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Transient MHD Couette Flow of a Partially Ionized Liquid Metal in Rectangular Channel with Induced Magnetic Field

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Abstract

The focus is on the influence of time development on a partially ionized liquid metal in the presence of an applied uniform magnetic field. The rectangular channel is assumed to have a conducting, moving top plate and a stationary conducting bottom and side walls. Numerical solution by an implicit finite-difference method of the Crank Nicolson type is employed. Results presented illustrate the influence of time variation and magnetic parameter for the two components of the fluid designated as ionized (i) and neutral (n)when the induced magnetic field is not negligible. The graphs presented for the velocities clearly show the effect of the parameters under consideration on the two components of the fluid.

Keywords: Transient MHD; Couette flow; liquid metal; partial ionization; finite difference.

1.0 Introduction

Magnetic fields influence are many and man-made flows. They are routinely used in industry to heat, pump , stir and levitate liquid metals. Liquid metal, such as mercury, liquid sodium and lithium is one of the simplest examples of an electrically conducting fluid. Liquid metals have been discovered to sustain high current density than the conventional brush technology which perhaps accounted for its wide application in the area of homopolar electrical machines and nuclear fusion technology. For instance, coolant used in nuclear reactors have high heat extraction rate, the high melting point and boiling point which eliminates the possibility of local boiling makes liquid metal more attractive to high temperature application in nuclear fusion reactors. The design of electromagnetic pump is another important application of liquid metal MHD. The pump consists of mutually perpendicular magnetic and electrical fields arranged normal to the axis of a duct. Filling the duct with liquid metal causes current to flow and the resulting Lorentz force provides the necessary pumping action.

The application of liquid metal in various aspect of technology are numerous hence the interest of many researchers. Some have considered liquid metal flow problems (with applied magnetic field) and boundary conditions containing combination of moving and fixed perfectly conducting or insulating walls while others focused on radiation, heat transfer, steady and time dependent flows, ionization and partial ionization. The concept of partial ionization in liquid metals MHD system and conductivity was popularized by Lielpeteris and Moreau [1], who carried out extensive research on the subject.The theory of partial ionization in liquid metal is centered upon one of the concepts: impurities of other metallic components within the system, establishment of pressure gradient within the flow pipes and generation of Eddy-current within the moving fluid system [2]. Thus in looking at partial ionization of liquid metals one is indirectly considering a two or multi-component system. There are quite considerable numbers of publications involving liquid metal MHD, Neil and Mohammed [3] undertook the study of fully developed, liquid metal, open channel flow in a nearly coplanar magnetic field. They were able to show that in completely insulated channels, there is a parabolic velocity jet at the free surface which is essentially a parallel layer restrained only by friction at the side walls. Furthermore, that in channels with nonzero electrical conductivity, the inviscid core flow is accompanied by boundary-layer velocity jets that can carry an appreciable portion of the flow and when the channel is at an oblique angle α to the magnetic field, the inviscid core flow can behave in two different fashions. Fredrick and Samuel [4] investigated the transient Magnetohydrodynamics liquid metal flow in a rectangular channel with a moving conducting wall. The equations for this study was formulated and solved for a liquid metal flowing in a rectangular channel which has a perfectly conducting moving top wall and a perfectly conducting stationary bottom wall in the presence of an applied external magnetic field aligned perpendicular to the conducing walls. Their results show that fast transient is

