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Application of Hydrostatic Equilibrium in Leachate Flow Control in Landfills

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Abstract

Surface waste dump is generally associated with various environmental and health problems, hence landfill method of waste disposal wasnecessary. Leachate from landfills contaminates aquifer and renders underground water unfit for consumption. This work focuses on how to strategically locate landfills in regions where there will be hydrostatic equilibrium; i.e where there is zero underground water flow. A model experiment was set up using five different porous sand samples of different porosities, packed into a cylindrical pipe inclined between $0^0 \leq \theta \leq 25^0$through which water was made to flow with a known piezometric height. Values of hydrostatic angles were determined for each sample at volume flux V = 0. The graph of porosity Φ against hydrostatic angle θ showed that both are linearly related with a relationΦ = 0.148656 * θ + 0.139473with coefficient of determination of 0.991637. It was also observed that there exist, acritical point where at a particular angle all the samples has the same volume flux and vis-a-vis the same volume rate of flow.

Keywords: Leachate, aquifer, hydrostatic equilibrium, porosity and volume flux

1.0 Introduction

Environmental pollution is a major problem in both the developing and the developed world [1]. 1996). Contamination of our surroundings comes from a variety of sources, ranging from underground petrol tanks at filling stations to nuclear weapons facilities. Industrial plants and garbage dumps can also cause environmental problems.

Just as it does on the surface of the earth, water also flows underground, but only through pores in the soil and underlying geologic structure [2, 3].Water also moves at various speeds through the ground depending upon its flow path. Near surface flows move the fastest and normally supply most of the water that discharges at springs. The velocity with which water circulates in the ground gradually decreases with depth and the movement of deep groundwater may be extremely slow in the range of inches or feet per day [4,5, 6,7]. However, the velocity at which it can move is inversely proportional to the size of the openings through which it moves [6]. Although, these pores are very small and account for only a small portion of the underground volume it is possible for water to move large distances underground [2].

The conceptual model of a typical contaminant spill into porous media, has been put forward in [8, 9, 10, 11]. In some cases, the contaminant is dissolved in water and thus travels in a fractured aquifer, aquitard or acquicludes [12, 13] as a solute in somewhat unpredictable directions depending on the fracture planes that are intersected [14]; as in [15]. Fluid flow in the fractured porous media is of significance not only in the context of contaminant transport, but also in the production of oil from reservoirs, the generation of steam for power from geothermal reservoirs, and the prediction of large geotechnical structures, such as dams or foundations [15].Thus, the results of this study has a wide range of applications

This work aims at determining: the volume flux and volume rate of flow, which are important parameters in knowing the extent of spread of contaminants, and also to determine the angle of hydrostatic equilibrium at that medium.

2.0 Theoretical Background

Darcy Equation can be written in a more specific form;

$$V_l = -\frac{k}{\mu}\nabla(p - \rho g z) \tag{1}$$

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which can be re-expressed as;

$$V_l = -\frac{k}{\mu}\left(\frac{dp}{dl} - \rho g \frac{dz}{dl}\right) \qquad (2)$$

where V_l is the volume flux across a unit area of the porous medium in unit time along flow path *l;*

$\frac{dp}{dl}$ is the pressure gradient along *l* at the point to which V_l refers;

$\frac{dz}{dl} = \sin\theta$, where θ is the angle between *l* and the horizontal.

$$V_l = \frac{Q}{A}$$

It can also be deduced from (2) that;

$$\frac{dp}{dl} = \rho g \sin\theta - V\frac{\mu}{k} \qquad (3)$$

For an horizontal flow,

$\frac{dz}{dl} = 0$ i.e. $\theta = 0$

If a sample is completely saturated with an incompressible fluid, then

$$\frac{dp}{dl} = -V_l\frac{\mu}{k} \qquad (4)$$

For hydrostatic equilibrium to be attained $V_l = 0$, hence

$$\frac{dp}{dl} = \rho g \sin\theta$$

But for a horizontal flow

$$\frac{dp}{dl} = 0$$

3.0 Methodology

A modeled experiment was performed in the laboratory using riverbed sand of varying porosities. A cylindrical, rigid plastic pipe of diameter $3.45\text{x}10^{-2}$m and 2.20m long drilled at 0.20m interval was half- filled with water and was made to stand vertically with side holes blocked with plasticine. One end of it was screened and filled with sand in this condition so as to allow for uniform compaction. It was then set up in a horizontal position and joined to it was a similar pipe 0.30m long to make an elbow joint. A hole was drilled at 0.06m from the center of the horizontal pipe in the adjoining pipe to allow for run off of excess water. This height (0.06m) created the pressure head. Using an adapted manometer, pressure along the horizontal pipe (that is, $\theta=0^0$) was measured at 0.20m intervals and their corresponding distances recorded. The experiment was repeated for tilt angles $\theta=5.0^0, 10.0^0, 15.0^0, 20.0^0$ and 25.0^0.

