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# The Spectral Variation of Transverse Magnetic Scattering Amplitude Coefficient With Size Parameters In Electromagnetic Wave By An Infinitely Circular Cylinder

*Suleiman A. B and Zarma S. S*

**Department of Physics, Federal University, Dutse P. M. B 7156, Dutse Jigawa State Nigeria.**

## *Abstract*

***The transverse magnetic scattering coefficient and the transverse internal field coefficient as applied to electromagnetic wave scattering has been studied and their relationship with size parameters for n= 20 and refractive index of 1.5 for both real and imaginary parts have been presented in this paper. The computer generated results of spectral variation of the Mie coefficients in the range of size parameters between 10 and 20 were graphically presented to have two resonances. The peaks of the morphology dependence resonance (MDR) are manifestation of the resonance behavior of the Mie coefficients. It was observed that for real index of refraction, the results for the coefficientsd$_n$ and b$_n$ show that $\Re eb_n$ and $\Re ed_n$ lie between zero (0) and one (1), while Imb$_n$ and Imd$_n$ lie between – 0.5 and + 0.5. Resonances occur for all values of b$_n$ and d$_n$ equal to 1 + i0.***

**Keywords:** Transverse Magnetic, Scattering Amplitude Coefficient, Infinitely Circular cylinder, Morphology Dependence Resonance

## 1.0 Introduction

In 1881 Lord Rayleigh first treated the classical electromagnetic problem of the incidence of plane electric waves on an insulating dielectric cylinder. He investigated the diffraction of a plane wave at normal incidence by a homogeneous dielectric cylinder and his solution was generalized for obliquely incident plane waves when the magnetic vector of the incident wave is transverse to the axis of the cylinder [1].

More recent investigation in the area of scattering by the arrays of cylinders have been conducted in which the spatial and spectral domain forms of the greens function for the diffraction of plane wave at arbitrary incident in the x-y plane on a grating oriented along the x-axis were developed [2]. During the same year parallel investigationtook place where techniques for representing in absolutely convergent forms of the lattice sums in doubly periodic electromagnetic diffraction problems were studied[3].

The use of poison's summation to obtain effective formulas for sums arising in scattering problems for the case of an infinite number of cylinders ordered periodically along the line in the form of an infinite array were considered [4].The layered multiple scattering method for anti-plane wave scattering from multiple gratings consisting of parallel cylinders were also treated [5].The direct Neuman iteration technique in order to acquire the exact solutionfor the scattering coefficients of an infinite grating in the form of an infinite series were employed [6] and an analogue of Twersky's solution[7] is acquired for obliquely incident plane electromagnetic waves.

The Mie solutions or Lorentz-Mie-Debye theory provides an analytical solution of Maxwell equations for the scattering of electromagnetic radiation by spherical particles in terms of infinite series [8,9]

Light scattering and absorption by particles have many important applications. These include estimating thesmoke capacity in a nuclear winter (cold dark period after nuclear explosions) scenario, effects on the propagation of millimeter and radar waves caused by aerosols, laser light scattering diagnostics of biological cells, polymers, and colloidal aggregates; radiation properties of flame soot agglomerates; detection of smokes emitted from automobile exhausts or coal fired power plants [10].

The other point of view includes the terrestrial atmosphere which is among the best media for direct application of this field. Its diverse optical phenomena and their variations during day light, twilight and night conditions must have been subjected to observation and interpretation from the beginning of human life. The particular aspect of the terrestrial problem in which scattering is involved include the cloudless sunlit atmosphere, the particulate layers in the stratosphere and mesosphere, water

Corresponding author: Suleiman A.B., E-mail: suleiman_abdussalam@yahoo.com, Tel.: +2348039644595

droplet and ice cloud and disturbed condition produced by major volcanic eruption, forest fires and man-made pollutions.
In the field of radio meteorology, the radar reflectively of precipitation particles represents a particular set of scattering. Others are in laser communications and in atmospheric transmissions; this application is beset with difficulties such as loss of power due to atmospheric attenuation, divergence of the beam and loss of the coherence. Saturn rings are also of particular interest since they are related to scattering phenomena. Saturn's rings still represent another fascinating problem that bears re-examination in the field of electromagnetic scattering.
The nature of interplanetary and interstellar dust and interplanetary condensation is of fundamental importance in the theory of star and planet formations [11].