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believed to be associated with the propagation of Alfvan waves and a slow transient is the result of viscous and electrical diffusion. Gita, Samuel and Patricia [5] did a study on Magnetohydrodynamics Liquid-metal flows in a rectangular channel with an Acid Magnetic Field, a moving conducting wall and free surfaces. The rectangular Channel was assumed to have a homogeneous external (axial) magnetic field parallel to the moving, perfectly conducting, top wall and stationary, perfectly conducing bottom wall. The two stationary side walls were also perfect conductors and the gap between the moving wall and each side wall was an insulating free surface. Series solution was used to obtain the variables of interest. It was found out that since there was no pressure gradient, the flow along the channel was driven by the viscous effects of the moving wall and the Lorentz body force. Kishore [6], examined the effect of magnetic parameter M, Grashof number Gr, Eckert number E and time t on velocity and temperature fields. Alice and Mark [7] studied MHD heat transfer in elongated Rectangular ducts for liquid metal blankets. Their results show that as the aspect ratio increase (i.e. the ratio of the side wall to Hartman wall length) the peak velocity and side layer flow quantity increase, which leads to enhancement of the average heat transfer coefficient along the side layer. Also the heat transfer analysis indicate that non uniformity along the heated wall and the peak wall temperature, both increase as the aspect ratio increases due to smaller velocities in the corners and near the interface between the side layer and the core. They also discover that at peak velocity, elongated ducts always have higher peak temperatures. Kaddeche, Henry, Patelat and Hadid [8] investigated the stabilizing effects of a constant vertical magnetic field in a heated planar liquid metal layer. The steady shear flow driven in the bounded layer by the imposed horizontal temperature and gradient involved two types of instabilities, stationary transverse and oscillatory longitudinal instabilities. Their approximate analytical linear stability analysis shows that the vertical magnetic field has a great stabilizing effect on both types of instabilities as Gr, and exp ($Ha^2/21.6$). These results are of great importance for crystal growers as the vertical field is seen to delay the onset of instabilities.Solutions to problems in liquid metal MHD are achieved using different models and methods.Seung-Hasan and Seong-O [9] developed a 2 dimensional axisymmentry MHD analysis method based on Maxwell equation, finite difference method was used to obtain solution to some MHD parameters and Electromagnetic pump. Gautam [10] investigated the numerical simulation of wall bounded liquid metal flow in the presence of a magnetic Dipole. Gnaneswara [11]studied heat and mass transfer effects on unsteady MHD flow of a chemically reacting fluid, using the finite difference method of Crank Nicolson type. He observed that the presence of chemical reaction parameter lead to decrease in velocity field.Chukwuocha [2] undertook a study on partially ionized MHD liquid metal flow in rectangular channel, where he was able to establish among other results, based on steady flows, the velocity profiles of the two components of the fluid (the ionized and the neutral components) using an analytical series solution.However, the study did not consider ,the possibility of the flow being time-dependent. This was addressed by Musa and Chukwuocha [12] where they examined the transient MHD Couette flow of a partially ionized liquid metal in rectangular channel, where it was established that time variations has significant effect on the components of the fluids. However, this later study did not consider ,the effect of induced magnetic field, which is now being addressed here.

2.0 Formulation of the Problem.

The channel is filled with a liquid metal in a uniform magnetic field B_0 applied in the z-direction. The perfectly conducting top wall is assumed to move with constant velocity U_0. The moving wall extends from z = a to z = b (a ≤ z ≤ b) at y=1. The perfectly conducting bottom wall is stationary with velocity U = 0. The flow is assumed to be unsteady, incompressible, fully developed and laminar in the positive x-direction. The pressure gradient is in the x – direction, but assumed to be zero. The configuration of the problem assumes a rectangular coordinate system as shown below.

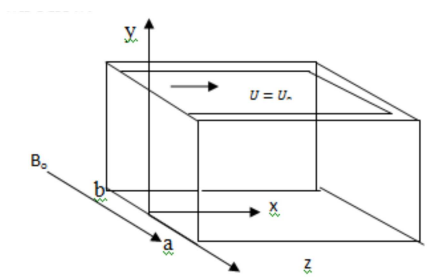

Figure 1: Typical rectangular channel configuration

For a conducting fluid, the formulation takes the conventional form of basic fluid mechanics equations (Navier Stokes) and the effect of applied magnetic field on the fluid is coupled into the momentum equation through the electromagnetic body force. The incompressible MHD equations governing the liquid-metal flow in vector form are;

Maxwell's equations

$$\vec{\nabla}' \times \vec{E} = -\frac{\partial B}{\partial t} \qquad (1)$$

$\vec{\nabla}' \times \vec{H} = \vec{j}$ (2)

$\vec{\nabla}'.\vec{j} = 0$ (3)

$\vec{\nabla}'.\vec{B} = 0$ (4)

Ohm's law Equation

$j = \rho(E + \vec{V} \times B)$ (5)

Continuity Equation

$\nabla'.V' = 0$ (6)

Momentum Equation

$\rho\left(\frac{\partial V'}{\partial t} + (\vec{V}'.\vec{\nabla})\vec{V}'\right) = -\vec{\nabla}p + \mu_f\vec{\nabla}^2V' - \rho\nabla\psi + \rho_e\vec{E} + \vec{j} \times \vec{B}$ (7)

Where

B^I= Magnetic flux density in Weber/m^2 or teslas.

