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Alternative to Thakur and Gupta Linear Combined Product - Ratio Type Estimator of Population Mean in Simple Random Sampling

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Abstract

In this study, an alternative linear combined product - ratio type estimator was proposed. This proposed estimator combined classical product estimator with classical ratio estimator. Four data sets were used to determine the efficiency of this proposed linear combined product – ratio type estimator and this proposed linear combined product - ratio type estimator was found to be better, preferred and efficient to all other estimators considered provided , $0 < \alpha < 1$, $c_y = c_x$. $c_y < c_x$ and the correlation coefficient between the study variable and variable of interest must be negative, $-1 < \rho < 0$. For all the data used, the proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ has the least mean square error than estimator, $\bar{y}_{TG}$. Therefore, $\bar{y}_{praa}$ is an alternative linear combined product – ratio type estimator recommended to estimator, $\bar{y}_{TG}$.

Keywords: Product, ratio, linear, combined, alternative, mean square error

1.0 Introduction

Let N and n be the population and sample sizes respectively, $\bar{X}$ and $\bar{Y}$ be the population means for the auxiliary variable (X) and the variable of interest (Y), $\bar{x}$ and $\bar{y}$ be the sample means based on the sample drawn. Then classically [1,2],

$$\bar{y}_p = \frac{\overline{yx}}{\bar{X}},$$

$$bias(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}[\rho c_x c_y] \quad \ldots \quad (1)$$

and

$$mse(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y)] \quad \ldots \quad (2)$$

respectively.

The literature on survey sampling describes a great variety of techniques for using auxiliary information to obtain more efficient estimators. Product method of estimation is a good example in this context. If the correlation between the study variable y and the auxiliary variable x is negative, the product method of estimation is quite effective.

In sample surveys, supplementary information is often used for increasing the precision of estimators [3 - 7].

Many authors [8 – 10] have used auxiliary information for improved estimation of population mean of study variable y.

2.0 On the Combined Product - Ratio Type Estimator [11].

Suppose a pairs (x, y) (i=1, 2, ….., n) observations are taken on n units sampled form N population units using simple

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random sampling without replacement scheme, $\bar{X}$ and $\bar{Y}$ are the population means for the auxiliary variable (X) and variable of interest (Y), and $\bar{x}$ and $\bar{y}$ are the sample means based on the sample drawn.

A linear combined product – ratio type estimator of the form:

$$\bar{y}_{TG} = f\frac{\overline{yx}}{\bar{X}} + (1-f)\frac{\bar{y}\bar{X}}{\bar{x}} \qquad \ldots (3)$$

was proposed by [1] where, $f = \frac{n}{N}$, $\bar{x} = \bar{X}(1+\Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$

The bias and mean square error of this proposed linear combined product-ratio type estimator, $\bar{y}_{TG}$ are given as:-

$$bias(\bar{y}_{TG}) = (\frac{N-n}{Nn})\bar{Y}(v^2c^2{}_x + v\rho c_x c_y) \qquad \ldots (4)$$

and

$$mse(\bar{y}_{TG}) = (\frac{N-n}{Nn})\bar{Y}^2[v^2c^2{}_x + c^2{}_y - 2v\rho c_x c_y] \qquad \ldots (5)$$

where $v = \rho\frac{c_y}{c_x}$

3.0 On the Proposed Linear Combined Product – Ratio Type Estimator

The classical product estimator used in this study is defined as $\bar{y}_p = \alpha(\frac{\overline{yx}}{\bar{X}})$ while that of classical ratio estimator used is defined as: $\bar{y}_r = (1-\alpha)(\frac{\bar{y}\bar{X}}{\bar{x}})$ such that $\alpha + (1-\alpha) = 1$. Their biases and mean square errors are given respectively as:

$$bias(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}[\rho c_x c_y] \qquad \ldots (6)$$

$$mse(\bar{y}_p) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y + 2\rho c_x c_y] \qquad \ldots (7)$$

$$bias(\bar{y}_r) = (\frac{N-n}{Nn})\bar{Y}[c_x{}^2 - \rho c_x c_y] \qquad \ldots (8)$$

and

$$mse(\bar{y}_r) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x + c^2{}_y - 2\rho c_x c_y)] \qquad \ldots (9)$$

