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On Soft Multiset Operations

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Abstract

Soft set and Soft multiset are new emerging mathematical tools that deal with uncertainties about vague concepts. These two concepts have been studied by scholars in practice and theory. In this paper, Soft multiset operations were presented with relevant examples. We also show with an example that the common multiset universe $U = P(U_1) \uplus P(U_2) \uplus P(U_3)$ is closed under AND operation but not closed under OR operation.De Morgan's law holds in soft multiset theory with respect to various operations on soft multiset.

Keywords: Multi-value class,Multiset, operations, soft multiset,Soft set.

1.0 Introduction

Most of the problems we are confrontedwith in engineering, medical sciences, economics, environments, social sciences, etc., have various uncertain attributes. Molodtsov [1] initiated the concept of soft set theory, as a general mathematical tool for dealing with such uncertainties. This theory is free from the inadequacy of the parameterization tool of other nonstandard set theory. The origin of soft set theory could be traced to the work of Pawlak [2] in 1993 titled Hard and Soft set in Proceedings of the International EWorkshop on rough sets and knowledge discovery at Banff. His notion of soft sets is a unified view of classical, rough and fuzzy sets. This might have motivated D. Molodtsov's work in 1999 titled *soft set theory: first result.*There in , the basic notions of the theory of soft sets and some of its possible applications were presented. For positive motivation, the work discusses some problems of the future with regards to the theory. This theory, to some extent, is free from the inadequacies of the parameterization tool of other nonstandard set theory.

Soft sets could be regarded as neighborhood systems, and they are a special case of context-dependent fuzzy sets. In soft set theory the problem of setting the membership function in fuzzy set, among other related problems does not arise. This makes the theory very convenient and easy to apply in practice as in [3-6]. After Molodtsov's work, different operations were defined in [7-10] and derivatives of soft set in [11-14] were studied.

2.0 Soft Multiset Theory

Let $\{U_i:\ i \in I\}$ be a collection of universes such that there exist at least U_j,$U_k$and $U_j \cap U_k \neq \emptyset$. Suppose$U = \uplus_{i\in I} P(U_i)$, where$P(U_i)$ denotes the power set of U_i, and E be a set of parameters. A pair (F, A), where$A \subseteq E$, is called a soft multiset over U.$F$is a mapping given by$F: A \longrightarrow U$. That is, a soft multiset over U is a parameterized family of submultisets of U such that for $e \in A,\ F(e)$ is considered as the set of e–approximate element of the soft multiset (F, A).

Example 2.3:

Let $C_i: i \in N$ be a collection of candidates seeking for employment in anew generation Bank X, and$U_i: i \in N$ be a collection of candidates with B.Sc degree in Accounting, nine credits in O level and M.Sc degree in Accounting.

Suppose

$U_1 = \{C_1,\ C_2,\ C_3\}$be a set of candidates with B.Sc degree in Accounting,

$U_2 = \{C_2,\ C_4,\ C_6,\}$be a set of candidates with nine credits in O level,

$U_3 = \{C_2,\ C_4,\ C_5,\}$be a set of candidates with M.Sc degree in Accounting.

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$$U = P(U_1) \uplus P(U_2) \uplus P(U_3)$$

$$= \left\{ \begin{matrix} \{C_1\}, \{C_2\}, \{C_3\}, \{C_1, C_2\}, \{C_1, C_3\}, \{C_2, C_3\}, \{C_1, C_2, C_3\}, \emptyset, \{C_2\}, \{C_4\}, \{C_6\}, \{C_2, C_4\}, \{C_2, C_6\}, \{C_4, C_6\}, \\ \{C_2, C_4, C_6\}, \emptyset, \{C_2\}, \{C_4\}, \{C_5\}, \{C_2, C_4\}, \{C_2, C_5\}, \{C_4, C_5\}, \{C_2, C_4, C_5\}, \emptyset, \end{matrix} \right\}$$

$$= \left\{ \begin{matrix} \{C_1\}, 3\{C_2\}, \{C_3\}, 2\{C_4\}, 3\emptyset, \{C_6\}, \{C_5\}, \{C_1, C_2\}, \{C_1, C_3\}, \{C_2, C_3\}, 2\{C_2, C_4\}, \{C_1, C_2, C_3\}, \{C_2, C_6\}, \{C_4, C_6\}, \\ \{C_2, C_4, C_6\}, \{C_2, C_5\}, \{C_4, C_5\}, \{C_2, C_4, C_5\} \end{matrix} \right\} \text{this is a multiset}$$

Let E be a set of decision parameters related to the above universes, where
E = {e_1 = intelligent, e_2 = eloquent, e_3 = beautiful, e_4= tall, e_5 = neat, e_6 = slim , e_7 = vocal}.
Let A = { e_1 = intelligent, e_2 = eloquent, e_3 = beautiful, e_4= tall, e_5 = neat }.