H^I= Magnetic field in ampere/m

μ = Permeability in Henry/m

ρ_e= charge unit volume in coulomb/m^2

J = conduction current

V^I =Velocity vectors

ρ_0=fluid density

The fluid is considered to flow in the x-direction and the applied magnetic field is uniform and directed along the z-axis. Since $V \times B$ has a component in the z- direction, it is expected that current flow in the z–direction giving support to a magnetic field in the x – direction. Since the current need a return path, there must generally be y components of current as well. Thus, the liquid metal flow is in the x-direction ($V_x \neq 0, V_y = V_z = 0$) and the magnetic field is in the z- direction, $\vec{B} = (0,0,B)$, therefore $V \times B = -jV_xB_z$ which is in the y-direction. Equation (1) is transformed and equates to corresponding components of the transformed equation(2) noting that H_z is constant and was denoted by $H_z = \frac{B_0}{\mu}$. Using Ohm's law we have.

$\frac{\partial H_x}{\partial z} = \sigma E_y - \sigma B_0 U$ (8)

$-\frac{\partial H_x}{\partial y} = \sigma E_z$ (9)

Taking a partial derivatives of equations (8) and (9) with respect to z and y, subtracting the results gives

$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + \sigma B_0 \frac{\partial U}{\partial z} = \sigma \frac{\partial B_x}{\partial t}$ (10)

or $\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + \sigma B_0 \frac{\partial U}{\partial z} = \sigma\mu_0 \frac{\partial H_x}{\partial t}$ (11)

where $B = \mu_0 H$

Also the momentum equation in rectangular coordinates, noting that the problem is time dependent becomes;

$$\frac{\partial u'}{\partial t'} = \mu_f\left[\frac{\partial^2 u_i'}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right] + \beta_o \frac{\partial H_x'}{\partial z} \quad (12)$$

Following [13] and [2] equations (11) and (12) can be written for a two components liquid metal flows as

$$\frac{\partial^2 H_x'}{\partial y'^2} + \frac{\partial^2 H_x'}{\partial z'^2} + \sigma\beta_o \frac{\partial u_{i,n}'}{\partial z'} \mp f_c(u_n' - u_i') = \sigma\mu_o \frac{\partial H_x'}{\partial t'} \quad (13)$$

$$\rho\frac{\partial u_{i,n}'}{\partial t'} = \mu_f\left(\frac{\partial^2 u_{i,n}'}{\partial y'^2} + \frac{\partial^2 u_{i,n}}{\partial z'^2}\right) + \sigma B_o^2 u_{i,n} \pm f_c(u_n' - u_i') \quad (14)$$

Where $f_c(u_n - u_i)$ is the coupling frequency term.

In this study the following non-dimensional parameters are introduced

$$\left.\begin{array}{l} U = \frac{U'}{U_0},\ X = \frac{x'}{L},\ Y = \frac{y'}{L},\ Z = \frac{z'}{L}, \qquad t = \frac{t'}{L^2\rho/\mu_f}, \\ H_x = \frac{Hx'}{u_0\sqrt{\sigma u_f}},\ M = B_0 L\sqrt{\frac{\sigma}{\mu_f}},\ ,\ p_m = \frac{\sigma\mu_f u_o}{\rho}\ F_c = \frac{f_c' L^2}{\mu_f} \end{array}\right\} \quad (15)$$

Non dimensionalizing equations (13) and (14) using (15) and separating into the components, we have:

For ionized

$$P_m \frac{\partial H_x}{\partial t} = \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + M \frac{\partial u_i}{\partial z} - Fc(u_n - u_i) \tag{16}$$

$$\frac{\partial u_i}{\partial t} = \frac{\partial^2 u_i}{\partial y^2} + \frac{\partial^2 u_i}{\partial z^2} + M \frac{\partial H_x}{\partial z} + Fc(u_n - u_i) \tag{17}$$

and for Neutral

$$P_m \frac{\partial H_x}{\partial t} = \frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + M \frac{\partial u_n}{\partial z} + Fc(u_n - u_i) \tag{18}$$

$$\frac{\partial u_n}{\partial t} = \frac{\partial^2 u_n}{\partial y^2} + \frac{\partial^2 u_n}{\partial z^2} + M \frac{\partial H_x}{\partial z} - Fc(u_n - u_i) \tag{19}$$

These equations are solved subject to the following boundary conditions

$$\left.\begin{array}{l} t \geq 0 : u_{i,n} = 1 \quad at \quad y = 1 \\ t < 0 : u_{i,n} = 0 \quad at \quad y = 0 \\ \dfrac{\partial u_i}{\partial y} = 0, at 0 \leq y \leq 1 \\ \dfrac{\partial u_n}{\partial y} = 0, at 0 \leq y \leq 1 \\ \left.\begin{array}{l} \dfrac{\partial H_x}{\partial y} = 0 \quad at \quad y = 1 \\ \dfrac{\partial H_x}{\partial y} = 0 \quad at \quad y = 0 \end{array}\right\} 0 \leq z \leq 1 \\ H(y,z) = 0 \end{array}\right\} \tag{20}$$