## 2.0 Theoretical Background

The expressions for the Mie coefficients $a_n$ and $b_n$ contain the cylindrical Bessel function $J_n(mx)$. High order or large argument cylindrical Bessel function may result in over flow. This statement was supported by (Huifen*et al*, 2006).

Solutions for the coefficients gives:

$$\left.\begin{aligned} Case\ \ I:\ \ b_n &= \frac{mJ_n^{'}(y)J_n(x)-J_n(y)J_n^{'}(x)}{mJ_n^{'}(y)H_n(x)-J_n(y)H_n^{'}(x)}, \\ Case\ \ II:\ a_n &= \frac{J_n^{'}(y)J_n(x)-mJ_n(y)J_n^{'}(x)}{J_n^{'}(y)H_n(x)-mJ_n(y)H_n^{'}(x)}, \end{aligned}\right\} \tag{2.0}$$

Where primes denotes derivatives and $x=\rho a,\ y=\gamma\rho a.$

The over flow can be avoided if both the denominator and numerator of these equations. (i.e. 2.0) are divided by the cylindrical Bessel function $J_n(mx)$. This produces equation (2.1) and (2.2).

$$a_n = \frac{\left[\frac{A_n}{m}(mx)+\frac{n}{x}\right]J_n(x)-J_{n-1}(x)}{\left[\frac{A_n}{m}(mx)+\frac{n}{x}\right]H_n^{(1)}(x)-H_{n-1}^{(1)}(x)} \tag{2.1}$$

$$b_n = \frac{\left[mA_n(mx)+\frac{n}{x}\right]J_n(x)-J_{n-1}(x)}{\left[mA_n(mx)+\frac{n}{x}\right]H_n^{(1)}(x)-H_n^{'}(x)-H_{n-1}^{(1)}(x)} \tag{2.2}$$

where

$$A_n(mx)=\frac{\psi_n^{'}(mx)}{\psi_n(mx)}=\frac{d[\log\psi_n(mx)]}{d(mx)} \tag{2.3}$$

The function $A_n(mx)$ is called the logarithmic functiondenoted by (Deimendjian, 1969). It has the following form:

$$A_n(mx)=-\frac{n}{mx}+\frac{J_{n-1}(mx)}{J_n(mx)}=-\frac{n}{mx}+\frac{1}{\frac{n}{mx}-A_{n-1}(mx)} \tag{2.4}$$

The scattered field expansion coefficients $a_n$ *and* $b_n$ are calculated in a program. The expression for each of these can be rewritten by defining the logarithmic derivative [14].

$$A_n(mx)=\frac{J_n^{'}(mx)}{J_n(mx)} \tag{2.5}$$

By making use of the recursion formulae for Bessel function of arbitrary order and argument,

$$\psi'_n(mx)=mxJ_{n-1}(mx)-nj_n(mx)\ , \tag{2.6}$$

the function $A_n(mx)$ can be written in the form

$$A_n(mx) = \frac{\psi_n^{'}(mx)}{\psi_n(mx)} = \frac{mxj_{n-1}(mx) - nj_n(mx)}{mxj_n(mx)} \quad (2.7)$$

$$= \frac{-n}{mx} + \frac{j_{n-1}(mx)}{j_n(mx)}$$

Thus

$$A_{n-1}(mx) = \frac{-(n-1)}{mx} + \frac{j_{n-2}(mx)}{j_{n-l}(mx)} \quad (2.8)$$

$$= \frac{-(n-1)}{mx} + \frac{\frac{2n-1}{mx} j_{n-1}(mx) - J_n(mx)}{j_{n-1}(mx)}$$

or

$$\frac{J_n(mx)}{J_{n-1}(mx)} = \frac{n}{mx} - A_{n-1}(mx) \quad (2.9)$$

Substituting equation (2.9) into (2.7) we obtain a recursion relation for $A_n(mx)$:

$$A_n(mx) = \frac{-n}{mx} + \frac{1}{\frac{n}{mx} - A_{n-1}(mx)} \quad (2.10)$$

From equation (2.7) we have:

$$A_0(mx) = \frac{J_1(mx)}{J_0(mx)} \quad (2.11)$$

$A_n(mx)$ can be calculated using downward recursion starting at a maximum index $n_{mx}$ [14].