The proposed linear combined product – ratio type estimator is given as:

$$\bar{y}_{praa} = \alpha\frac{\overline{yx}}{\bar{X}} + (1-\alpha)\frac{\bar{y}\bar{X}}{\bar{x}} \qquad \ldots (10)$$

which is the linear combination of classical product estimator, $\bar{y}_p$, and the classical ratio estimator, $\bar{y}_r$, where $\bar{x} = \bar{X}(1+\Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that $|\Delta_{\bar{y}}| < 1$ and $|\Delta_{\bar{x}}| < 1$

The bias and mean square error of this proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ are given as:-

$$bias(\bar{y}_{praa}) = (\frac{N-n}{Nn})\bar{Y}[c^2{}_x(1-\alpha) - \rho c_x c_y(1-2\alpha)] \qquad \ldots (11)$$

and

$$mse(\bar{y}_{praa}) = (\frac{N-n}{Nn})\bar{Y}^2[c^2{}_x(3\alpha^2-1)+c^2{}_y+\rho c_x c_y(2\alpha-1)] \qquad \ldots (12)$$

Provided $0<\alpha<1$ and $-1<\rho<0$.

The proof of (11) and (12) are as shown below:

4.0 Derivation of Bias and Mean Square Error of $\bar{y}_{praa}$

Let,

$\bar{y}_{praa} = \alpha\frac{\bar{y}\bar{x}}{\bar{X}} + (1-\alpha)\frac{\bar{y}\bar{X}}{\bar{x}}$, where, $\bar{x} = \bar{X}(1+\Delta_{\bar{x}})$, $\bar{y} = \bar{Y}(1+\Delta_{\bar{y}})$, $\Delta_{\bar{y}} = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$, $\Delta_{\bar{x}} = \frac{\bar{x}-\bar{X}}{\bar{X}}$ such that $\left|\Delta_{\bar{y}}\right|<1$ and $\left|\Delta_{\bar{x}}\right|<1$ as earlier defined.

Therefore, using power series expansion,

$$\bar{y}_{praa} = \alpha\frac{\bar{y}\bar{x}}{\bar{X}} + (1-\alpha)\frac{\bar{y}\bar{X}}{\bar{x}}$$

$$= \alpha[\frac{\bar{Y}(1+\Delta_{\bar{y}})\bar{X}(1+\Delta_{\bar{x}})}{\bar{X}} + (1-\alpha)\frac{\bar{Y}(1+\Delta_{\bar{y}})}{\bar{X}(1+\Delta_{\bar{x}})}\bar{X}]$$

$$= \alpha\bar{Y}(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}}) + (1-\alpha)\bar{Y}(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}})^{-1}$$

$$= \alpha\bar{Y}(1+\Delta_{\bar{y}})(1+\Delta_{\bar{x}}) + (1-\alpha)\bar{Y}(1+\Delta_{\bar{y}})(1-\Delta_{\bar{x}}+\Delta^2{}_{\bar{x}})$$

$$= \bar{Y}[\alpha(1+\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta_{\bar{x}}) + (1-\alpha)(1-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta^2{}_{\bar{x}})$$

$$= \bar{Y}[\alpha+\alpha\Delta_{\bar{x}}+\alpha\Delta_{\bar{y}}+\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+1-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}$$

$$-\alpha+\alpha\Delta_{\bar{x}}-\alpha\Delta_{\bar{y}}-\alpha\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}]$$

$$= \bar{Y}[\alpha\Delta_{\bar{x}}+\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+1-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}$$

$$-\alpha\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}]$$

Let, $E(\Delta_{\bar{y}}) = E(\Delta_{\bar{x}}) = 0, E(\Delta_{\bar{x}}{}^2) = \frac{S^2{}_x}{\bar{X}^2} = c^2{}_x, E(\Delta_{\bar{y}}{}^2) = \frac{S^2{}_y}{\bar{Y}^2} = c^2{}_y$ and $E(\Delta_x\Delta_y) = \frac{S_{xy}}{\bar{X}\bar{Y}} = \rho c_x c_y$. Then,

$$Bias(\bar{y}_{praa}) = \bar{Y}E[(\alpha\Delta_{\bar{x}}+\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+1-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}$$

$$-\alpha\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta_{\bar{y}}\Delta^2{}_{\bar{x}})-\bar{Y}]$$

$Bias(\bar{y}_{praa}) = \bar{Y}E[\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}\Delta_{\bar{y}}]$, ignoring higher orders.

$$Bias(\bar{y}_{praa}) = \bar{Y}E[2\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta^2{}_{\bar{x}}]$$