3.0 Solution of the Problem

Equations (16) to (19) are to be solved for the ionized (u_i) and neutral (u_n) components subject to the boundary conditions (20). To achieved this a finite difference method of Crank Nicolson type is employed. The finite difference equations corresponding to equations (16) to (19) are as follows:

For ionized components we have,

$$P_m \left(\frac{H_{i,j}^{l+1} - H_{i,j}^{l}}{\Delta t} \right) = \frac{1}{2} \left(\frac{H_{i,j+1}^{l+1} - 2H_{i,j}^{l+1} + H_{i,j-1}^{l+1}}{\Delta y^2} + \frac{H_{i,j+1}^{l} - 2H_{i,j}^{l} + H_{i,j-1}^{l}}{\Delta y^2} \right)$$

$$+ \frac{1}{2} \left(\frac{H_{i+1,j}^{l+1} - 2H_{i,j}^{l+1} + H_{i,j-1}^{l+1}}{\Delta z^2} + \frac{H_{i+1,j}^{l} - 2H_{i,j}^{l} + H_{i-1,j}^{l}}{\Delta z^2} \right) \tag{21}$$

$$+ \frac{M}{2} \left(\frac{u_{i+1,j}^{l+1} - u_{i-1,j}^{l+1}}{2\Delta z} + \frac{u_{i+1,j}^{l} - u_{i-1,j}^{l}}{2\Delta z} \right) + Fc \left(\frac{u_{i,j}^{l+1} + u_{i,j}^{l}}{2} \right)$$

$$\frac{u_{i,j}^{l+1}-u_{i,j}^{l}}{\Delta t}=\frac{1}{2}\left(\frac{u_{i,j+1}^{l+1}-2u_{i,j}^{l+1}+u_{i,j-1}^{l+1}}{\Delta y^2}+\frac{u_{i,j+1}^{l}-2u_{i,j}^{l}+u_{i,j-1}^{l}}{\Delta y^2}\right)$$

$$+\frac{1}{2}\left(\frac{u_{i+1,j}^{l+1}-2u_{i,j}^{l+1}+u_{i-1,j}^{l+1}}{\Delta z^2}+\frac{u_{i+1,j}^{l}-2u_{i,j}^{l}+u_{i-1,j}^{l}}{\Delta z^2}\right) \tag{22}$$

$$+\frac{M}{2}\left(\frac{H_{i+1,j}^{l+1}-H_{i-1,j}^{l+1}}{2\Delta z}+\frac{H_{i+1,j}^{l}-H_{i-1,j}^{l}}{2\Delta z}\right)+Fc\left(\frac{u_{i,j}^{l+1}+u_{i,j}^{l}}{2}\right)$$

And for neutral components

$$P_m\left(\frac{H_{i,j}^{l+1}-H_{i,j}^{l}}{\Delta t}\right)=\frac{1}{2}\left(\frac{H_{i,j+1}^{l+1}-2H_{i,j}^{l+1}+H_{i,j+1}^{l+1}}{\Delta y^2}+\frac{H_{i,j+1}^{l}-2H_{i,j}^{l}+H_{i,j-1}^{l}}{\Delta y^2}\right)$$

$$+\frac{1}{2}\left(\frac{H_{i+1,j}^{l+1}-2H_{i,j}^{l+1}+H_{i-1,j}^{l+1}}{\Delta z^2}+\frac{H_{i+1,j}^{l}-2H_{i,j}^{l}+H_{i-1,j}^{l}}{\Delta z^2}\right) \tag{23}$$

$$+\frac{M}{2}\left(\frac{v_{i+1,j}^{l+1}-v_{i-1,j}^{l+1}}{2\Delta z}+\frac{v_{i+1,j}^{l}-v_{i-1,j}^{l}}{2\Delta z}\right)+Fc\left(\frac{u_{i,j}^{l+1}+u_{i,j}^{l}}{2}\right)$$

$$\frac{v_{i,j}^{l+1}-v_{i,j}^{l}}{\Delta t}=\frac{1}{2}\left(\frac{v_{i,j+1}^{l+1}-2v_{i,j}^{l+1}+v_{i,j-1}^{l+1}}{\Delta y^2}+\frac{v_{i,j+1}^{l}-2v_{i,j}^{l}+v_{i,j-1}^{l}}{\Delta y^2}\right)$$