The hydraulic conductivity was calculated from the relation:

$$K = \frac{Q.L}{A(H+L).t} \qquad (5)$$

[16]

Where Q = volume of water passing through the sample; m^3, t = time,sec;A = cross-sectional area of the sample (m^3), L = length of the soil samples; m and;H = height of the constant head m.

For each of the samples, the percolate, or the quantity of water drained in one minute were measured using a very narrow measuring cylinder and the volume of the percolate recorded. The measurement was continued until a constant volume (of water drained) was attained for the duration of consideration, that is one minute.

The diameter of the transparent cylindrical tube was measured using a pair of vernier calipers and an average value was taken. Using $A = \pi\left(\frac{d}{2}\right)^2$, where d is the diameter of the tube, the cross-sectional area for each of the samples were calculated and the results obtained in S.I. unit.

The water characteristics that affect the hydraulic conductivity are the density, ρ_w and the viscosity, μ. The changes in characteristics in soil are caused due to changes in temperature and electrolyte concentration. Under such conditions, the relation gives the hydraulic conductivity [7, 17, 18]:

$$K = \frac{k\rho_w g}{\mu} \tag{6}$$

Where k = intrinsic permeability or permeability, cm^2 or m^2 if in S.I. unit.

Therefore, Permeability,

$$k = \frac{\mu}{\rho_w g} K \tag{7}$$

g is the acceleration due to gravity taken as $9.80665ms^{-2}$.

From the respective values of hydraulic conductivities, the permeability of the media were calculated using equation (6) above.

4.0 Results

Table 1: Values of porosity, hydraulic conductivity and permeability for all samples

Sample	Porosity*	Hydraulic Conductivity (m/s)	Permeability (m^2)
A	0.361±0.001	1.100 E-4	1.210 E-11
B	0.375±0.001	1.430 E-4	1.457 E-11
C	0.417±0.006	2.024 E-4	2.062 E-11
D	0.448±0.020	2.510 E-4	2.558 E-11
E	0.467±0.010	3.433 E-4	3.498 E-11

*(Entov et al, 1981)

Table 2: Experimentally determined values of volume flux rate for samples at various angles vis-à-vis the corresponding angles (in degrees)

Angle (degree)	Vol. Flux rate (m/s) *E-5, (A)	Vol. Flux rate (m/s) *E-5,(B)	Vol. Flux rate (m/s) *E-5, (C)	Vol. Flux rate (m/s) *E-5,(D)	Vol. Flux rate (m/s) *E-5, (E)
0	-0.08±0.18	-0.11±0.14	-0.17±0.15	-0.24±0.13	-0.33±0.25
5	0.58±0.18	0.68±0.17	0.92±0.13	1.08±0.07	1.45±0.13
10	1.26±0.07	1.48±0.06	1.97±0.05	2.43±0.08	3.25±0.06
15	2.06±0.08	2.42±0.06	3.38±0.06	4.10±0.10	5.45±0.09
20	2.65±0.20	3.16±0.01	4.46±0.07	5.38±0.04	7.29±0.07
25	3.29±0.08	3.92±0.11	5.34±0.02	6.59±0.14	8.97±0.14

Table 3: Direct determination of volume flux for samples A, C and E

Angle(geree)	vol. Flux (m/s)*E-4,A	vol. Flux (m/s)*E-4,C	vol. Flux (m/s)*E-4,E
0	-0.5947	-0.80543	1.14058
5	1.3856	1.4435	1.5014
10	1.9336	2.4727	2.6969
15	2.486	2.7802	3.1009
20	3.0475	3.3995	3.7336
25	3.9073	4.1558	4.5356

