Journal of the Nigerian Association of Mathematical Physics
***Volume 33, (January, 2016*), pp 119 – 124**

Alternative Linear Combined Product Estimator of Population Mean in Simple Random Sampling

A.A. Adewara

Department of Statistics, University of Ilorin, Ilorin. Kwara State. Nigeria.

Abstract

In this study, a linear combined product estimator was proposed. This proposed estimator combined classical product estimator with our usual regression estimator. Two data sets were used to determine the efficiency of this proposed linear combined product estimator and this proposed linear combined product estimator was found to be more efficient than the classical product estimators [1, 2] provided $0 < \alpha < 1$ ***and*** $\frac{\bar{X}}{\bar{X} + \rho} \geq 1$.

Keywords: Linear, combined, product, estimator, alternative, mean square error

1.0 Introduction

Let N and n be the population and sample sizes respectively, $\bar{X}$ and $\bar{Y}$ be the population means for the auxiliary variable (X) and the variable of interest (Y), $\bar{x}$ and $\bar{y}$ be the sample means based on the sample drawn. Then classically [3, 4],

$$\bar{y}_p = \frac{\overline{yx}}{\bar{X}},$$

$$bias(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}[\rho c_x c_y] \quad \ldots \quad (1)$$

and

$$mse(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y)] \quad \ldots \quad (2)$$

respectively.

The literature on survey sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Product method of estimation is a good example in this context. If the correlation between the study variable y and the auxiliary variable x is negative, the product method of estimation is quite effective.

In sample surveys, supplementary information is often used for increasing the precision of estimators [5 - 11).

2.0 On the Product Estimators [1, 2]

Suppose pairs (x, y) (i=1, 2, ….., n) observations are taken on n units sampled form N population units using simple random sampling without replacement scheme, $\bar{X}$ and $\bar{Y}$ are the population means for the auxiliary variable (X) and variable of interest (Y), and $\bar{x}$ and $\bar{y}$ are the sample means based on the sample drawn.

A family of estimators was suggested in [12] as: $$t = \bar{y}\left[\frac{a\bar{x}+b}{\alpha(a\bar{x}+b)+(1-\alpha)(a\bar{x}+b)}\right]^{g}$$ … (3)

where $a \neq 0$, b are either real numbers or a functions of the known parameters of the auxiliary variable x such as standard derivation σ_x, coefficient of variation, c_x, Skewness $\beta_{1(x)}$, Kurtosis $\beta_{2(x)}$ and correlation coefficient ρ.

Corresponding author: A.A. Adewara, E-mail: aaadewara@gmail.com, Tel.: +2348033813093

(i). When α=1 a=1, b= 0, g=-1, we have the usual product estimator, $\bar{y}_p = \frac{\bar{y}\bar{x}}{\bar{X}}$ with

$$mse(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y]$$ … (4)

(ii). when α=1 a=1, b=C_x, g=1, we have the product estimator[1],

$$mse(\bar{y}_{PD}) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y + c^2{}_x(\frac{\bar{X}}{\bar{X}+c_x}) + 2(\frac{\bar{X}}{\bar{X}+c_x})\rho c_x c_y)]$$ … (5)

(iii). when α=1 a=-1, b=ρ ,g=1, we have the product estimator [2], $\bar{y}_{ST} = \bar{y}[\frac{\bar{x}+\rho}{\bar{X}+\rho}]$ with

$$mse(\bar{y}_{ST}) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y + c^2{}_x(\frac{\bar{X}}{\bar{X}+\rho}) + 2(\frac{\bar{X}}{\bar{X}+\rho})\rho c_x c_y)]$$ … (6)

3.0 On the Proposed Linear Combined Product Estimator

The classical product estimator used in this study is defined as $\bar{y}_p = \alpha(\frac{\bar{y}\bar{x}}{\bar{X}+\rho})$ while our traditional regression estimator used is defined as: $\bar{y}_{reg} = \beta[\bar{y} + b(\bar{X}-\bar{x})]$ such that $\alpha + \beta = 1$. Their biases and mean square errors are given respectively as:

$$bias(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}[\rho c_x c_y]$$

and

$$mse(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y)]$$ (as given in eq. (2) and (3) above) ,

$$bias(\bar{y}_{reg}) = 0 \quad \ldots (7)$$

and

$$mse(\bar{y}_{reg}) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_y(1-\rho^2)] \quad \ldots (8)$$

The proposed linear combined product estimator is given as:

$$\bar{y}_{paa} = \alpha(\frac{\overline{yx}}{\bar{X}+\rho}) + \beta[\bar{y} + b(\bar{X} - \bar{x})] \quad \ldots (9)$$

which is the linear combination of classical product estimator, $\bar{y}_p$, and the traditional regression estimator, $\bar{y}_{reg}$, where $\bar{x} = \bar{X}(1+\Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$