$$+\frac{1}{2}\left(\frac{v_{i+1,j}^{l+1}-2v_{i,j}^{l+1}+v_{i-1,j}^{l+1}}{\Delta z^2}+\frac{v_{i+1,j}^{l}-2v_{i,j}^{l}+v_{i-1,j}^{l}}{\Delta z^2}\right) \tag{24}$$

$$+\frac{M}{2}\left(\frac{H_{i+1,j}^{l+1}-H_{i-1,j}^{l+1}}{2\Delta z}+\frac{H_{i+1,j}^{l}-H_{i-1,j}^{l}}{2\Delta z}\right)+Fc\left(\frac{u_{i,j}^{l+1}+u_{i,j}^{l}}{2}\right)$$

To simplify equations (21)to (24) further we defined

$$\left.\begin{aligned}&D_1=-\frac{\Delta t}{2P_m\Delta y^2},\quad D_2=-\frac{\Delta t}{2P_m\Delta z^2};\quad D_3=-\frac{M\Delta t}{4P_m\Delta z},\quad D_4=\frac{F_c\Delta t}{2P_m}\\&D_5=-\frac{\Delta t}{2\Delta y^2},\quad D_6=-\frac{\Delta t}{2\Delta z^2};\quad D_7=-\frac{M\Delta t}{4\Delta z},\quad D_8=\frac{F_c\Delta t}{2}\end{aligned}\right\} \tag{25}$$

Substituting (25) into (21) to (24) and then equating each components to F_1 and F_2 we have:

$$F_1=H_{i,j}^{l}-H_{i,j}^{l+1}+D_1\left(H_{i,j+1}^{l+1}-2H_{i,j}^{l+1}+H_{i,j-1}^{l+1}+H_{i,j+1}^{l}-2H_{i,j}^{l}+H_{i,j-1}^{l}\right)$$

$$+D_2\left(H_{i+1,j}^{l+1}-2H_{i,j}^{l+1}+H_{i-1,j}^{l+1}+H_{i+1,j}^{l}-2H_{i,j}^{l}+H_{i-1,j}^{l}\right) \tag{26}$$

$$+D_3\left(u_{i+1,j}^{l+1}-u_{i-1,j}^{l+1}+u_{i+1,j}^{l}-u_{i-1,j}^{l}\right)+D_4\left(u_{i,j}^{l+1}+u_{i,j}^{l}\right)$$

$$F_2 = u_{i,j}^{l+1} - u_{i,j}^{l} + D_5\left(u_{i,j+1}^{l+1} - 2u_{i,j}^{l+1} + u_{i,j-1}^{l+1} + u_{i,j+1}^{l} - 2u_{i,j}^{l} + u_{i,j-1}^{l}\right)$$

$$+ D_6\left(u_{i+1,j}^{l+1} - 2u_{i,j}^{l+1} + u_{i-1,j}^{l+1} + u_{i+1,j}^{l} - 2u_{i,j}^{l} + u_{i-1,j}^{l}\right) \tag{27}$$

$$+ D_7\left(H_{i+1,j}^{l+1} - H_{i-1,j}^{l+1} + H_{i+1,j}^{l} - H_{i-1,j}^{l}\right) + D_8\left(u_{i,j}^{l+1} + u_{i,j}^{l}\right)$$

And

$$F_1 = H_{i,j}^{l+1} - H_{i,j}^{l} + D_1\left(H_{i,j+1}^{l+1} - 2H_{i,j}^{l+1} + H_{i,j-1}^{l+1} + H_{i,j+1}^{l} - 2H_{i,j}^{l} + H_{i,j-1}^{l}\right)$$

$$+ D_2\left(H_{i+1,j}^{l+1} - 2H_{i,j}^{l+1} + H_{i-1,j}^{l+1} + H_{i+1,j}^{l} - 2H_{i,j}^{l} + H_{i-1,j}^{l}\right) \tag{28}$$

$$+ D_3\left(v_{i+1,j}^{l+1} - v_{i-1,j}^{l+1} + v_{i+1,j}^{l} - v_{i-1,j}^{l}\right) + D_4\left(u_{i,j}^{l+1} + u_{i,j}^{l}\right)$$

