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Application of Linear Programming Techniques to the production Activities of 7up Bottling Company in Edo State

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Abstract

In this work, we developed the mathematical Model of the production activities of the Seven Up Bottling Company, Benin City in Nigeria. Using the Simplex Method of solution, Excel Solver and Linear Programming Solver (LPS), the most favourable different quantities of brands of soft drinks to be produced, to obtain the best or optimal profit were obtained. The quantities obtained compared favourably with that of the company.

1.0 Introduction

Operation Research is a branch of Mathematics which is concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solution. The process begins by carefully observing and formulating the problem, including gathering all the relevant data. The next step is to construct a scientific model that attempts to abstract the essence of the real model.

Management and executive decisions revolve around the problem of how best to make use of the available resources, such as money, man power, production facilities, Machines Capacities, Time, Raw Materials and so on[1]. Since the resources have economic value and are limited and the sales of the production of an organization or company are expected to lead to profit, then it becomes necessary to determine the best possible way that maximum profit can be generated.

Linear Programming Techniques derive its name from the fact that the functional relationships in the Mathematical model are linear and solution consists of pre-determined mathematical steps usually referred to as a programme.

As a result of the usefulness of linear programming models more areas of its applications are being introduced.

Linear Programming can be gainfully applied in banking institutions, schools, Agricultural sectors, transportation problems, the oil company among others[2]. However, from report of various surveys, many production companies are yet to know fully the importance of linear programming application to their Operations system. All production companies are faced with the problem of how to maximize profit from available resources. This is because they are not familiar with the use of linear programming.

In most production companies, management base their decision on the total input used in the production and the total output produced[3]. This system of decision making always have a set-back in that it brings about a reduction in the accuracy of forecasting for the future such as price fluctuation and shortage of raw materials

The problem of decision making therefore brings about the application of linear programming model which is now seen as a concept which all decision matters have to understand before achieving an effective decision.

2.0 Method

A linear programming problem consists of three parts namely the objective function, the constraints and the non-negative constraints. The Objective function, which is a linear function of the decision variables is either to be maximized or minimized subject to a set of linear constraints which constraint the technological specifications of the problem in relation to the given resources or requirements. There could be a non-negative constraints, which implies that negative production does not exist in practice. Thus, in a linear programming problem, we seek values of the variables x_1, which are non-negative and satisfy a set of m linear constraints and also maximize or minimize a linear function of the variables. Mathematically, a linear programming problem has the form,

Maximize or minimize $Z = \sum_{j=1}^{n} c_j x_j$ (1)

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \ (\leq, =\geq)\ b_i,\ i = 1, 2, \ldots, m \qquad (2)$$

$$and\ x_j \geq 0,\ j = 1, 2, \ldots, n \qquad (3)$$

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In each constraint, only one of the signs ($\leq$, =, $\geq$)holds, but may vary from one constraint to another.
Thus a programming problem is linear if

1) The Objective function is linear
2) The left hand side of the constraint in (2)is linear

Linear programming is a method of solving problems associated to some situation with quantifiable entities (input and output values) by converting those values into a mathematical model that satisfies the requirement of a linear programme in order to obtain the best value/outcome from the situation

Linear programming is essentially a method of determining an optimum program of the candidates or inter-dependent activities which are competing for limited resources under assumptions of linearity[4].

A remarked that linear programming can described as the process of transforming a real life problem into a mathematical model, together with the process of designing algorithms with which the mathematical model can be analyzed and/or solved, resulting in a proposal that may support the procedure of solving the real-life problem[5].

3.0 Application of Linear Programming Formulation in a Large Scale

We wish to know how best we can maximize the profit of Seven Up Bottling Company with the available data provided by the company as at 2008 quarterly report is shown in the tables below. The analysis is carried out to know the quantity of each brands of soft drink (7Up, Mirinda and Pepsi) that should be produced in order to maximize profit. The

Amount of resources available

RESOURCES	**QUANTITY AVAILABLE**
Water (Litres)	195,520.00
Sugar (kg)	450,330.00
Carbonate (mg)	425,125.40
No. of Crown (kg)	20,745.00
Concentrate (mol/litre)	23,454.90

Unit variable cost of producing one unit of each production.
Product cost per crate (35cl)

Mirinda	#724.55
7up	#904.75
Pepsi	#911.75

Selling price and profit of a unit of each product (per Crate 35cl).