Table 4: Hydrostatic angle and porosity for each of the sample

Hydrostatic Angle	Porosity	Sample
1.50	0.361	A
1.60	0.375	B
1.80	0.414	C
2.10	0.448	D
2.20	0.467	E

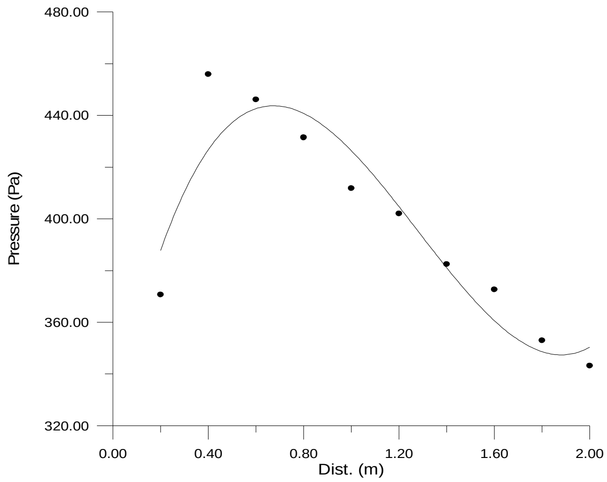

Fig. 1: Graph of pressure versus distance for horizontal flow for sample A, with entry length taking into account.

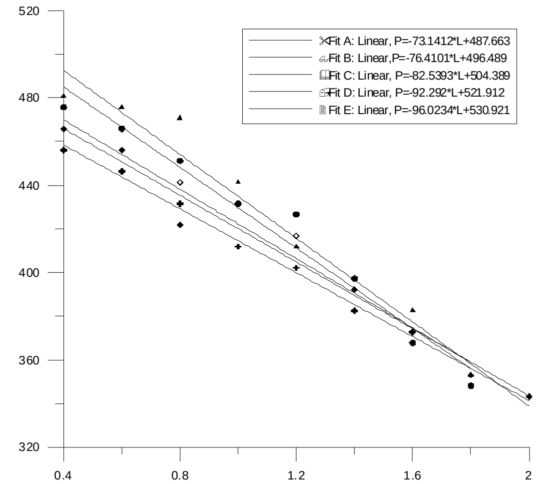

Fig. 2: Graph of pressure versus distance for horizontal flow (at angle $\theta=0^0$), for samples A - E

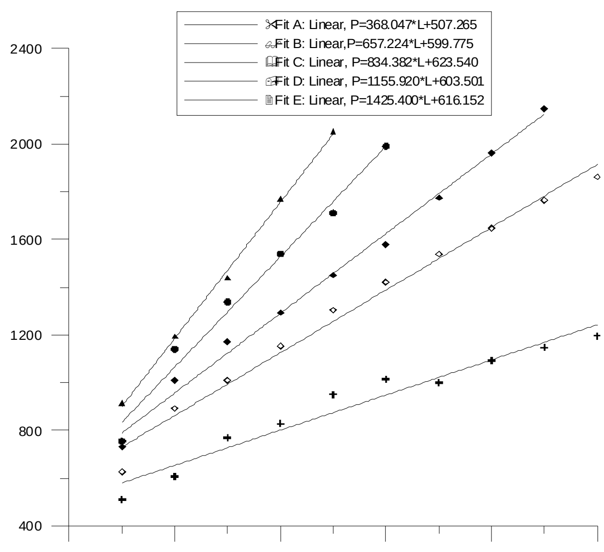

Fig. 3: Graph of pressure versus distance for inclined flow for sample A at angles $\theta=5^0$ - 25^0.

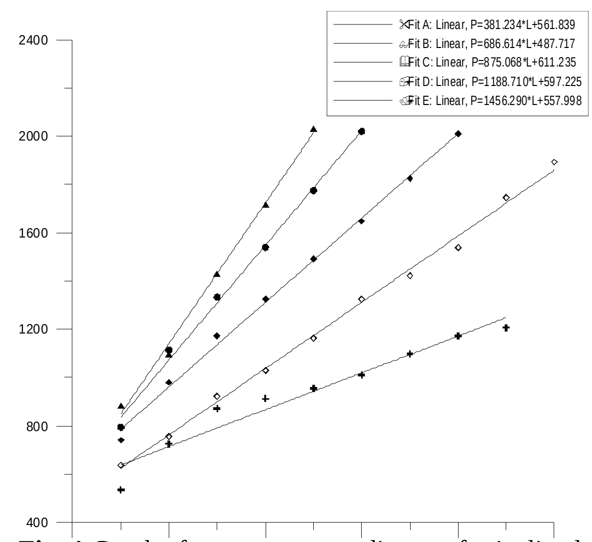

Fig. 4: Graph of pressure versus distance for inclined flow for sample B at angles $\theta=5^0$ - 25^0.