The bias and mean square error of this proposed linear combined product estimator, $\bar{y}_{paa}$ are given as:-

$$bias(\bar{y}_{paa}) = (\frac{N-n}{Nn})(\frac{\bar{X}}{\bar{X}+\rho})\alpha\bar{Y}(\rho c_x c_y) \quad \ldots (10)$$

and

$$mse(\bar{y}_{paa}) = (\frac{N-n}{Nn})(\frac{\bar{X}}{\bar{X}+\rho})^2\alpha^2\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y] \quad \ldots (11)$$

provided $0 < \alpha < 1$ and $(\frac{\bar{X}}{\bar{X}+\rho}) \geq 1$.

The proof of (10) and (11) are as shown below:

4.0 Derivation of Bias and Mean Square Error of $\bar{y}_{paa}$

Let,

$$\bar{y}_{paa} = \alpha(\frac{\overline{yx}}{\bar{X}+\rho}) + (1-\alpha)[\bar{y} + b(\bar{X} - \bar{x})]$$

, where, $\bar{x} = \bar{X}(1+\Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$ as earlier defined.

Therefore, using power series expansion,

$$\bar{y}_{paa} = \alpha(\frac{\overline{yx}}{\bar{X}+\rho}) + \beta[\bar{y} + b(\bar{X} - \bar{x})] = \alpha(\frac{\overline{yx}}{\bar{X}+\rho}) + (1-\alpha)[\bar{y} + b(\bar{X} - \bar{x})]$$

$$= \alpha(\frac{\bar{Y}(1+\Delta_{\bar{y}})\bar{X}(1+\Delta_{\bar{x}})}{(\bar{X}+\rho)} + (1-\alpha)[\bar{Y}(1+\Delta_{\bar{y}}) + b(\bar{X} - \bar{X}(1+\Delta_{\bar{x}}))]$$

$$= \alpha\bar{Y}(\frac{\bar{X}}{\bar{X}+\rho})(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}}) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} + b\bar{X} - b\bar{X} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\bar{Y}\Delta_{\bar{y}}$$

$$\alpha b\bar{X} - \alpha b\bar{X} - \alpha bX\Delta_{\bar{x}}$$

$$= \alpha\bar{Y}(\frac{\bar{X}}{\bar{X}+\rho})(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}}) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\bar{Y}\Delta_{\bar{y}} +$$

$$- \alpha bX\Delta_{\bar{x}}$$

$$= \alpha\bar{Y}(\frac{\bar{X}}{\bar{X}+\rho})(1+\Delta_{\bar{y}}+\Delta_{\bar{x}}+\Delta_{\bar{y}}\Delta_{\bar{x}}) + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\bar{Y}\Delta_{\bar{y}} +$$

$$- \alpha bX\Delta_{\bar{x}}$$

$$= \alpha\bar{Y}k + \alpha\bar{Y}k\Delta_{\bar{y}} + \alpha\bar{Y}k\Delta_{\bar{x}} + \alpha\bar{Y}k\Delta_{\bar{y}}\Delta_{\bar{x}} + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha\bar{Y} - \alpha\bar{Y}\Delta_{\bar{y}} +$$

$$- \alpha bX\Delta_{\bar{x}}$$

where $k = (\frac{\bar{X}}{\bar{X}+\rho})$

$$Bias(\bar{y}_{paa}) = E[\alpha\bar{Y}k + \alpha\bar{Y}k\Delta_{\bar{y}} + \alpha\bar{Y}k\Delta_{\bar{x}} + \alpha\bar{Y}k\Delta_{\bar{y}}\Delta_{\bar{x}} + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha$$

$$- \alpha bX\Delta_{\bar{x}} - \bar{Y}]$$

Let, $E(\Delta_{\bar{y}}) = E(\Delta_{\bar{x}}) = 0, E(\Delta_{\bar{x}}{}^2) = \frac{S^2{}_x}{\bar{X}^2} = c^2{}_x, E(\Delta_{\bar{y}}{}^2) = \frac{S^2{}_y}{\bar{Y}^2} = c^2{}_y$ and $E(\Delta_x\Delta_y) = \frac{S_{xy}}{\bar{X}\bar{Y}} = \rho c_x c_y$. Then,

$$Bias(\bar{y}_{paa}) = E[\alpha\bar{Y}k\Delta_{\bar{y}}\Delta_{\bar{x}}]$$

$$Bias(\bar{y}_{paa}) = (\frac{N-n}{Nn})\alpha\bar{Y}k(\rho c_x c_y)$$

$$= (\frac{N-n}{Nn})\alpha\bar{Y}(\frac{\bar{X}}{\bar{X}+\rho})(\rho c_x c_y)$$ (as in eq. (10) above)