PRODUCT	**SELLING PRICE**	**PROFIT**
Mirinda	#890	#165.45
7up	#1080	#175.25
Pepsi	#1080	#160.25

Raw materials combination

PRODUCT	**WATER(LITRE)**	**SUGAR (kg)**	**CARBONATE (mg)**	**CROWN(kg)**	**CONCENTRATE (mol/litres)**
Mirinda	0.625	0.95	0.02	0.055	0.76
7up	0.4	0.55	0.025	0.55	0.082
Pepsi	0.72	1.1	0.04	0.055	0.067

Let the brands of soft drinks be the number of variables
Let X_1 be the quantity of Mirinda to be produce
Let X_2 be the quantity of 7Up to be produce
Let X_3 be the quantity of Pepsi to be produce
Let raw materials be the number of constraints and amount of resources in the right hand side of each constraint.
The linear programming formulation is given as:

Max $Z = 165.45X_1 + 175.25X_2 + 160.25X_3$

S. t.

$0.625X_1 + 0.4X_2 + 0.72X_3 \leq 195,520$
$0.95X_1 + 0.55X_2 + 1.1X_3 \leq 450,330$
$0.02X_1 + 0.025X_2 + 0.04X_3 \leq 425,125.45$
$0.055X_1 + 0.55X_2 + 0.055X_3 \leq 20,745$
$0.76X_1 + 0.082X_2 + 0.067X_3 \leq 23,454.90$

Introducing Slack Variables X_4, X_5, X_6, X_7 and X_8 we have the following tables of iterations;

Tableau I

Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS	RATIO
X_4	0.625	0.4	0.72	1	0	0	0	0	195,520	488,800
X_5	0.95	0.55	1.1	0	1	0	0	0	450,330	818.781,81
X_6	0.02	0.025	0.04	0	0	1	0	0	425,125.45	17,005,018
X_7	0.055	(0.55)P.e	0.055	0	0	0	1	0	20,745	37,718.18
X_8	0.76	0.082	0.067	0	0	0	0	1	23,454.90	286,035
Z	-165.45	-175.25	-160.25	0	0	0	0	0	0	0

Tableau 2

Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
X_4	0.59	0	0.68	1	0	0	-0.73	0	180,433
X_5	0.90	0	1.05	0	1	0	-1.00	0	429,585.01
X_6	0.02	0	0.04	0	0	1	-0.05	0	424,182.5
X_2	0.1	1	0.1	0	0	0	1.82	0	37,718.18
X_8	(0.75)p.e	0	0.06	0	0	0	-0.15	1	20,362.01
$Z_j - c_j$	-147.93	0	-142.73	0	0	0	318.96	0	6,610,111.0

Tableau 3

Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
X_4	0	0	(0.63)p.e	1	0	0	-0.61	-0.78	164,414.88
X_5	0	0	0.98	0	1	0	-0.82	-1.20	405,150.60
X_6	0	0	0.04	0	0	1	-0.05	-0.03	423,639.52
X_2	0	1	0.09	0	0	0	1.84	-0.13	35,003.25
X_1	1	0	0.08	0	0	0	-0.2	1.33	27,149.35
$Z_j - c_j$	0	0	-130.90	0	0	0	289.37	196.75	10,626,314.346

Tableau 4

Basic	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	RHS
X_3	0	0	1	1.59	0	0	-0.97	-1.24	260,976
X_5	0	0	0	-1.59	1	0	0.13	0.02	149,394.12
X_6	0	0	0	-0.06	0	1	-0.01	0.02	413,200.48
X_2	0	1	0	-0.14	0	0	1.93	-0.02	11,515.41
X_1	1	0	0	-0.13	0	0	-0.12	1.43	6,271.27
$Z_j - c_j$	0	0	0	208.13	0	0	162.40	34.43	44,788,072.4

The most negative is -175.25 on the column X_2
The get the ratio, divide RHS by the corresponding X_2 column
The pivot element is 0.55
X_7 will leave the basic while
X_2 will enter the basic

The most negative -147.93 which is X_1 column enters while X_8 row leaves. Therefore the pivot element will be 0.75.

