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A Mathematical Model for Estimating Pressure Drops as a Function of Well Trajectory

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Abstract

A pressure drop model is a vital tool used by the oil and gas industries to predict the performance of any wellbore system and this is well documented in the literatures. It is critical therefore to evaluate energy potential of every well drilled in a reservoir and this defines the life span of the well as well as the reservoir system.
In order to evaluate the performance of a well, the geometry will play a significantly role especially for a deviated well. In this study, development of analytical approach is adopted to investigate pressure drops behavior in deviated wells. It presents formulation of a new mathematical model to predict pressure drops in deviated wellbores. The existing pressure drop models were modified to accommodate multiple well geometries such as build-and-hold, build-hold-and-drop, and continuous build. The modified model enabled efficient assessment of the effect of pressure drops on deviated wellbores. It can therefore be used to analyze the performance of any deviated wellbore system which will further enhance the effective pressure maintenance in the well?.

Key words: Well trajectory; pressure drop; well performance; pressure maintenance.

Nomenclature

A=tubing cross sectional area, ft^2 $\bar{\rho}$ = fluids mixture density, Ib/cuft ΔP= pressure drop in pipe, psi
H_L=pressure head loss, psi q_L= liquid flow rate, bbl/dayθ= inclination angle, degree
L=tubing length, ftV= fluids velocity, ft/s ϕ = lead angle, degree
d=tubing internaldiameter, in P= pressure, psi $\Re_B$ = build rate, degree/100ft
F_s= force due to Shear Stress, Ib g= acceleration due to gravity, ft/s^2 L_1= measured depth for well vertical section, ft
F_g= force due to gravity, Ibg_c= unit conversion for acceleration, 32.17Ib_m-ft/Ib-s^2 $\Re_D$ = drop rate, degree/100ft
F= force due to friction, Ib ΔZ= change in TVD, ft L_2= measured depth for well build section, ft
f = fanning friction factorμ= fluids viscosity, cpL_3 = measured depth for well tangent section, ft
N_{Re}= Reynolds Number τ= shear stress, Ib/ft^2 L_4=measured depth for drop section, ft

1.0 Introduction

It is critical to know the energy required to transport fluids from its original location at the bottom of the well to the surface along the tubing. This energy required must be sufficient to overcome friction losses in the wellbore system and to lift reservoir fluids to the surface.The equations presented by Beggs and Brill [1] apply to flow in a pipe at any angle of inclination. The method has been found to slightly over predict pressure gradient in vertical wells in some cases. However, it gives good results for pipeline calculations. Asheim [2] also used the program called MONA to reproduce pressure drops measured in Prudhoe Bay field surface lines. These results were compared with those from the combined models formulated by Beggs and Brill [1], Dukler[3] and Eaton [4]and it was discoveredthat the former performed slightly better than latter. Hassan and kabir [5]proposed a model that is to be used especially for directional or deviated wells. The model predicts the flow pattern and the pressure gradient. A number of authors [6-10] presentedothermethods eitherexperimentally or semi theoretically in order to analyze the energy equation.
The energy losses or pressure losses must therefore be considered during the transportation of fluids along the tubing to ascertain the amount of energy the system must have to be at its optimum performance. This insight allows for efficient

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decision making in the areas of custody transfer and also to reduce the uncertainties in future economic of the well.
Well performance models are very essential in many activities such as well design, production optimization, field development, and reservoir management. A reliable and accurate well performance model is crucial for these tasks [11].
From this excerpt, it is further reaffirmed that the consideration of the wellbore performance model is a vital tool for reservoir pressure maintenance. This study presents a mathematical model to investigate the pressure drops behavior in deviated wells.

2.0 Problem Description

A Field with an existing well is to be developed with more wells. In order to achieve this, it was discovered that only deviated wells can be drilled to maximize the reserves given the Geology of the Formation. The big question is what type of deviated well will be suitable to maximize the reserves by considering pressure management culture?

3.0 Research Objective

The primary objective of this research is to develop a mathematical model of pressure drops for the three standard existing trajectories such as build-and-hold, build-hold-and-drop, andcontinuous build in literatures [12].

