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Controllability Results for Retarded Functional Differential Systems of Sobolev Type in Banach Spaces with Multiple Delays in the Control

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Abstract

In this paper, the Retarded Functional Differential Systems of Sobolev Type in Banach Space with Multiple Delays in the Control of a particular form is presented for controllability analysis. For purposes of clarity, we defined and extracted the following terminologies as they relate to the system .The solution is given as an integral formula The solution of the system is given by the integral equation. Necessary and sufficient conditions for controllability of the Retarded Functional Differential Systems with multiple delays in the control of Sobolev type in Banach Spaces are established. Here, the schauder fixed point theorem was used to establish the controllability of the system .Mild Solution of the System is also obtained. The results are obtained using compact Semigroup and the Schauder fixed point theorem.

Key words: CompactSemigroup, Mild Solution, Sobolev System, Retarded Functional Differential System. Banach Space, Multiple Delays, Controllability

1.0 Introduction

Controllability is one of the fundamental concepts in mathematical control theory [1-4]. This is a qualitative property of dynamical control systems and is of particular importance in control theory. Systematic study of controllability was started at the beginning of sixties, when the theory of controllability based on the description in the form of state space for both time-invariant and time varying linear control systems was worked out.

Roughly speaking, controllability generally means, that it is possible to steer dynamical control system from an arbitrary initial state to an arbitrary final state using the set of admissible controls. In literature there are many different definitions of controllability, which strongly depend on class of dynamical control systems [4,5,6].

In the recent past, various controllability problems for different types of nonlinear dynamical systems have been considered in many publications and monographs. The extensive list of these publications can be found in the monograph [4,7]. However, it should be stressed, that the most literature in this direction has been mainly concerned with controllability problems for ; finite-dimensional nonlinear dynamical systems with unconstrained controls without delays and linear infinite-dimensional dynamical systems with constrained controls and without delays [2,8,9].

Several authors [2,10,11] have extended the concept to infinite dimensional systems in Banach Spaces with bounded operators. Lasieka and co-workers established sufficient conditions for controllability of linear and nonlinear systems in Banach Spaces [12]. The controllability and approximate controllability of delay volterra systems were investigated by using fixed point theorem [13]. The controllability and local null controllability of nonlinear integrodifferential systems and functional differential systems in Banach spaces were studied and it was shown that the controllability problem in Banach spaces can be converted into one of a fixed-point problem for a single-valued mapping [14, 15]. While Balachandranandco-workers studied the Controllability of Sobolev type partial functional differential systems in Banach spaces [16].

ThepurposeofthispaperisstudythecontrollabilityofretardedfunctionaldifferentsystemsofSobolevtypein Banachspaceswithmultipledelaysin thecontrol.

The equation considered here serves as an abstract formulation of sobolev type partial functional differential equations which arise in many physical phenomena [17,18]. Considerthenonlinear partial functionaldifferentialsystemofthe form

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$$(P\,x(t))' + Ax(t) = f(t, x_t) + \sum_{j=0}^{m} B_j u(t - hj), t > 0 \qquad (1.1)$$

$$x\,(t) = \Phi(t): -r \le t \le 0$$

$where\ the\ state\ x(.)takes\ values\ in\ a\ Banach\ space\ X = R^n$

$and\ the\ control\ function\ u\ (.)$

$is\ given\ in\ L_2(J, U)\ the\ Banach\ space\ of\ admissiblecontrol\ functions\ with\ U$

$a\ Banach\ space. B\ is\ a\ bounded\ linear\ operator\ from\ U\ into\ Y, a\ Banach\ space.$

$The\ nonlinear\ function\ f: J\ x\ C \to Y\ is\ continuous.$

$$Here\ , J = (t_0, t_1) and\ for\ a\ continuous\ function\ x: J^* = [-h, t_1] \to X,$$

$$x_t\ is\ that\ element\ of\ C = C\ ([-h, 0): X) defined\ by\ x_t(s) = x(t + s), -h \le s \le 0.$$

$The\ domain\ D(p) of\ P\ becomes\ a\ Banach\ space\ with\ norm$

$$\left||x|\right|_{D(p)} = \left||px|\right|_y, x\epsilon D(p)\ and\ (p) = C\ ([-h, 0): D(p)).$$

1.1 Preliminaries and Definitions

$The\ operators\ A: D(A) \subset X \to Y\ and\ P: D(p) \subset X \to Y\ satisfy\ the$

$following\ conditions\ (C_i) for\ i = 1, 2, 3, 4, 5, 6:$

(1) A and P are closed linear operators

(2) $D\ (P) \subset D(A)\ and\ P\ is\ bijective\ linear\ operator$

(3) $P^{-1}: Y \to D(P)\ is\ compact.$

(4) $For\ each\ t\epsilon\ [t_0, t_1]; t_1 > t_0, and\ for\ some\ \lambda\epsilon\ \rho(-Ap^{-1}), the\ resolvent\ set\ of\ -Ap^{-1},$

$we\ have\ that\ the\ resolvant\ R(\lambda: -Ap^{-1})\ is\ a\ compact\ operator.$