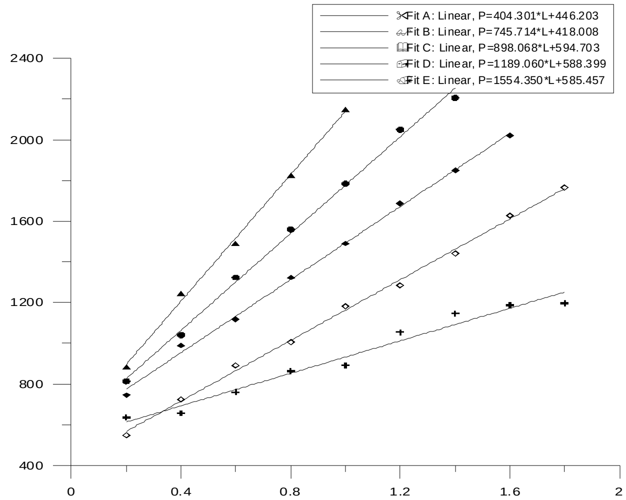

Fig. 5: Graph of pressure versus distance for inclined flow for sample C at angles $\theta=5^0$ - 25^0.

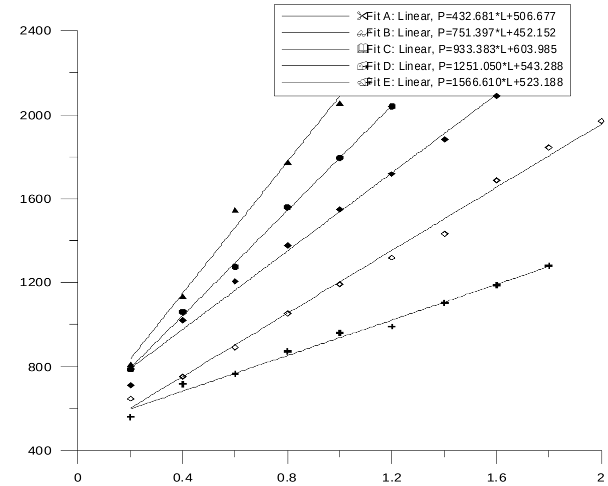

Fig. 6: Graph of pressure versus distance for inclined flow for sample D at angles $\theta=5^0$ - 25^0.

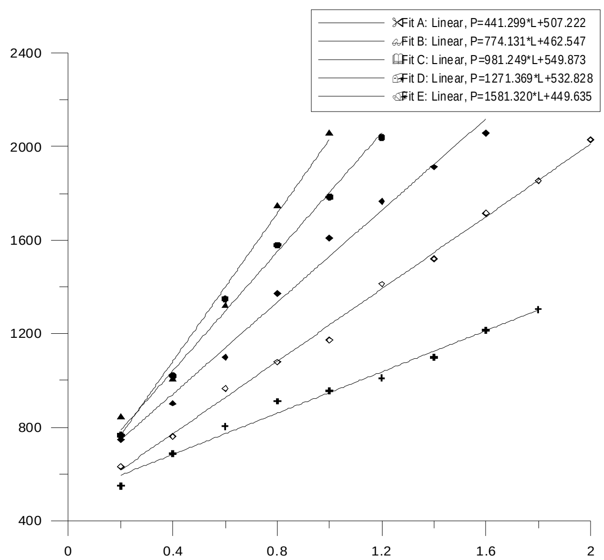

Fig. 7: Graph of pressure versus distance for inclined flow for sample E at angles $\theta=5^0$ - 25^0.