4.0 Methodology

The problem of pressure maintenance culture of the reservoir if drained with deviated wellbores was addressed by considering the pressure drop issues in the wellbores of the three selected standard existing trajectories (build-and-hold, build-hold-and-drop, and continuous build) in literatures[12]. An analytical approach was adopted to develop a mathematical model of pressure drops for each of the aforementioned trajectory. Developed model is homogeneous-flowmodel because it treats multiphase flow as a homogeneous mixture and does not considered the effect of liquid holdup (slip) for fluids upward movement.
This model was developed from first principle using Bernoulli`s equation for fluids flow in pipes and combined with such correlations as Darcy weisbach`s equation for head loss due to friction in pipes, Reynolds's equation for turbulent flow, Shear stress impacting on the wall of the pipe during turbulent flow, volumetric flow rate, inclination angles from the wellbores trajectories, and trajectories measured depth.
The developed pressure drops model was modified for each of the selected trajectory.

5.0 Physical Model Description

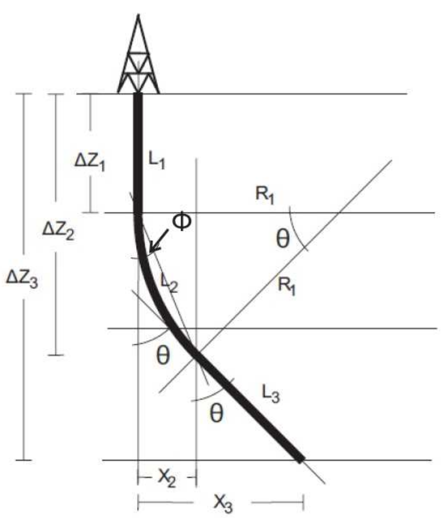

Figure 1: Diagrammatic representation of build-and-hold Trajectory

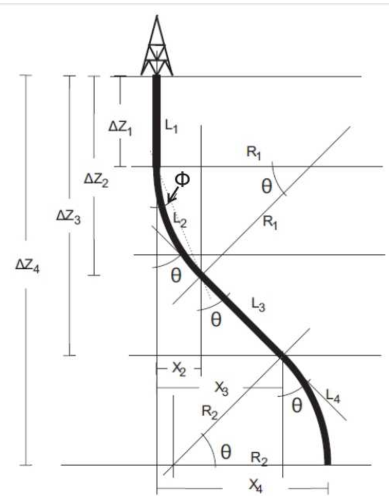

Figure 2: Diagrammatic representation of build-hold-and-drop trajectory

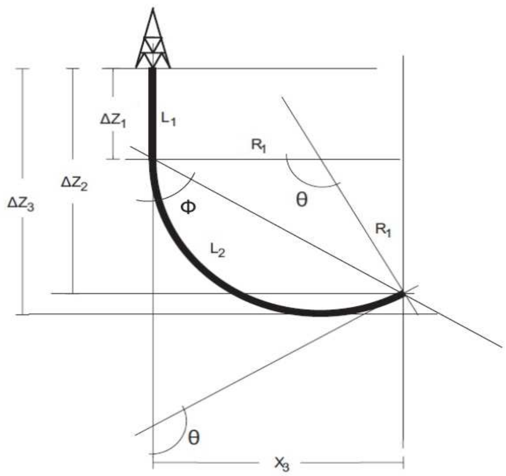

Figure 3: Diagrammatic representation of continuous build trajectory.

Figures 1, 2, and 3 illustrate typical deviated wells and their geological paths that convey fluids from the bottom of the well to the surface.
L_1= measured depth of surface (vertical) section of the well, ft
L_2= measured depth of build section of the well, ft
L_3= measured depth of tangent section of the well,ft
L_4= measured depth of drop section of the well, ft
X_3=horizontal departure, ft
ΔZ =True Vertical Depth(TVD), ft
θ = angle of inclination,degree
ϕ = angle between build section and TVD (lead angle). degree

6.0 Model Assumptions

The following assumptions were made during the model formulation:

1. The density of the fluids in the pipe is constant
2. Multiphase-flow system is adopted
3. The fluids are taken as homogeneous mixture
4. Turbulent flow regime is considered to avoid fluids loading at the toe of the well.
5. The temperature of the system is constant.
6. No liquid holdup (slip) for upward movement.
7. Steady state flow in pipes is adopted.