$$Mse(\bar{y}_{paa}) = E[\alpha\bar{Y}k + \alpha\bar{Y}k\Delta_{\bar{y}} + \alpha\bar{Y}k\Delta_{\bar{x}} + \alpha\bar{Y}k\Delta_{\bar{y}}\Delta_{\bar{x}} + \bar{Y} + \bar{Y}\Delta_{\bar{y}} - b\bar{X}\Delta_{\bar{x}} - \alpha$$

$$- \alpha bX\Delta_{\bar{x}} - \bar{Y}]^2$$

$$Mse(\bar{y}_{paa}) = E[\alpha\bar{Y}k\Delta_{\bar{y}} + \alpha\bar{Y}k\Delta_{\bar{x}}]^2$$

$$Mse(\bar{y}_{paa}) = (\frac{N-n}{Nn})\alpha^2\bar{Y}^2k^2(c^2{}_x + c^2{}_y + 2\rho c_x c_y)$$

$$= (\frac{N-n}{Nn})\alpha^2\bar{Y}^2(\frac{\bar{X}}{\bar{X}+\rho})^2(c^2{}_x + c^2{}_y + 2\rho c_x c_y)$$ (as in eq. (11) above)

5.0 Data Used

The proposed linear combined product estimator, $\bar{y}_{paa}$ is said to be better and more efficient than the classical product estimators[1, 2]. Results obtained will be used to justify this claim.
The question is "at what value of α using the two data sets will this proposed linear combined product estimator, $\bar{y}_{paa}$ be better"?.

Table 1: Summary of the Data Sets Used

Population	I	II
Source	[13]	[5]
N	16	100
4	4	30
$\bar{X}$	75.4313	0.2
$\bar{Y}$	7.6375	0.3
ρ	-0.6823	-0.05
c_x	0.0986	0.0036
c_y	0.2278	0.0036
$\frac{\bar{X}}{\bar{X}+\rho}$	1.009127881	1.33333

6.0 Results

The results obtained are shown in Tables 2(a) and 2(b).

Table 2(a):- Mean Square Error obtained on $\bar{y}_p$, $\bar{y}_{ST}$, $\bar{y}_{PD}$ and $\bar{y}_{paa}$ using Population I

Estimator	MSE
Classical Product $(\bar{y}_p)$	0.3387
Singh and Taylor (2003) $(\bar{y}_{ST})$	0.3394
Pandey and Dubey (1998) $(\bar{y}_{PD})$	0.3376
Proposed linear Combined Product $(\bar{y}_{paa})$	(i).when $\alpha = 1, MSE(\bar{y}_{paa}) = 0.3450$ (ii). when $\alpha = 0.9, MSE(\bar{y}_{paa}) = 0.2794$ (iii). when $\alpha = 0.8, MSE(\bar{y}_{paa}) = 0.2208$ (iv). when $\alpha = 0.7, MSE(\bar{y}_{paa}) = 0.1690$ (v). when $\alpha = 0.6, MSE(\bar{y}_{paa}) = 0.1242$ (vi). when $\alpha = 0.5, MSE(\bar{y}_{paa}) = 0.1037$ (vii). when $\alpha = 0.4, MSE(\bar{y}_{paa}) = 0.0689$ (viii). when $\alpha = 0.3, MSE(\bar{y}_{paa}) = 0.0310$ (ix). when $\alpha = 0.2, MSE(\bar{y}_{paa}) = 0.0138$ (x). when $\alpha = 0.1, MSE(\bar{y}_{paa}) = 0.00345$

7.0 Discussion

From the estimates in Table 2(a):

(i). when $\alpha = 1$, $MSE(\bar{y}_{PD}) < MSE(\bar{y}_P) < MSE(\bar{y}_{ST}) < MSE(\bar{y}_{paa})$ and

(ii). when $0 < \alpha < 1$, $MSE(\bar{y}_{paa}) < MSE(\bar{y}_{PD}) < MSE(\bar{y}_P) < MSE(\bar{y}_{ST})$

Table 2(b):- Mean Square Error obtained on $\bar{y}_p$, $\bar{y}_{ST}$, $\bar{y}_{PD}$ and $\bar{y}_{paa}$ using Population II