$$F_2 = v_{i,j}^{l+1} - u_{i,j}^{l} + D_5\left(v_{i,j+1}^{l+1} - 2v_{i,j}^{l+1} + v_{i,j-1}^{l+1} + v_{i,j+1}^{l} - 2v_{i,j}^{l} + v_{i,j-1}^{l}\right)$$

$$+ D_6\left(v_{i+1,j}^{l+1} - 2v_{i,j}^{l+1} + v_{i-1,j}^{l+1} + v_{i+1,j}^{l} - 2v_{i,j}^{l} + v_{i-1,j}^{l}\right) \tag{29}$$

$$+ D_7\left(H_{i+1,j}^{l+1} - H_{i-1,j}^{l+1} + H_{i+1,j}^{l} - H_{i-1,j}^{l}\right) + D_8\left(u_{i,j}^{l+1} + u_{i,j}^{l}\right)$$

The boundary conditions (20) are applied to the nodes that captured the boundaries.

Equations (26) to (29) are apply to all nodes (i,j) in the solution domain which consists of a rectangular region with y varying from 0 to 1 and z from 0 to z-maximum (=1), where equations (26) and (27) are for ionized components while (28) and (29) are for neutral components. The region to be examined in (y, z) space is covered by grids system to enhance accuracy with a time step of 0.0001 for the transient state. The grid spacing parallel to both axes were denoted by Δy, Δz and Δt corresponding to y, z and t directions respectively. In order to determine the solution of the finite difference equation in each node, a MATLAB CODE was employed in which a Newton-Raphson search algorithm is fused into the non linear Crank Nicolson finite difference equation with a technique of updating individual nodal solution discretely and testing the convergence of the entire system simultaneously using the infinity norm of the function vectors $\|F\| \leq \varepsilon$ where we set $\varepsilon = 10^{-10}$. The problem under consideration is time dependent hence a time step $\Delta t$is chosen. For each time level $(l\Delta t)$ the Newton-Raphson algorithm is applied till convergence to obtain $f_{i,j}^{l}$ and the scheme is repeated for the next time level $(l+1)\Delta t$ to obtain $f_{i,j}^{l+1}$ values. The time stepping is terminated when the desired time level t is attained or until steady state.

4.0 Results and Discussion

The velocity profiles of the ionized components with an induced magnetic field (H_x) for a rectangular channel with a moving perfectly conducting top wall arepresented in Figures 2, 3 and 4.The channel parameters were chosen to illustrate variations of the velocity profiles and the time components of the velocities. The Hartmann's number (M) used are 1,5,10,12 and 15 with a Magnetic Prandtl number P_m of 0.01 (value for liquid metal). Transients considered were t=0.01, 0.03 and 0.07. Steady state solution was attained at t ≥ 0.07. The velocity profile along the moving interface is equal to 1 and along the stationary walls at the bottom and sides are zero in satisfaction of the boundary conditions. For the region considered on the y and z axis, it is observed that the velocity drops steadily from its moving plate edge value to zero at the stationary walls as stated in the boundary conditions. This also implies that the stress free surface does not drive the fluid and the stationary perfectly conducting side wall retards the fluid. As the time increases the effect of the induced magnetic field become apparent as shown in Figures 2, 3 and 4. As the Hartmann's number is increased, it was observed that the velocity profiles decreases. This decrease in the velocity profile is found to be the effect of magnetic field which in an electrically conducting fluid produces a Lorentz force that acts against the flow. This observation was found to be in perfect agreement with earlier result as reported in[6]. The neutral velocity was discovered to have the same velocity pattern as that of the ionized at similar transient values, see Figures 5 and 6. A single velocity curve was maintained at all values of the Hartman's number indicating that the induced magnetic field has little or no effect on the neutral velocity. However as time increases the neutral velocity profiles also increased. Furthermore, it was also noticed that increase in Pm lead to a decrease in the velocity profile of the ionized components in agreement with [7] and no effect on the neutral components.