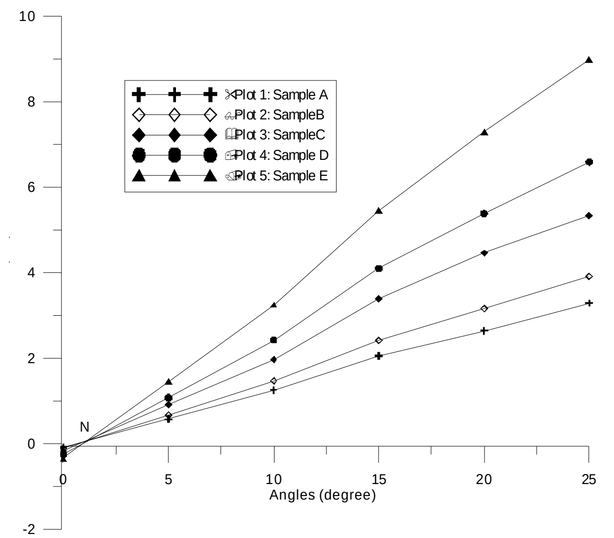

Fig. 8: Graph of volume fluxes versus angle of flow for samples A - E

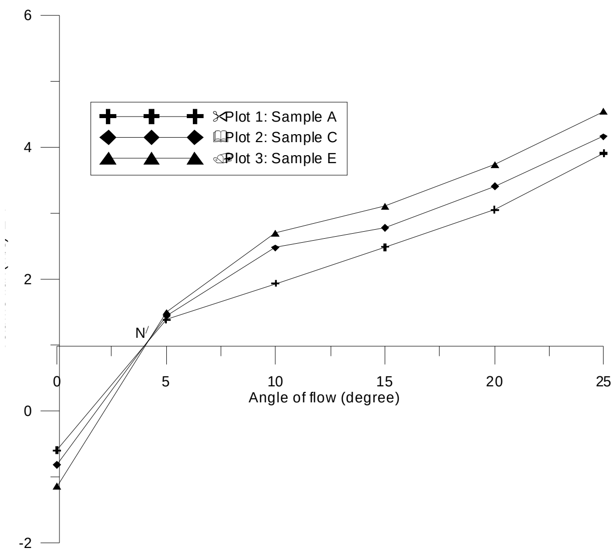

Fig. 9: Graph of volume flux versus angle of flow for samples A, C and E by direct determination

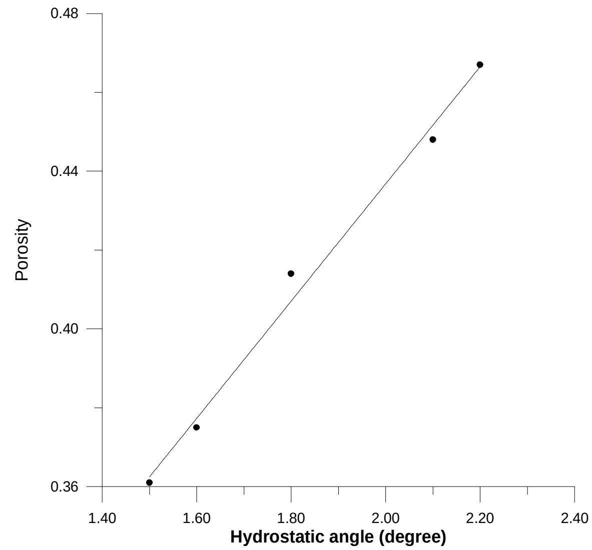

Fig. 10: Graph of porosity versus hydrostatic angle

$$\Phi = 0.148656 * \theta + 0.139473, \text{ Correlation coefficient} = 0.991637 \qquad (8)$$

5.0 Discussion and Summary

The values of pressure as read by the manometer was recorded, at interval of 0.20m both for horizontal flow ($\theta = 0^0$) and for inclined ones ($5^0 \leq \theta \leq 25^0$). For horizontal flow in which entry length were considered, the result showed that there exist a point of inflexion for all cases as seen in Figure1. The fits of plots for graphs of pressure-distance for horizontal flow for which entry lengths were truncated is shown in Figure 2. The figure revealed that pressure decreases along the direction of flow and both are linearly related in all the samples. The values for volume flux were determined from equation (4) and the gradient of pressure –distance graphs were equated to $-\frac{v\mu}{k}$ from which volume fluxes were determined.