$Bias(\bar{y}_{praa}) = \bar{Y}(\frac{N-n}{Nn})[c^2{}_{\bar{x}}(1-\alpha)-\rho c_x c_y(1-2\alpha)]$ (as in eq. (11))

$$Mse(\bar{y}_{praa}) = \bar{Y}^2E[(\alpha\Delta_{\bar{x}}+\alpha\Delta_{\bar{y}}\Delta_{\bar{x}}+1-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\Delta^2{}_{\bar{x}}-\Delta_{\bar{x}}\Delta_{\bar{y}}+\Delta_{\bar{y}}\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}$$

$$-\alpha\Delta^2{}_{\bar{x}}+\alpha\Delta_{\bar{x}}\Delta_{\bar{y}}-\alpha\Delta_{\bar{y}}\Delta^2{}_{\bar{x}})-\bar{Y}]^2$$

$Mse(\bar{y}_{praa}) = \bar{Y}^2E[(\alpha\Delta_{\bar{x}}-\Delta_{\bar{x}}+\Delta_{\bar{y}}+\alpha\Delta_{\bar{x}}]^2$, ignoring higher orders.

$$Mse(\bar{y}_{praa}) = \bar{Y}^2(\frac{N-n}{Nn})[c^2{}_x(3\alpha^2-1)+c^2{}_y+\rho c_x c_y(2\alpha-1)]$$

(as in eq. (12))

By partial differentiation,

$$\frac{\partial Mse}{\partial\alpha}[c^2{}_x(3\alpha^2-1)+c^2{}_y+\rho c_x c_y(2\alpha-1)] = 6\alpha c^2{}_x + 2\rho c_x c_y$$

This implies that,

$$6\alpha c^2{}_x = -2\rho c_x c_y$$

$$\alpha = \frac{-2\rho c_x c_y}{6c^2{}_x} = \frac{-2\rho c_x c_y}{6c^2{}_x} = -\frac{\rho c_x c_y}{3c^2{}_x}.$$

Therefore,

$$\alpha = -\frac{\rho c_x c_y}{3c^2{}_x} \qquad \ldots \qquad (13)$$

5.0 Data Used

The proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ is said to be better, efficient and preferred to linear combined product - ratio type estimator, $\bar{y}_{TG}$ [11]. Results obtained on the data sets below will be used to justify this claim.

Table 1: Summary of the data sets used.

Parameters	Population I	Population II	Population III	Population IV
Source	[3]	[12]	[13]	[14]
Population (N)	100	30	20	30
Sample (n)	30	6	8	6
$\bar{X}$	0.2	75.4313	18.8	0.8077
$\bar{Y}$	0.3	7.6375	19.55	0.6860
ρ	-0.05	-0.6823	-0.9199	-0.4996
c_x	0.0036	0.0986	0.1281	0.7493
c_y	0.0036	0.2278	0.1554	0.700123
v	0.05	-1.57634	-0.74657	-0.4996
α	0.0167	0.5254	0.3459	0.1556

6.0 Results

The results obtained are shown in Table 2:

Table 2:- Bias and Mean Square Error obtained on $\bar{y}$, $\bar{y}_p$, $\bar{y}_{TG}$ and $\bar{y}_{praa}$

Population	I	II	III	IV
$bias(\bar{y})$	0	0	0	0
$bias(\bar{y}_p)$	$-4.536x10^{-08}$	-0.0156	-0.0249	-0.0239
$bias(\bar{y}_{TG})$	0	-0.0115	$-1.6292x10^{-05}$	-0.0041
$bias(\bar{y}_{praa})$	$9.3589x10^{-08}$	0.00441	0.02343	0.02450
$mse(\bar{y})$	$2.7x10^{-08}$	0.4036	0.5985	0.0308
$mse(\bar{y}_p)$	$5.2x10^{-08}$	0.2408	0.0927	0.0330
$mse(\bar{y}_{TG})$	$2.4x10^{-08}$	0.7794	0.5985	0.0426
$mse(\bar{y}_{praa})$	$1.36x10^{-09}$	0.3846	0.4474	0.0094

7.0 Discussion

From the results shown in Table 2, for:

(i). Population I, $mse(\bar{y}_{praa}) = 1.36x10^{-09}$ and $mse(\bar{y}_{TG}) = 2.4x10^{08}$. $mse(\bar{y}_{praa}) < mse(\bar{y}_{TG})$

(ii). Population II, $mse(\bar{y}_{praa}) = 0.3846$ and $mse(\bar{y}_{TG}) = 0.7794$, $mse(\bar{y}_{praa}) < mse(\bar{y}_{TG})$

(iii). Population III, $mse(\bar{y}_{praa}) = 0.4474$ and $mse(\bar{y}_{TG}) = 0.5985$, $mse(\bar{y}_{praa}) < mse(\bar{y}_{TG})$

(iv). Population IV, $mse(\bar{y}_{praa}) = 0.0094$ and $mse(\bar{y}_{TG}) = 0.0426$, $mse(\bar{y}_{praa}) < mse(\bar{y}_{TG})$

Also for:

(i). Population I, $c_y = c_x$, hence, $\bar{y}_{praa}$ is the most efficient estimator among the four estimators considered.