Estimator	MSE
Classical Product $(\bar{y}_p)$	$5.2x10^{-08}$
Singh and Taylor (2003) $(\bar{y}_{ST})$	$7.2x10^{-08}$
Pandey and Dubey (1998) $(\bar{y}_{PD})$	$5.1x10^{-08}$
Proposed linear Combined Product $(\bar{y}_{paa})$	(i).when $\alpha = 1, MSE(\bar{y}_{paa}) = 9.2x10^{-08}$ (ii). when $\alpha = 0.9, MSE(\bar{y}_{paa}) = 7.5x10^{-08}$ (iii). when $\alpha = 0.8, MSE(\bar{y}_{paa}) = 5.9x10^{-08}$ (iv). when $\alpha = 0.7, MSE(\bar{y}_{paa}) = 4.5x10^{-08}$ (v). when $\alpha = 0.6, MSE(\bar{y}_{paa}) = 3.3x10^{-08}$ (vi). when $\alpha = 0.5, MSE(\bar{y}_{paa}) = 2.3x10^{-08}$ (vii). when $\alpha = 0.4, MSE(\bar{y}_{paa}) = 1.5x10^{08}$ (viii). when $\alpha = 0.3, MSE(\bar{y}_{paa}) = 8.3x10^{-09}$ (ix). when $\alpha = 0.2, MSE(\bar{y}_{paa}) = 3.7x10^{-09}$ (x). when $\alpha = 0.1, MSE(\bar{y}_{paa}) = 9.2x10^{-10}$

8.0 Discussion

From the estimates in Table 2(b):

(i). when $\alpha = 1$, $MSE(\bar{y}_{PD}) < MSE(\bar{y}_P) < MSE(\bar{y}_{ST}) < MSE(\bar{y}_{paa})$,

(ii). when $\alpha = 0.9$, $MSE(\bar{y}_{PD}) < MSE(\bar{y}_P) < MSE(\bar{y}_{ST}) < MSE(\bar{y}_{paa})$,

(iii). when $\alpha = 0.8$, $MSE(\bar{y}_{PD}) < MSE(\bar{y}_P) < MSE(\bar{y}_{paa}) < MSE(\bar{y}_{ST})$ and

(ii). when $0.1 \le \alpha < 0.8$, $MSE(\bar{y}_{paa}) < MSE(\bar{y}_{PD}) < MSE(\bar{y}_p) < MSE(\bar{y}_{ST})$

9.0 Conclusion

Therefore, from the results obtained above, the proposed linear combined product estimator, $\bar{y}_{paa}$, is preferred to that of the classical product estimators [1, 2] provided $0 < \alpha < 1$ and $(\frac{\bar{X}}{\bar{X} + \rho}) \ge 1$.

10.0 References

[1]. Pandey, B.N. and Dubey, Vyas (1988): Modified product estimator using coefficient of variation of auxiliary variate, Assam Statistical Rev., 2(2), 64 – 66.

[2]. Singh, H.P. and Tailor, R. (2003): Use of known correlation coefficient in estimating the finite population mean. Statist. in Trans 6(4). 555-560.

[3]. Cochran, W.G. (1977). Sampling Techniques. 3rd Edition. John Wiley, New York.

[4]. Okafor, F.C. (2002). Sample Survey Theory With Applications. First Edition, Afro-Orbit Publications Ltd. University of Nigeria,Nsukka, Nigeria.

Journal of the Nigerian Association of Mathematical Physics Volume 33, (January, 2016), 119 – 124

[5]. Adewara, A.A. (2006): Effect of improving both the auxiliary and variable of interest in ratio and product estimators. Proc. Pakistan Academy Science. 43(4), 275 – 278

[6]. Ogunyinka, Peter I.1 and Sodipo, A. A. (2013): Efficiency of Ratio and Regression Estimators Using Double Sampling. Journal of Natural Sciences Research. 3(7), 202 – 207.

[7]. Onyeka A.C. (2012): Estimation of population mean in poststratified sampling using known value of some population parameter(s). Statistics in Transition-New Series. 13, 65-78

[8]. Pandey H, Yadav S.K. and Shukla A.K. (2011): An improved general class of estimators estimating population mean using auxiliary information. International Journal of Statistics and Systems. 6, 1-7

[9]. A.A. Adewara (2015): Alternative to Tin Ratio Type Estimator. Al-Hikmah Journal of Pure & Applied Sciences, 2, 7 -13.

[10]. Solanki R.S., Singh H.P. and Rathour, A. (2012): An alternative estimator for estimating the finite population mean using auxiliary information in sample surveys. International Scholarly Research Network: Probability and Statistics. 2012, 1-14

[11]. Subramani, J. and Kumarapandiyan, G. (2012): Estimation of Population Mean Using Co Efficient of Variation and Median of an Auxiliary Variable. International Journal of Probability and Statistics. 1(4), 111-118

[12]. Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007): A general family of estimators for estimating population mean using known value of population parameter(s). For East Jour. of theor. Statist. 22(2), 181-191.

[13]. Maddala, G.S. (1977): Econometrics. “McGraw Hills Pub. Co.” New York.