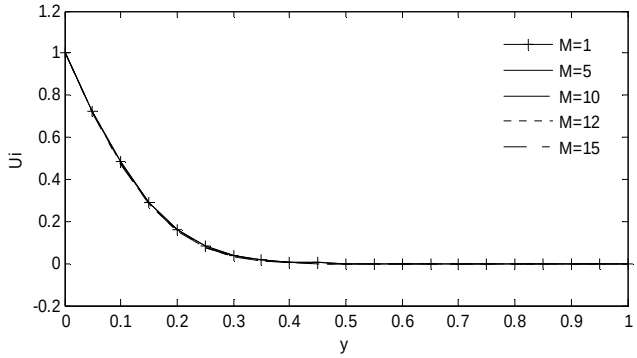

Figure 2: Ionized velocity profiles with induced magnetic field for different M at t=0.01, Pm=0.01

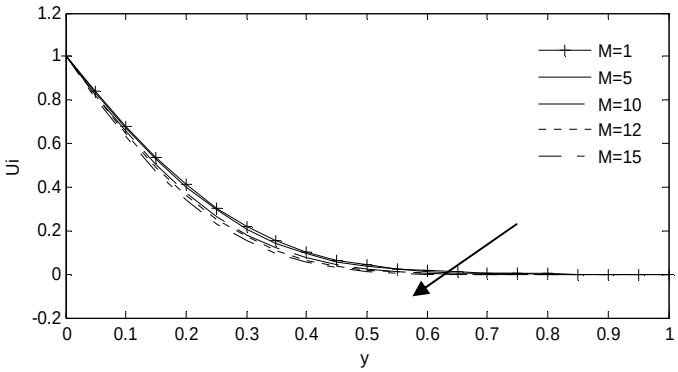

Figure 3: Ionized velocity profiles with induced magnetic field for different M at t=0.03, Pm=0.01

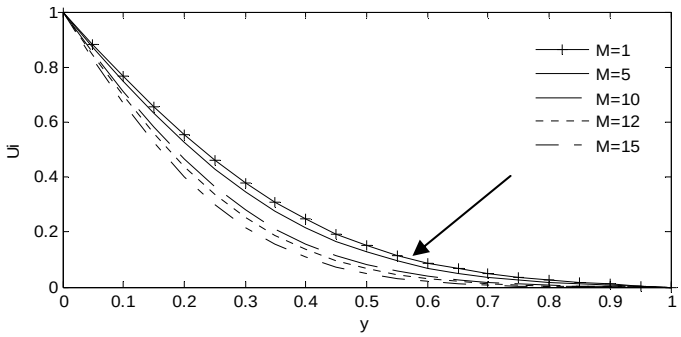

Figure 4: Ionized velocity profiles with induced magnetic field for different M at t=0.07, Pm=0.01

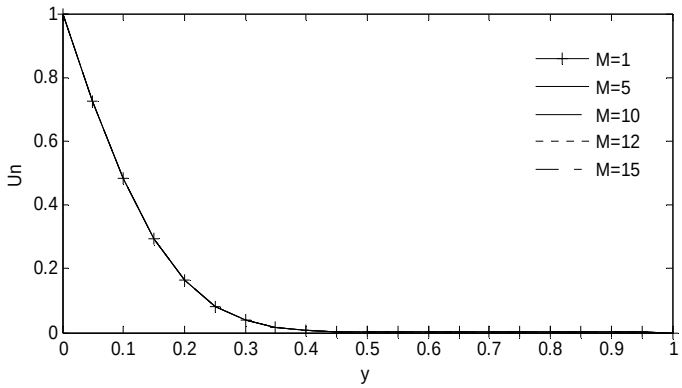

Figure 5: Neutral velocity profile with induced magnetic field for different M at t=0.01, Pm=0.01

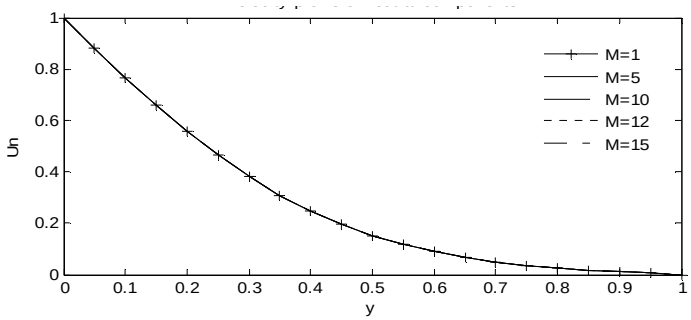

Figure 6: Neutral velocity profile with induced magnetic field for different M at t=0.07, Pm=0.01

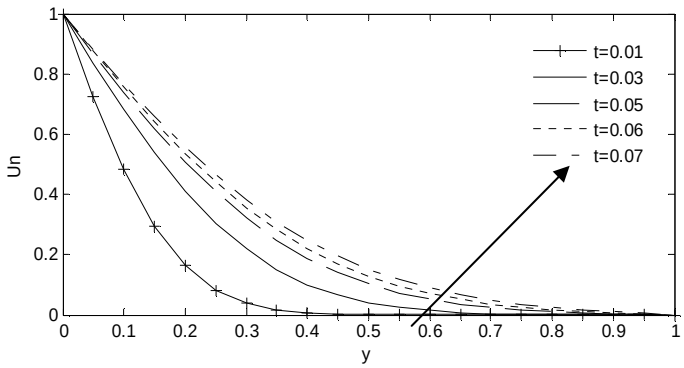

Figure 7: Neutral velocity profiles with induced magnetic field for various time t at M=5, Pm=0.01