(ii). Population II, $c_y > c_x$, hence, $\bar{y}_{praa}$ is better than $\bar{y}_{TG}$ and $\bar{y}$ but not better than $\bar{y}_p$.

(iii). Population III, $c_y > c_x$, hence, $\bar{y}_{praa}$ is better than $\bar{y}_p$ but not better than $\bar{y}$ and $\bar{y}_{TG}$.

(iv). Population IV, $c_y < c_x$, hence, $\bar{y}_{praa}$ is the most efficient estimator among the four estimators considered.

8.0 Conclusion

The proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ is said to be better , efficient and preferred to all other estimators considered here provided $0 < \alpha < 1, -1 < \rho < 0$, $c_{\bar{y}} = c_{\bar{x}}$ and $c_y < c_x$. Also, the proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ is said to be better , efficient and preferred to linear combined product - ratio type estimator, $\bar{y}_{TG}$ provided $0 < \alpha < 1, -1 < \rho < 0$, $c_y = c_x$, $c_y < c_x$ and $c_y > c_x$ [11]. For all the data used, the proposed linear combined product – ratio type estimator, $\bar{y}_{praa}$ has the least mean square error than estimator, $\bar{y}_{TG}$, [11]. Therefore, $\bar{y}_{praa}$ is an alternative linear combined product – ratio type estimator recommended to estimator, $\bar{y}_{TG}$ [11].

9.0 References

[1]. Cochran, W.G. (1977). Sampling Techniques. 3rd Edition. John Wiley, New York.

[2]. Okafor, F.C. (2002). Sample Survey Theory With Applications. First Edition, Afro-Orbit Publications Ltd. University of Nigeria,Nsukka, Nigeria.

[3]. Adewara, A.A. (2006): Effect of improving both the auxiliary and variable of interest in ratio and product estimators. Proc. Pakistan Academy Science. 43(4), 275 – 278

[4]. Singh, R., Malik, S., Chaudhary, M.K., Verma, H. and Adewara, A.A. (2012): A general family of ratio type-estimators in systematic sampling, Jour. Reliab.and Stat. Stud., 5(1), p. 73-82.

[5]. Ogunyinka, Peter I.1 and Sodipo, A. A. (2013):Efficiency of Ratio and Regression Estimators Using Double Sampling. Journal of Natural Sciences Research. 3(7), 202 – 207.

[6]. Onyeka A.C. (2012): Estimation of population mean in poststratified sampling using known value of some population parameter(s). Statistics in Transition-New Series. 13, 65-78

[7]. Pandey H, Yadav S.K. and Shukla A.K. (2011): An improved general class of estimatorsestimating population mean using auxiliary information. International Journal of Statistics and Systems. 6, 1-7

[8]. A.A. Adewara (2015): Alternative to Tin Ratio Type Estimator. Al-Hikmah Journal of Pure & Applied Sciences,2, 7 -13.

[9]. Solanki R.S., Singh H.P. and Rathour, A. (2012): An alternative estimator for estimating the finite population mean using auxiliary information in sample surveys. International Scholarly Research Network: Probability and Statistics. 2012, 1-14

[10]. Subramani, J. and Kumarapandiyan, G. (2012): Estimation of Population Mean Using Co Efficient of Variation and Median of an Auxiliary Variable. International Journal of Probability and Statistics. 1(4), 111-118

[11]. Narendra Singh Thakur and Kokil Gupta (2013): A study of linear combination based factor – type estimator for estimating population mean in sample surveys. Journal of Reliability and Statistical Studies. 6(2), 81 – 90.

[12]. Maddala, G.S. (1977): Econometrics. “McGraw Hills Pub. Co.” New York.

[13]. Pandey, B.N. and Dubey, Vyas (1988): Modified product estimator using coefficient of variation of auxiliary variate, Assam Statistical Rev., 2(2), 64 – 66.

[14]. Steel, R.G.D. and Torrie, J.H. (1960): Principles and procedures of statistics, MCGraw hill book.