7.0 Model Description

Using the existing correlations and principles shown in equation (1-5) to develop the pressure drops model, it was derived as:

$$144\Delta P = \frac{5.171\times10^{-6}\, fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\,\rho q_L{}^2 g_c}{d^4 g} - \frac{\rho\Delta Z}{g_c} \text{ ----------------------} \quad (1)$$

Equation (1) shows the generalized model that was developed for this analytical study. A stepwise formulation of the model is shown in**appendix A.** furthermore, equation (1) was modified for each case considered; giving rise to equations (7), (8), and (9) for build-and-hold, build-hold-and-drop, continuous build trajectories respectively.
Equation (1) was actualized with the help of some correlations and principles, these are:
(i)**Bernoulli`s Equation[13]**

The equation is for Fluids flow through Pipes.

$$\frac{P}{\rho g} + \frac{V^2}{2g} + Z - H_L = \text{constant} \text{ ---} \quad (2)$$

(ii) **Darcy Weisbach`s Equation[14]**

The equation is for Head Loss due to Friction for Fluid flow through Pipes.

$$H_L = \frac{fLV^2}{2dg}$$ -- (3)

(iii) Volumetric Flow Rate Equation

This equation describes the volume of fluids being transported at a particular time.

$$q = A.V$$ -- (4)

(iv) ReynoldsEquation [15]

This equation is for Reynolds Number for turbulent flow.

$$N_{\text{Re}} = \frac{928\rho Vd}{\mu}$$ -- (5)

(v) Shear Stress Equation

The equation describes the force impacting on the Wall of the Pipes by the fluids

$$\tau = \frac{96V\mu}{d}$$ -- (6)

8.0 Modified Models

The modified models are shown below:

(1) Pressure Drops Model Build-and-Hold Trajectory

$$144\Delta P = 5.171*10^{-6}*\frac{fL\overline{\rho q}_L \tau g_c}{g\mu d^2} - \frac{7.102*10^{-5}*\overline{\rho q}_L^2 g_c}{gd^4} - \frac{\overline{\rho}(L_1 + L_2 COS\phi + L_3 COS\theta)}{g_c}$$ ------------- (7)

(2) Pressure Drops Model for Build-Hold-and-Drop Trajectory

$$144\Delta P = \frac{5.171\times10^{-6}\ fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\rho q_L{}^2 g}{d^4 g} - \frac{\rho(L_1 + L_2\cos\phi + L_3\cos\theta + L_4\cos\phi)}{g_c}$$ -(8)

(3) Pressure Drops Model for Continuous Build Trajectory

$$144\Delta P = \frac{5.171\times10^{-6}\ fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\rho q_L{}^2 g}{d^4 g} - \frac{\rho(L_1 + L_2\cos\phi)}{g_c}$$ -------------------------------- (9)

9.0 Conclusion

The following observations have been obtained from this study:

(1) A modified pressure drops model for thethree trajectories was successfully developed.
(2) The designed model allows for efficient decision making in the areas of custody transfer.
(3) The modified models allow for proper pressure evaluation of any deviated wellbore.
(4) Efficient reservoir pressure maintenance can be achieved with this model.

The observations obtained from this study show that the modified pressure drops model developed has met all the conditions assumed during the formulation.

10.0 Appendix A: Model Formulation for Pressure Drop in the Tubing.

The sum of all the forces acting on a particular section of the tubing in the wellbore is equaled to zero:

$$\sum F_s + F_g + F = 0$$ -- (A-1)

Bernoulli equation for fluids flow through pipes is considered for building conservation of energy from this section of tubing where all the forces are acting:

$$\frac{P}{\rho} + \frac{V^2}{2g} + Z - H_L = \text{Constant}$$ -- (A-2)

$$H_L = \left(\frac{P_1}{\rho g} - \frac{P_2}{\rho g}\right) + \left(\frac{V^2{}_1}{2g} - \frac{V^2{}_2}{2g}\right) + \left(Z_1 - Z_2\right) \text{ ------- (A-3)}$$

$$H_L = \frac{\Delta P}{\rho g} + \frac{\Delta V^2}{2g} + \Delta Z \text{ ------- (A-4)}$$

From Darcy weisbach equation for Head loss (H_L) due to friction for fluids flow through pipes

$$H_L = \frac{fLV^2}{2dg}$$

$$\frac{fLV^2}{2dg} = \frac{\Delta P}{\rho g} + \frac{V^2}{2g} + \Delta Z \text{ ------- (A-5)}$$

From volumetric flow rate, q=A.V,

$$\Delta P = \frac{fL\rho q_L{}^2}{2dgA^2} - \frac{\rho q_L{}^2}{2gA^2} - \rho\Delta Z \text{ ------- (A-6)}$$

Considering Reynolds number for turbulent flow in pipes:

$$N_{\mathrm{Re}} = \frac{928\rho Vd}{\mu}$$

$$\Delta P = \frac{fL\rho q_L{}^2}{\dfrac{2N_{\mathrm{Re}}\mu g A^2}{928\rho V}} - \frac{\rho q_L{}^2}{2gA^2} - \rho\Delta Z \text{ ------- (A-7)}$$