The Soft multiset(F, A) is a parameterized family $\{F(e_i),\ i = 1, 2, \dots, 5\}$of subsets of the set U and gives us a collection of approximate description of the candidates for the selection to the new generation Bank X.

Example 2.4. Let $(F, A)$and (G, B) be two soft multiset over U and let A, B⊆ E, where A ={ e_1 = intelligent, e_2 = eloquent, e_3 = beautiful, e_4= tall, e_5 = neat, e_6 = slim}, and B ={ e_1 = intelligent, e_2 = eloquent, e_3 = beautiful, e_4= tall }.

Suppose that
$F(e_1) = 3\{C_2\}$,$(e_2) = 2\{C_2, C_4\}$, $F(e_3) = \{C_2, C_4, C_5\}$, $F(e_4) = \{C_1, C_3\}$, $F(e_5) = \{C_4\}$, $F(e_6) = \{C_1, C_2\}$and$G(e_1) = \{C_2\}$, $G(e_2) = 2\{C_2, C_4\}$, $G(e_3) = \{C_2, C_4, C_5\}$ $G(e_4) = \{C_1, C_3\}$, then
(F, A) ={$(e_1, 3\{C_2\})$, $(e_2, 2\{C_2, C_4\})$,$(e_3,\{C_2, C_4, C_5\})$,$(e_4, \{C_1, C_3\})$,$(e_5, \{C_4\})$, $(e_6, \{C_1, C_2\})$} and (G, B) = {$(e_1, \{C_2\})$, $(e_2, 2\{C_2, C_4\})$,$(e_3,\{C_2, C_4, C_5\})$,$(e_4, \{C_1, C_3\})$}

Definition 2.5: Multivalue-class.
The class of all value set of a soft multiset (F, A) is called the value class of the soft multiset and is denoted by
$C^*_{(F,A)} = \{V_1, V_2, \dots, V_n\}$. Obviously
$C^*_{(F,A)} \subseteq U$. Also, if there exists at least one i such that $V_i = V_j$,
$\forall i, j = 1,2, \dots, n$, then the value-class of the soft multiset (F, A) is called Multi value-class of the soft multiset(F, A) and is denoted by$C^m_{(F,A)}$. Similarly$C^m_{(F,A)} \subseteq U$.

Example 2.6. Considering example 2.3. The multi value-class of the soft multiset (F, A) denoted by $C^m_{(F,A)} = \{3\{C_2\}, 2\{C_2, C_4\}, \{C_2, C_4, C_5\}, \{C_1, C_3\}, \{C_4\}, \{C_1, C_2\}\}$

Definition 2.7. Soft submultiset.
Let (F, A) and (G, B) be two soft multisets over U, we say that (G, B) is a soft multi subset of (F,A) written as $(G, B) \widetilde{\subseteq} (F, A)$ if

i. B ⊆ A
ii. $M_{(G,B)}(x) \leq M_{(F,A)}(x)$ for all $x \in U$.

Note an object in a soft multiset is an approximation, not just an ordinary element.

Example 2.8 Considering example 2.3. We see that B ⊆ Aand $M_{(G,B)}(x) \leq M_{(F,A)}(x)$. Therefore, (G, B) is a soft multisubset of (F, A).

Definition 2.9 Equality of two soft multisets.
Two Soft multisets (F, A) and (G, B) over U are said to be equal if and only if (F, A) is a soft multi subset of (G, B) and (G, B) is a soft multi subset of (F, A).

Definition 2.10. NOT Set of a set parameters.
Let E be a set of parameters. The NOT set of E denoted by$\urcorner E$ is defined by$\urcorner E = \{\urcorner e_1, \urcorner e_2, \dots, \urcorner e_n\}$ where$\urcorner e_i = note_i, \forall i$.

Proposition 2.11

1. $\urcorner(\urcorner A) = A$
2. $\urcorner(A \cup B) = (\urcorner A \cup \urcorner B)$
3. $\urcorner(A \cap B) = (\urcorner A \cap \urcorner B)$

Definition 2.12. Similar Soft multisets
Two Soft multisets $(F, A)\ and\ (G, B)$ are said to be 'Cognate' or similar if
$\forall x\ (x \in (F, A) \Leftrightarrow x \in (G, B))$ where$x$ is an approximation. Therefore, similar Soft multisetshave equalroot sets but need not be equal themselves.

Example 2.13. $F(e_1) = 3\{C_2\}$, $(e_2) = 2\{C_2, C_4\}$, $F(e_3) = \{C_2, C_4, C_5\}$, $F(e_4) = \{C_1, C_3\}$ and $G(e_1) = \{C_2\}$, $G(e_2) = \{C_2, C_4\}$, $G(e_3) = \{C_2, C_4, C_5\}$, $G(e_4) = \{C_1, C_3\}$. Therefore, (F, A) is similar to (G, B.)