Figures 3, 4, 5, 6 and 7 are pressure-distance graphs for tilted flow varied between 5^0 and 25^0 at the spacing of $5^0$for A, B, C, D and E respectively. In all these samples the gradient of the plots increases with the increasing angle of tilt and also with increasing porosities. Using equation (3), the fits for inclined flow were determined. The gradients of pressure-distance graph were equated to $\rho g \sin \theta - \frac{v\mu}{k}$. The pressure in this case increases with the distance of flow and also the volume flux increases as the angle of tilt increases. The values for volume rate of flow were obtained by using the relation $Q = VA$[19] where A is the cross-sectional area and $A = \pi (\frac{d}{2})^2$ and d is the diameter of the pipe used given as 3.45×10^{-2} m. The values of the volume flux obtained in Table 2 at various angles for different porosities, are from the values of gradients in Figures 3-7. The volume flux decreases with increasing distance at angle of flow $\theta = 0^0$.

It is also very obvious from Table 2that seepage velocity decreases with distance of flow at$\theta=0^0$but increases with increasing angle of tilt while it increases with porosity and angle of flow between 5^0 and 25^0. At $\theta=0^0$ volume flux decreases with distance but at $\theta \geq 5^0$ (and for increasing values of porosities) it increases in value.

Table 3 shows the values of volume flux, per minute for samples A, C and E. These values were obtained in a very direct way (by collecting the volume of water discharged per minute and not by measuring the values of pressure along the pipe) so as to cross check the possibility of compaction in the actual experiment that was performed overtime in Tables 2. Fig. 8 shows the graph of volume flux versus angle of flow plotted from Table 2. There exist a point N which is a critical point where at a particular angle all the samples has the same value of volume flux and vis-à-vis the same volume rate of flow. It can be seen that the values obtained at N are independent of porosity and permeability of the medium. At this point, $\theta = 1.13^0$ and volume flux is 6.99×10^{-7}m/s.

The observations from horizontal and inclined flow shows that pressure decreases along the direction of flow for $\theta=0^0$ and above this (i.e. inclined flow), pressure increases with the distance of flow. It is now very obvious that there must exist an angle of tilt for which there must be no flow where volume flux and volume rate of flow are zero. This condition is called the hydrostatic equilibrium. This explains why angle of tilt was considered up to 25^0, it was obvious that it was not known originally at what angle the hydrostatic equilibrium would be attained.

To establish this angle, the volume flux and volume rate of flow were plotted against angle of flow figures 42 and 43 respectively. It was observed that, this angle indeed exists and were found to be 0.63^0, 0.67^0, 0.75^0, 0.88^0 and 0.92^0 respectively (the values of θ for which V = 0). In addition to this (figures 42 and 43) it was observed that there exist a point N, where for all samples considered the value of volume flux and volume rate remain constant despite the fact that the samples used were of different porosities and permeabilities. This point N we call a critical point and was obtained to be 6.99×10^{-7}m/s and 6.50×10^{-10}m^3/s for volume flux and volume rate of flow respectively at angle $\theta = 1.13^0$.

We suspect that there may have been compaction of the medium by virtue of the fact that the experiment was performed over a long period of time. The chance is that, as the water flows through, the sand get more and more compacted. Since we considered flow up to angle $25\ ^0$ tilt, there is the possibility that flow at that angle will be subjected to more compaction than flow at $\theta = 0^0$.

To examine this problem closely, a separate experiment was performed with samples A, C and E tilted at the same angles as the original experiment but instead of measuring pressure at intervals, discharge rate were observed per minute. The result is displayed in Tabled 3. Using $Q = VA$, the volume fluxes were determined. A point N′ was also observed, which has value higher than in Fig. 8, which suggests that compaction actually occured. A method of correction of the compaction factor was adopted. It was assumed that since we had made use of the same sample prepared the same way but only in the latter case compaction was avoided, then the point N and N′in both cases are equivalent. To establish that the values in Figures 8 and 9 are related, the ratio of values at V = 0 for samples A, C and E in both cases i.e (1.50^0, 1.79^0, 2.17^0) and (0.63^0, 0.75^0, 0.92^0) were found to be 2.40, 2.38 and 2.36 having average value of 2.38. We refer to this value as the ‘compaction coefficient’ and it was used to normalize the compaction effect. Using this assumption the values for which V= 0 in Fig. 8 were obtained to be 1.5^0, 1.6^0, 1.8^0, 2.1^0, and 2.2^0 as displayed is Table 4. In the same vein, the values of volume flux and volume rate of flow at the critical point are 1.04×10^{-4}m/s and 9.73×10^{-8}m^3/s respectively. $\theta = 4.2^0$at this point.