$$n_{mx} = \max(n_c, |mx|) + 15 \quad (2.12)$$

Starting the recursion with:

$$A_n(mx) = 0.0 + i0.0$$

when $n = n_{mx}$ yields accurate values of:

$$A_n(mx) \quad for \quad n < n_C$$

## 3.0 Computation

The program[15] is used to calculate internal or scattered field quantities at the morphology-dependent resonances (MDR) of the cylinder, while the number of terms should be increased. To obtain results for the narrowest resonances, it is recommended that $n_c$ to be increased to $|mx|$ as a starting point. However, Bessel functions are important for many problems of wave propagation. Example the function is used to expand a potential in some cylindrical region

$0 \leq \rho \leq a$.

The second order differential equation,

$$x^2 \frac{d^2}{dx^2} j_\gamma(x) + x\frac{d}{dx} j_\gamma(x) + (x^2 - \gamma^2) j_\gamma(x) = 0, \quad (3.1)$$

is called Bessel's equation. Usually $x = \rho\alpha$. Where $\alpha$ the radial variable in cylindrical coordinates is $j_\gamma$ is called the "Bessel function of the first kind". When $m$ is not an integer, (3.2) has two linearly independent solutions given by $j_\gamma(x)$ and $j_{-\gamma}(x)$.

The second-order differential equation

$$x^2 \frac{d^2}{dx^2} R_\gamma(x) + x\frac{d}{dx} R_\gamma(x) - (x^2 + \gamma^2) R_\gamma(x) = 0 \quad (3.2)$$

The Bessel function of the second kind defined by $N_\gamma\ (x)$ can form another solution of (3.2). $N_\gamma$ is called Neumann function or Bessel function of the second kind. In addition to $j_\gamma$ and $N_\gamma$ there are other solutions to Bessel's equation:

$$H_\gamma^{(1)} = j_\gamma(x) + i\,N_\gamma(x)$$
$$H_\gamma^{(2)} = j_\gamma(x) + i\,N_\gamma(x)$$

These are called Hankel functions or "Bessel functions of the third kind".

Just as $j_\gamma$ and $N_\gamma$ are analogous to sine and cosine, $H_\gamma^{(1)}$ and $H_\gamma^{(2)}$ are analogous to exponentials of the form $\ell^{\pm\ \{\ (ix-\delta)/\sqrt{x}\ \}}$.

Observe that if $\gamma = n$ is an integer, then $j_\gamma(x)$ and $j_{-\gamma}(x)$ are no longer linearly independent.

This is because:

$$j_{-n}(x) = (-1)^n\, j_n(x)$$

is called the "Modified Bessel equation".

This is equivalent to Bessel equation with $x$ replaced by $ix$. Accordingly we define

$$I_m(x) = \frac{j_m(ix)}{i^m} \qquad K_\gamma = \frac{\pi(i)^{\gamma+1}}{2}\left[j_\gamma(ix) + i\,N_\gamma(ix)\right]$$

For large $x$ then,

$$I_\gamma \to \frac{1}{\sqrt{x}}e^{x} \tag{3.3}$$

$$K_\gamma \to \frac{1}{\sqrt{x}}e^{-x} \tag{3.4}$$

Since these functions do not oscillate they have no simple orthogonality relations.

## 4.0 Results and Discussion

**Coefficient Amplitude and Size Parameter**

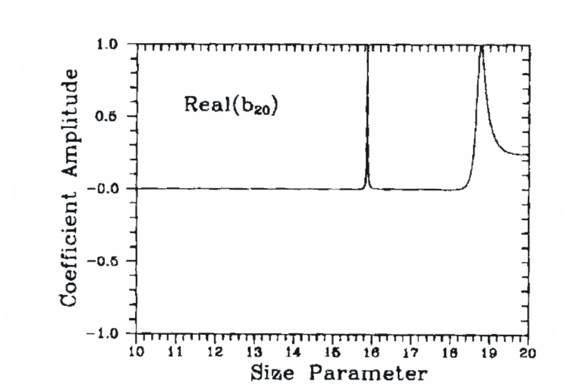


**Fig. 1a**Real part of the TM scattering coefficient as a function of size parameter for $n = 20$ and an index of refraction of $1.5$