X_4 leaves the basis because it has the minimum value while X_3 enters the basis since it has the highest negative value.
The solution is feasible and optimal since there is no negative in zj-cy and RHS.
X_1*=6,271.26
X_2*=11,515.41
X_3*=260,976
Z*= 44,788,072.4
The problem was also solved using Excel Solver and Linear Programming Solver (LiPS) and the results shown below. The manual solution as well as that obtained from Excel Solver and Linear Programming Solver agreed and compared favourably with the company output of these products quarterly.

4.0 Using Excel Solver

The three reports generated when excel solver is used to calculate linear program are:

- Answer report,
- Sensitivity report and
- Limit report

The answer report gives details of the solutions (in this case, profit is maximized at 44,651,735.5 when 6,788.199088 units of Mirinda, 11,089.12208 units of 7UP and 259,502.3982 units of Pepsi are produced and information concerning the status of each constraint with accompanying slack values is provided. The outputs of the Answer Report are shown below:

Microsoft Excel 15.0 Answer Report
Worksheet: [linear program msc project.xlsx]Sheet1
Report Created: 31-Jan-16 10:06:01 PM
Result: Solver found a solution. All Constraints and optimality conditions are satisfied.
Solver Engine
Engine: GRG Nonlinear
Solution Time: 0.078 Seconds.
Iterations: 4 Subproblems: 0
Solver Options
Max Time Unlimited, Iterations Unlimited, Precision 0.000001, Use Automatic Scaling
Convergence 0.0001, Population Size 100, Random Seed 0, Derivatives Forward, Require Bounds
Max Subproblems Unlimited, Max Integer Sols Unlimited, Integer Tolerance 1%, Assume NonNegative

Objective Cell (Max)

Cell	Name	Original Value	Final Value
$C$3	PROFIT	0	44651735.5

Variable Cells

Cell	Name	Original Value	Final Value	Integer
$C$13	X1	0	6788.199088	Contin
$C$14	X2	0	11089.12208	Contin
$C$15	X3	0	259502.3982	Contin

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$C$5	CONSTRAINT	195520	$C$5<=$E$5	Binding	0
$C$6		298000.4443	$C$6<=$E$6	Not Binding	4205329.556
$C$7		10793.08796	$C$7<=$E$7	Not Binding	414332.912
$C$8		20745	$C$8<=$E$8	Binding	0
$C$9		23455	$C$9<=$E$9	Binding	0

The sensitivity report for the manufacturing company provides information about how sensitive the solution is to changes. The output of the Sensitivity Report is shown below:

Microsoft Excel 15.0 Sensitivity Report
Worksheet: [linear program msc project.xlsx]Sheet1
Report Created: 31-Jan-16 10:06:02 PM

Variable Cells

Cell	Name	Final Value	Reduced Gradient
$C$13	X1	6788.199088	0
$C$14	X2	11089.12208	0
$C$15	X3	259502.3982	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
$C$5	CONSTRAINT	195520	206.7896065
$C$6		298000.4443	0
$C$7		10793.08796	0
$C$8		20745	162.8987997
$C$9		23455	35.85139378

Microsoft Excel 15.0 Limits Report
Worksheet: [linear program msc project.xlsx]Sheet1
Report Created: 31-Jan-16 10:06:02 PM

Cell	Objective Name	Value
$C$3	PROFIT	44651735.5

Cell	Variable Name	Value	Lower Limit	Objective Result
$C$13	X1	6788.199088	0	43528627.96
$C$14	X2	11089.12208	0	42708366.86
$C$15	X3	259502.3982	0	3066476.184

Using Linear Programming Solver (LPS)

*** Phase II --- Start ***

Basis	X1	X2	X3	s4	s5	s6	s7	s8	RHS
s4	5/8	0.4	0.72	1	0	0	0	0	195520
s5	0.95	0.55	1.1	0	1	0	0	0	450330
s6	0.02	1/40	0.04	0	0	1	0	0	425125
s7	11/200	0.55	11/200	0	0	0	1	0	20745
s8	0.76	41/500	67/1000	0	0	0	0	1	23454.9
Obj.	165.45	175.25	160.25	0	0	0	0	0	0