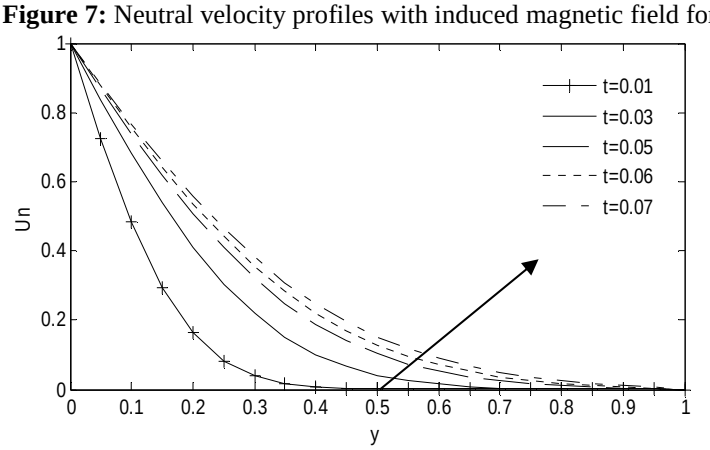

Figure 8: Neutral velocity profiles with induced magnetic field for various time t at M=15, Pm=0.01

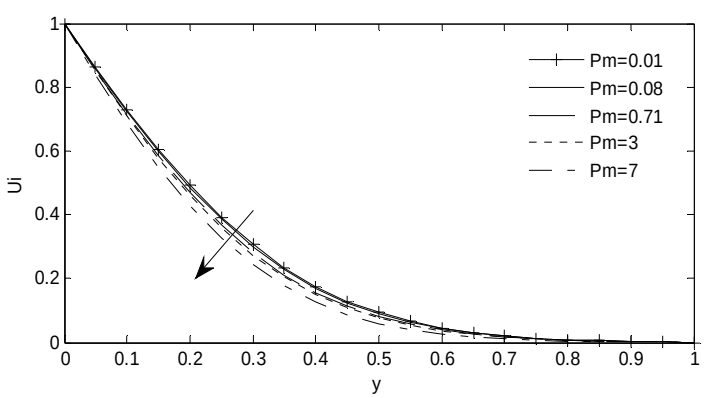

Figure 9: Velocity profile of ionized component at various values of P_m. when M=5,T=0.1

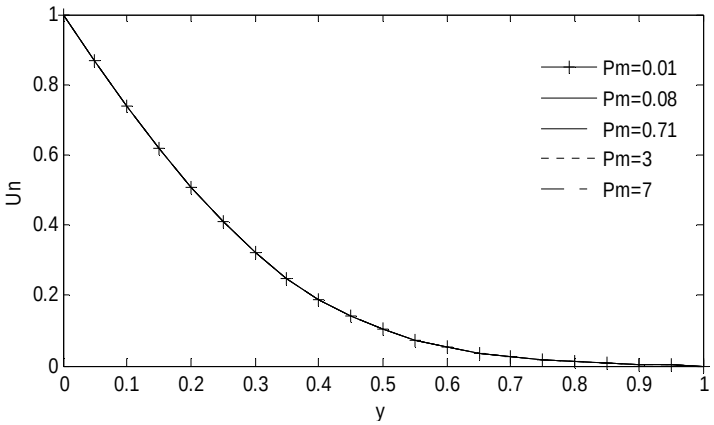

Figure 10: Velocity profile of neutral component at various values of P_m when M=5 and T=0.1

5.0 Conclusion

This paper studied the effect of time variations on partially ionized MHD liquid metal couette flow in a rectangular channel with an induced magnetic field. The momentum and magnetic field equations written in dimensionless form were separated into ionized and neutral components and coupled. Implicit finite difference method was employed to solve the equations. From the present numerical investigation the following conclusion have been drawn:

i. The magnetic field parameter M gets more significant as time increases in the ionized velocity profile than in the neutral velocity

ii. Increase in Magnetic Prandtl numbers lead to a decrease in ionized velocity profiles and no effect on the neutral velocity.

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