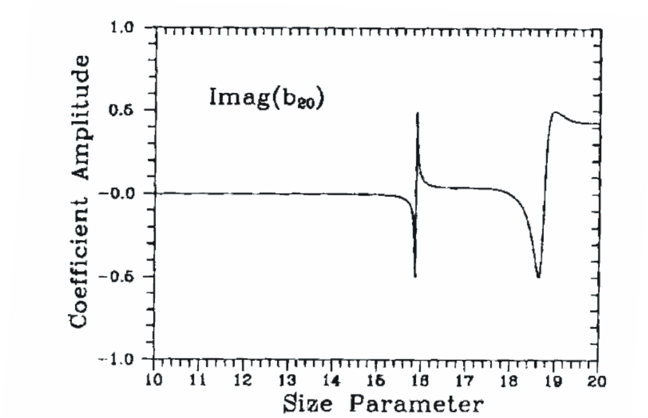


**Fig.1b** Imaginary part of the TM scattering coefficient as a function of size parameter for $n = 20$ and index of refraction of $1.5$

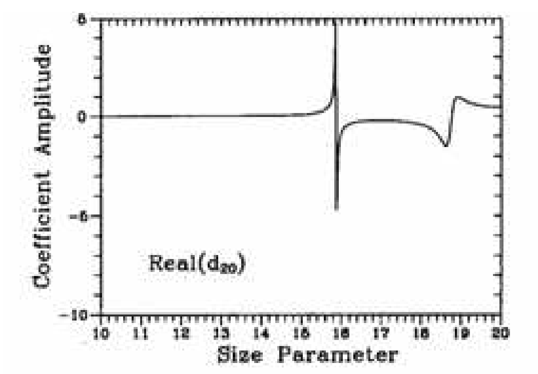


**Fig.1c**Real part of the TM internal field coefficient as a function of size parameter for $n = 20$ and index of refraction of 1.5.

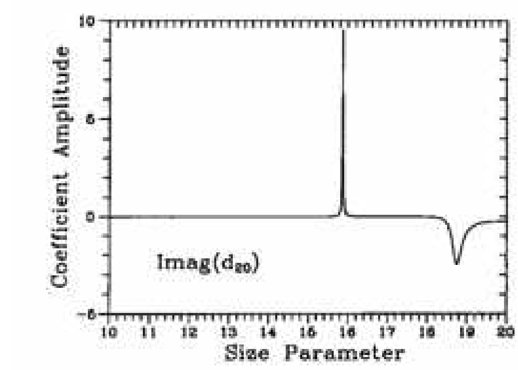


**Fig.1d** Imaginary part of the TM internal field coefficient as a function of size parameter for *n*=20 and index of refraction of 1.5

Superimposed on the scattering and extinction efficiency curves is the Morphology Dependent Resonances (MDR) which becomes more pronounced for lossless cylinder and narrower as the size parameter increases. The MDR peaks are manifestations of the resonance behavior of the Mie coefficients $a_n$ and $d_n$. We have seen that for real index of refraction, the results for $a_n$ and$d_n$ shows that $\Re eb_n$ and $\Re ea_n$ lie between zero and one while $Imb_n$ and $Imd_n$ lie between – 0.5 and + 0.5. Resonances occur for all values of $b_n$ and $a_n$ equal to 1 + i0.

## 5.0 Conclusion

This work has presented the spectral variation of $b_n$ and $d_n$amplitude coefficients, which has two resonances in the range of size parameters between 10 and 20, the behavior of $\mathrm{Re}(d_{20})$ and $\mathrm{Im}(d_{20})$is also presented. The main features of the resonance peaks are; their widths decrease as $n$ increases for a given $l$, and their width increase as $l$ increases for a given $n$. Also as the index of refraction increases, the position of the resonances shift to lower value of $x$, and their widths become narrower. Resonances having relatively narrow width occur for $n$ in the range $x$ to $mx$

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