and then take the transpose of the complex conjugate of R_n. The conjugate transpose of a rhotrix R_n is denoted by R_n^{θ} or R^* and is given by

$$R_n^{\theta} = \overline{R_n}^{T}$$

Where the entries of $\overline{R_n}$ are the complex conjugates of the corresponding entries ofR_n.

For example

If $R_3 = \left\langle \begin{array}{ccc} & a_1 + ib_1 & \\ a_2 + ib_2 & x_1 + iy_1 & a_3 + ib_3 \\ & a_4 + ib_4 & \end{array} \right\rangle$, then$\overline{R_3} = \left\langle \begin{array}{ccc} & a_1 - ib_1 & \\ a_2 - ib_2 & x_1 - iy_1 & a_3 - ib_3 \\ & a_4 - ib_4 & \end{array} \right\rangle$, and

$$R_3^{\theta} = \langle \overline{R_n} \rangle^{T} = \left\langle \begin{array}{ccc} & a_1 - ib_1 & \\ a_3 - ib_3 & x_1 - iy_1 & a_2 - ib_2 \\ & a_4 - ib_4 & \end{array} \right\rangle$$

2.0 Hermitian Rhotrix

A square matrix A is Hermitian if $A = A^{\theta}$. Similarly, a special Rhotrix is proposed and called ***Hermitian Rhotrix***. It is equal to its own conjugate transpose. A rhotrix $R_n$is said to be Hermitian if

$$R_n = R_n^{\theta}$$

The necessary and sufficient conditions for a rhotrix R_n to be Hermitianare:

1. The major and minor matrices embedded in R_n are Hermitian matrices.
2. The entries on the main diagonal of R_n are real.
3. The entries a_{ij} and c_{lk} are the complex conjugate of the entries a_{ji} and c_{kl} respectively.

Theorem 2.1

Let R_n and P_n be two complex rhotrices with R_n^{θ} and P_n^{θ} as the conjugates transpose of R_n and P_n respectively, then

$$(i) {R_n^{\theta}}^{\theta} = R_n (ii) (R_n + P_n)^{\theta} = R_n^{\theta} + P_n^{\theta} (iii) k{R_n}^{\theta} = \bar{k} R_n^{\theta} (iv) {R_n P_n}^{\theta} = P_n^{\theta} R_n^{\theta}$$

Proof

$$(i) {R_n^{\theta}}^{\theta} = \left[\overline{\overline{R_n}^T}\right]^T = R_n$$
$$(ii)(R_n + P_n)^{\theta} = (\overline{R_n + P_n})^T = R_n^{\theta} + P_n^{\theta}$$
$$(iii)(kR_n)^{\theta} = \left(\overline{kR_n}\right)^T = \bar{k}\left(R_n^{\theta}\right)$$
$$(iv)(R_nP_n)^{\theta} = (\overline{R_nP_n})^T = \left(P_n^{\theta}\right)\left(R_n^{\theta}\right)$$

3.0 Skew-Hermitian Rhotrix

A rhotrix $R_n = \langle a_{ij}, c_{lk}\rangle$ is said to be a Skew-Hermitian rhotrix if $a_{ij} = -\overline{a_{ji}}$ for all i and j, and $c_{lk} = -\overline{c_{kl}}$ for all l and k.
The necessary and sufficient condition for a rhotrix R_n to be Skew-Hermitian is that $R_n^{\theta} = -R_n$

Theorem 3.1

For any n-dimensional rhotrix R_n, if $R_nR_n^{\theta} = I$, then $R_n^{\theta}R_n = I$.

Proof

Given that $R_nR_n^{\theta} = I$ and let P_n be another rhotrix such that

$$R_nP_n = P_nR_n = I \tag{4}$$

Now

$$P_n = P_nI = P_n\left(R_nR_n^{\theta}\right)$$
$$= (P_nR_n)R_n^{\theta}$$

$$P_n = R_n^{\theta} \tag{5}$$

Multiplying both sides of (5) by R_n, obtain

$P_nR_n = R_n^{\theta}R_n = I$ From (4)

4.0 Conclusion

The concept of complex rhotrix was introduced.Two special types of complex rhotrices, the Hermitian and Skew-Hermitian rhotrices were established withsome properties. It is pertinent to note that application of rhotrices in particular the proposed Hermitian and Skew-Hermitian rhotrices in Control Theory and Electrical Engineering is being studied and some of its results will appear soon.

5.0 References

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