The relationship between porosity and the hydrostatic angle was established in Fig. 10 where a plot of porosity and hydrostatic angle was shown.

In order to verify the relationship that exist between the hydrostatic angle and porosity of a medium; the plot of porosity-hydrostatic angle (Fig.10) shows that they are linearly related as shown in equation (8) with correlation coefficient of 0.991637.

In locating a landfill, this condition is very necessary i.e zero flow situation which drastically minimizes migration of pollutants or tracer. It should be noted that other geological information are needed to achieve this.

6.0 Acknowledgement

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7.0 References

[1] John Bredehoeft, 1996. When Pollution goes with the Flow. Physics World. May 1996.47-48.

[2] Gleick, P.H., 1996. Water resources. In Encyclopedia of climate and Weather, ed. By Schneider, S.H., Oxford University Press, New York, vol.2 pp. 817-823, as in http://ga.water.usgs.gov/edu/watercyclegwdischarge.html. Retrieved on 7 October, 2014.

[3] Atkinson, B.K., 1989. Fractured Mechanics of Rock, Academic Press, New York. (As in Arturo Alenjadro, 1996).

[4] Blyth F.G.H. and de Freitas M. H., 1984. A Geology for Engineers.Seventh Edition. Edward Arnold A division of Holder and Stoughton London.

[5] Roberson J. A. and Crowe C.T., 1995. Hydraulic Engineering. 2nd edition. Wiley and Sons, INC. New York.

[6] Korfiatis P.,1989. Integrated approach to soil and groundwater pollution abatement and remediation. Edited by Joao B. et al. A Proceeding of the International Symposium, (SISIPPA), organized by the International Symposium On Integrated Approaches To Water Pollution Problems (SISIPPA). Published by ELSEVIER Applied Science London and New York.

[7] Jacob Bear and Verruijt Arnold, 1990. Modelling Groundwater Flow and Pollution. Reidel Publishing Company. Dordrecht, Holland.

[8] Abriola, L., 1989. Modelling migration of organic chemicals in groundwater systems – A review and assessment, Environmental Health Perspectives, 83, 117-143.

[9] Mercer, J.W., and Cohen, R.M., 1990. A review of immiscible fluids in the subsurface: properties, models, characterization and remediation: Journal of Contaminant Hydrology, vol. 6, p. 107-163. As in http://www.epa.gov/swerust1/pubs/fprref.pdf. Retrieved on 16th October, 2004.

[10] Kueper, B.H. and D.B. Mc Whorter, 1991. The behaviour of dense, nonaqueous phase liquids in fractured clay and rock, Ground Water, 29:716-728.

[11] Parker, B.L. et al, 1994. Diffusive disappearance of immiscible phase organic liquids in fractured geologic media. Ground Water 32(5): 805-820.

[12] Donald P. Goduto, 1999. Geotechnical Engineering-Principles and Practices. Prentice-Hall, INC. New Jersey.

[13] The U.S. Geological Survey Series of print publication "The Ground Water Atlass of the United States. http://capp.water.usgs.gov/gwa/ch_e/index.html. Retieved on Tue., October 15.

[14] Hsieh, P.A. Neuman S.P., Stiles G.K. and Simpson E.S et al, 1985. Field determination of the three-dimensional hydraulic conductivity of anisotropy media: 2. Methodology and application to fracture rocks, water Resources Research, 21:11, 1667-1676.

[15] Verruijt Arnold., 1982. Groundwater Flow. Second Edition. The Macmillan Press Limited London.

[16] Arturo Alejandro Keller, (1996), Single and Multiphase Flow In Fractured Media. Univ. CA at Santa Babara. Ph.D Thesis. http://www.bren.ucsb.edu/~keller/thesis/thesis.html Retrieved 9th October 2014.

[17] Ghildyal B. P. and Tripathi R. P., 1987. Soil Physics. Wiley Eastern Limited New Delhi.

[18] Atkinson J., 1993. The Mechanics of soil and Foundations. Mc GRAW-HILL Book Company England.

[19] Dicker D. and Sevian W.A., 1965. Transient Flow through Porous Mediums.Journal of Geophysical Research, Vol. 70, No. 20, 5043-5054