The conditions, (1), (2) and the close graph theorem imply the boundedness of the linear operator

$$Ap^{-1}: Y \to Y.$$

$\boldsymbol{Lemma}\ (\boldsymbol{See}\ [\mathbf{1}])$

$Let\ A\ be\ the\ infinitesimal\ generator\ of\ a\ uniformly\ continuous\ semigroup\ \Psi(t). If\ the\ resolvent\ R\ (\lambda: A) of\ A\ is\ compact\ f$

$$Thus, max\left||Q(t)|\right| < \infty :\ t\epsilon J\ \ and\ so\ denote\ M\ by$$

$$M = max\ ||Q(t)|| \qquad t\ \epsilon\ J$$

Definition 2.1 (Controllability)

$The\ system\ (1.1) is\ said\ to\ be\ controllable\ on\ the\ interval\ J. if\ for\ every\ continuous\ initialfunction\ \phi\ defined\ on\ [-h, 0] and$

$$such\ that\ the\ solution\ x(.) of\ \ the\ system\ (1.1)\ satisfies\ x(t_1) = x_1.$$

(5) B is bounded linear operator and the linear operator w from U into X defined by

$$\sum_{j=1}^{m} wu(t - hj) = \int_{t_0}^{t_1} P^{-1}\, Q(t_1 - s) \sum_{j=1}^{m} B_j(s) u(s - h_j) ds \qquad (2.0)$$

$$has\ a\ bounded\ inverse\ operator\ w^{-1} defined\ on\ L2\ (J, U)/kerw.$$

(6) $The\ function\ f\ satisfies\ the\ following\ two\ conditions$:

(i) $For\ each\ t\epsilon J, the\ function\ f\ (t.,): C\ \rightarrow\ Y\ is\ continuous, and\ for\ each\ x\ \epsilon\ C$

$the\ function\ f(.,x): j\ \rightarrow\ Y\ is\ strongly\ measurable.$

$For\ each\ natural\ number\ k, there\ is\ a\ function\ a_n\ \epsilon\ L'(J) such\ that$

$$\sup_{|x|\le k} |f(t,x_t)|\ \le\ a_k(t), \lim_{k\to\infty} \frac{1}{k}\int_{t_0}^{t_1} a_k(s)ds = g <\ \infty$$

$where\ g\ is\ a\ real\ number.$

The solution of system (1.1) is given by the integral equation below (see [1]).

$$x(t) =\ P^{-1}Q\ (t-s)P\ \phi(0) + \int_{t_0}^{t} P^{-1}Q\ (t-s)f(s,x_s)ds$$
$$+ \int_{t_0}^{t} P^{-1}Q\ (t-s)\sum_{j=1}^{m} B_j(s)\ u\left(s-h_j\right)ds, t > 0 \quad (2.1)$$

2.0 Main Results

Here, the schauder fixed point theorem is used to establish the controllability of the system (1.1) under the above conditions (1-6).

Theorem 3.1

If the assumptions (1) – (6) are satisfied, then the system (1.1) is controllable on J. iff

$$\boldsymbol{gM}||\boldsymbol{p^{-1}}||\ [\boldsymbol{1}\ +\ \boldsymbol{gM}\ ||\ \boldsymbol{B}||.||\boldsymbol{w^{-1}}||.||\boldsymbol{p^{-1}}||]\ <\ 1.$$

Proof

Using the condition (5), for an arbitrary function x (.) define the control

$$\sum_{j=1}^{m} u\left(t-h_j\right) =\ w^{-1}\left[x_1 - P^{-1}Q\ (t_1)P\ \phi(0) - \int_{t_0}^{t} P^{-1}Q\ (t-s)f(s,x_s)ds\right](t).$$

It shall be proved that when using this control, the operator Ψ defined by

$$(\psi x)(t) =\ P^{-1}Q\ (t)P\ \phi(0) + \int_{t_0}^{t} P^{-1}Q\ (t-s)f(s,x_s)ds$$
$$+ \int_{t_0}^{t} P^{-1}Q(t-s)\sum_{j=1}^{m} B_j\left(s\right)u\left(s-h_j\right)ds, for\ t > 0, \quad (2.3)$$

$(\psi x)(t) =\ Q(t), for - h\ \le t\ \le 0,$

$From\ C(J*,X)\ into\ itself, for\ each\ x\epsilon C(J$

$*,X), has\ a\ fixed\ point. This\ fixed\ point\ is\ then\ a\ solution\ of\ system$ (1.1).

Clearly, $(\psi x)(t_1) =\ P^{-1}Q(t_1)P\phi(0) +\ \int_{t_0}^{t_1} P^{-1}Q(t-s)f(s,x_s)ds$

$$+ \int_{t_0}^{t_1} P^{-1}Q(t-s)\sum_{j=1}^{m} B_j\left(s\right)\ u\left(s-h_j\right)ds. \quad (2.4)$$

$$= P^{-1}Q\ (t_1)P\ \phi(0) + \int_{t_0}^{t_1} P^{-1}Q(t\text{-}s)f(s,x_s)ds + \int_{t_0}^{t_1} P^{-1}Q\ (t\text{-}s)\sum_{j=1}^{m} B_j(s)\ w^{-1}\{x_1 - P^{-1}\ Q\ t_1)\ P\ \phi(0)$$