Variable to be made basic -> X2
Ratios: RHS/Column X2 -> { 488800 818782 1.7005e+007 37718.2 286035 }
Variable out of the basic set -> s7

*** Phase II --- Iteration 1 ***

Basis	X1	X2	X3	s4	s5	s6	s7	s8	RHS
s4	117/200	0	0.68	1	0	0	-8/11	0	180433
s5	179/200	0	209/200	0	1	0	-1	0	429585
s6	7/400	0	3/80	0	0	1	-1/22	0	424182
X2	0.1	1	0.1	0	0	0	20/11	0	37718.2
s8	0.7518	0	147/2500	0	0	0	-41/275	1	20362
Obj.	5917/40	0	5709/40	0	0	0	-3505/11	0	6.61011e+006

Variable to be made basic -> X1
Ratios: RHS/Column X1 -> { 308432 479983 2.4239e+007 377182 27084.3 }
Variable out of the basic set -> s8

*** Phase II --- Iteration 2 ***

Basis	X1	X2	X3	s4	s5	s6	s7	s8	RHS
s4	0	0	0.634246	1	0	0	-0.61126	-0.778132	164588
s5	0	0	39/40	0	1	0	-190/231	-25/21	405345
s6	0	0	0.0361313	0	0	1	-0.0419841	-25/1074	423709
X2	0	1	33/358	0	0	0	1.83801	-0.133014	35009.7
X1	1	0	14/179	0	0	0	-0.198312	1.33014	27084.3
Obj.	0	0	131.155	0	0	0	-289.301	-196.761	1.06166e+007

Variable to be made basic -> X3
Ratios: RHS/Column X3 -> { 259503 415738 1.17269e+007 379803 346293 }
Variable out of the basic set -> s4

*** Phase II --- Iteration 3 ***

Basis	X1	X2	X3	s4	s5	s6	s7	s8	RHS
X3	0	0	1	1.57668	0	0	-0.963759	-1.22686	259503
s5	0	0	0	-1.53726	1	0	0.117154	0.00571487	152330
s6	0	0	0	-0.0569673	0	1	-0.00716223	0.0210507	414332
X2	0	1	0	-0.145336	0	0	1.92685	-0.0199234	11089.1
X1	1	0	0	-0.123315	0	0	-0.122934	1.4261	6788.06
Obj.	0	0	0	-206.79	0	0	-162.899	-35.8514	4.46517e+007

>> Optimal solution FOUND
>> Maximum = 4.46517e+007

*** RESULTS ***

Variable	Value	Obj. Cost	Reduced Cost
X1	6788.06	165.45	0
X2	11089.1	175.25	0
X3	259503	160.25	0

Constraint	RHS	Slack	Dual Price
Constraint1	195520	0	206.79
Constraint2	450330	152330	0
Constraint3	425125	414332	0
Constraint4	20745	0	162.899
Constraint5	23454.9	0	35.8514

5.0 Conclusion

This project work is concerned mainly with the application of linear programming to production activities of the Seven- Up Bottling Company. The production activities was successfully model and we obtained three variables which represent the quantity of each brand of drink produced, and five constraints which represent the raw materials used. The solution was obtained using the simplex method, LPS solver and Excel Solver. The solution obtained compare favourably with that of the company quarterly production.

It is my wish that most management or production managers who based their decision mainly on the total input used in the production and total output produced before now should apply linear programming to determine the best way to allocate the company's scarce resources and maximize profit or minimize cost.

6.0 Recommendation

From the foregoing, the following recommendations are made to help organization (Seven-Up Blottling Company) in solving the problem of decision making:

1) The staff of the production department should be trained on the use of linear programming techniques to model their activities and obtain the best or optimal solution.
2) Staff should also be trained on the use of Linear Programming Solver, Excel Solver and some other available Linear Programming Package, so as to reduce the enormous calculation involves in the Simplex Method and increase the efficiency.
3) Other production sectors and/or business sectors should encourage the use of Linear Programming in their decision making.

7.0 Reference

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