Considering shear stress on the wall of the tubing for turbulent flow in pipes:

$$\tau = \frac{96V\mu}{d},$$

$$\Delta P = \frac{4.83\, fL\rho^2 q_L{}^2 \tau d}{N_{\mathrm{Re}}\mu^2 g A^2} - \frac{\rho q_L{}^2}{2gA^2} - \rho\Delta Z \text{ ------- (A-8)}$$

$$\Delta P = \frac{77.33\, fL\rho^2 q_L{}^2 \tau}{N_{\mathrm{Re}}\mu^2 g d^3 \pi^2} - \frac{8\rho q_l{}^2}{g\pi^2 d^4} - \rho\Delta Z \text{ ------- (A-9)}$$

$$\Delta P = \frac{5.718\times10^{-5}\, fL\rho^2 q_L{}^2 \tau}{N_{\mathrm{Re}}\mu^2 d\ \ g} - \frac{7.102\times10^{-2}\, \rho q_L{}^2}{gd^2} - \rho\Delta Z \text{ ------- (A-10)}$$

Divide through with unit conversion for acceleration due to gravity $g_c = 32.17 Ib_m - ft / Ib - s^2$

$$144\Delta P = \frac{5.718\times10^{-5}\, fL\rho^2 q_L{}^2 \tau g_c}{N_{\mathrm{Re}}\mu^2 d^3 g} - \frac{7.102\times10^{-5}\, \rho q_L{}^2 g_c}{gd^4} - \frac{\rho\Delta Z}{g_c} \text{ ------- (A-11)}$$

$$V = 0.01192 q_L / d^2$$

$$144\Delta P = \frac{5.171\times10^{-6}\, fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\, \rho q_L{}^2 g_c}{d^4 g} - \frac{\rho\Delta Z}{g_c} \text{ ------- (A-12)}$$

Appendix B: Model Formulation for the Three Trajectories

Case One: Build-and-Hold Trajectory

$$R_1 = \frac{180}{\pi \Re_B} \text{ ------- (B-13)}$$

$$\tan\alpha = \frac{(X_3 - R_1)}{(\Delta Z_3 - \Delta Z_1)} \text{ ------- (B-14)}$$

$$\alpha = \tan^{-1}\left(\frac{X_3 - R_1}{\Delta Z_3 - \Delta Z_1}\right) \text{ ------- (B-15)}$$

$$\beta = 90 - \alpha \text{ ------- (B-16)}$$

$$\beta = 90 - \tan^{-1}\left(\frac{X_3 - R_1}{\Delta Z_3 - \Delta Z_1}\right) \text{ ------- (B-17)}$$

$$\sin\Omega = \frac{R_1}{\sqrt{(X_3 - R_1)^2 + (\Delta Z_3 - \Delta Z_1)^2}} \text{ ------- (B-18)}$$

$$\Omega = \sin^{-1}\left(\frac{R_1}{\sqrt{(X_3 - R_1)^2 + (\Delta Z_3 - \Delta Z_1)^2}}\right) \text{ ------- (B-19)}$$

$$\gamma = 90 - \sin^{-1}\left(\frac{R_1}{\sqrt{(X_3 - R_1)^2 + (\Delta Z_3 - \Delta Z_1)^2}}\right) \text{ ------- (B-20)}$$

$$\theta = 180 - (\beta + \gamma) \text{ ------- (B-21)}$$

$$\theta = \tan^{-1}\left(\frac{X_3 - R_1}{\Delta Z_3 - \Delta Z_1}\right) + \sin^{-1}\left(\frac{R_1}{\sqrt{(X_3 - R_1)^2 + (\Delta Z_3 - \Delta Z_1)^2}}\right) \text{ ------- (B-22)}$$

$$L_1 = \Delta Z_1 \text{ ------- (B-23)}$$

$$L_2 = \frac{\theta}{\Re_B} \text{ ------- (B-24)}$$

$$L_3 = \frac{\Delta Z_3(\Delta Z_1 + R_1 \sin\theta)}{\cos\theta} \text{ ------- (B-25)}$$

$$\phi = \tan^{-1}\left(\frac{\cos\theta}{\sin\theta}\right) \text{ ------- (B-30)}$$

$$\Delta Z_2 = L_2 \cos\phi \text{ ------- (B-31)}$$

$$\Delta Z_3 = L_3 \cos\theta \text{ ------- (B-32)}$$

$$Z = \Delta Z_1 + \Delta Z_2 + \Delta Z_3 \text{ ------- (B-33)}$$

$$Z = L_1 + L_2 \cos\phi + L_3 \cos\theta \text{ ------- (B-34)}$$

$$144\Delta P = 5.171*10^{-6} * \frac{fL\overline{\rho q}_L \tau g_c}{g\mu d^2} - \frac{7.102*10^{-5} * \overline{\rho q}_L^2 g_c}{gd^4} - \frac{\overline{\rho}(L_1 + L_2 COS\phi + L_3 COS\theta)}{g_c} \text{ -- (B-35)}$$

Case Two: Build-Hold-and-Drop Trajectory
A single equation for hold inclination proposed by Wiggins et al [16]

$$R_1 = \frac{180}{\pi\Re_B} \text{ ------- (B-36)}$$

$$R_2 = \frac{180}{\pi\Re_D} \text{ ------- (B-37)}$$

$$R = R_1 + R_2 \text{ ---------- (B-38)}$$

$$TVD = \Delta Z_4 - \Delta Z_1 \text{ ---------- (B-39)}$$

$$X = X_4 - R \text{ ---------- (B-40)}$$

$$L = \sqrt{TVD^2 + X^2 - R^2} \text{ ---------- (B-41)}$$

$$\theta = \sin^{-1}\left[\frac{(TVD)R + XL}{R^2 + L^2}\right] \text{ ---------- (B-42)}$$

$$L_2 = \frac{\theta}{\Re_B} \text{ ---------- (B-43)}$$

$$L_3 = \frac{\Delta Z_3 - (\Delta Z_1 + R_1 \sin\theta)}{\cos\theta} \text{ ---------- (B-44)}$$

$$L_4 = \frac{\theta}{\Re_D} \text{ ---------- (B-45)}$$

$$\Delta Z_1 = L_1 \text{ ---------- (B-46)}$$

$$\tan\phi = \frac{\cos\theta}{\sin\theta} \text{ ---------- (B-47)}$$

$$\Delta Z_2 = L_2 \cos\phi \text{ ---------- (B-48)}$$

$$\Delta Z_3 = L_3 \cos\theta \text{ ---------- (B-49)}$$

$$\Delta Z_4 = L_4 \cos\phi \text{ ---------- (B-50)}$$

$$Z = \Delta Z_1 + \Delta Z_2 + \Delta Z_3 + \Delta Z_4 \text{ ---------- (B-51)}$$

$$Z = L_1 + L_2 \cos\phi + L_3 \cos\theta + L \cos\phi \text{ ---------- (B-52)}$$

$$144\Delta P = \frac{5.171\times10^{-6}\, fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\,\rho q_L{}^2 g}{d^4 g} - \frac{\rho(L_1 + L_2\cos\phi + L_3\cos\theta + L_4\cos\phi)}{g_c} \text{ -- (B-53)}$$

Case Three: Continuous Build Trajectory

$$R_1 = \frac{180}{\pi\Re_B} \text{ ---------- (B-54)}$$

$$\alpha = \tan^{-1}\left(\frac{X_3 - R}{\Delta Z_2 - \Delta Z_1}\right) \text{ ---------- (B-55)}$$

$$\beta = 90 - \tan^{-1}\left(\frac{X_3 - R}{\Delta Z_2 - \Delta Z_1}\right) \text{ ---------- (B-56)}$$

$$\theta + \beta = 180 \text{ ---------- (B-57)}$$

$$\theta = 90 + \tan^{-1}\left(\frac{X_3 - R}{\Delta Z_2 - \Delta Z_1}\right) \text{ ---------- (B-58)}$$

$$L_2 = \frac{\theta}{\Re_B} \text{ ---------- (B-59)}$$

$$\tan\phi = \frac{X_3}{\Delta Z_2 - \Delta Z_1} \text{ ---------- (B-60)}$$

$$\Delta Z_1 = L_1 \text{ ---------- (B-61)}$$

$$\Delta Z_2 = L_2 \cos\phi \text{ ---------- (B-62)}$$

$$Z = L_1 + L_2 \cos\phi \text{ ----- (B-63)}$$

$$144\Delta P = \frac{5.171\times10^{-6}\, fL\rho q_L \tau g_c}{\mu d^2 g} - \frac{7.102\times10^{-5}\,\rho q_L{}^2 g}{d^4 g} - \frac{\rho(L_1 + L_2\cos\phi)}{g_c} \text{ ----- (B-64)}$$

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