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Comparative Analysis of Mean Time to System Failure and Steady-State Availability between two SYSTEMS SUBJECTto Different Types of Failure

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Abstract

This paper presents a study of Mean time to system failure and Steady-state availability systems subject to different types of failure. System I has only one type of failure. When the system fails, it is minimally repaired with probability p and it is replaced with probability $q = 1 - p$. System II has two types of failure. If the failure is of type 1, the system is minimally repaired with probability p and it is replaced with probability $q = 1 - p$. If the failure is of type II, the system is always rectified by a minimal repair. Failure, repair and replacement rates of each of the two systems are assumed to be exponentially distributed. Explicit expression for Mean time to system failure and Steady-state availability are derived and numerical illustration is presented. Finally, comparisons are madebased on Mean time to system failure and Steady-state Availability and the results show that the optimal system is System I.

Keywords: Probability, Availability, Mean time to system failure, Repair, Replacement.

1.0 Introduction

System failures occur during operations and such failures bring about loss of revenue due to loss of production, and delay in supply. Failures can be removed by either repair or replacement. Many researchers developed probabilistic models to predict systems with high Mean time to system and failure and Steady-state Availability.El-Said [1] studied the cost analysis of a system with preventive maintenance by using Kolmogorov's forward equations method. Haggag [2] studied the cost analysis of two unit cold standby system involving preventive maintenance. Mokaddis and Malta[3] studied the cost analysis of two dissimilar unit cold standby redundant systems subject to inspection and random change of units. Hajeeh [4] studied the availability of a system subject to different repair options. Yusuf and Hussain[5]analyzed the reliability characteristics of 2-out of-3 system under a perfect repair option. Wang et al [6] performed comparative analysisof availability between two systems with warm standby units and different imperfect coverage.Yusuf and Bashir [7] studied the availability, busy period and profit analysis of two dissimilar systems. Bashir and Ibrahim [8] studied a series system consisting of a single unit subjected to three types of failure. Bashir et al [9] studied probabilistic models for a system with different deterioration stages. Three configurations were considered and ranked based on mean time to system failure and availability.Most of these researches did consider system failure in which repair or replacement can be done based on probability to be determined by the decision maker.

In the present paper, we consider two systems subject to different types of failure. System I has only one type of failure. When the system fails it is minimally repaired with probability p and it is replaced with probability $q = 1 - p$. System II has two types of failure. If the failure is of type 1,the system is minimally repaired with probability p and it is replaced with probability $q = 1 - p$. If the failure is of type II, the system is always rectified by a minimal repair. Explicit expression for Mean time to system failure and Steady state availability are derived and numerical example is given to compute the Mean time to system failure and Steady-state Availability. Finally the Mean time to system failure and Steady-state availability were ranked to determine the optimal system.

2.0 Notations

S_1 :The system is new and working.

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S_2 : The system is working but not as good as new.

S_3 :The systemin a failed state due to type 1 failure.

S_4 : The system in a failed state due to type 2 failure.

λ_1: Type 1 failure rate.

λ_2 : Type 2 failure rate.

β : Deterioration rate.

α_r : Replacement rate.

α_{m1}: Minimal repair rate due to type 1 failure.

α_{m2}: Minimal repair rate due to type 2 failure.

$AV_1(\infty)$:Steady state availability of system 1.

$AV_2(\infty)$:Steady state availability of system 2.

$MTSF_1$: Mean time to system failure of system 1.

$MTSF_2$: Mean time to system failure of system 2.

3.0 System Description and Assumptions.

System Iisnew and deteriorate with time and has only one type of failure. When the system fails it is minimally repaired with probability p and it is replaced with probability $q = 1 - p$. System II is also new and deteriorates with time and is subject to two types of failure. If the failure is of type I, the system is minimally repaired with probability p and it is replaced with probability 1-p. If the failure is of type II, the system is always rectified by a minimal repair. Failure, repair and replacement rates are assumed to be exponentially distributed.

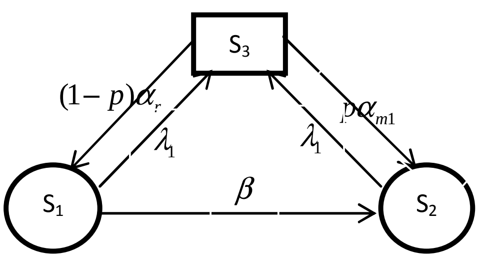

Figure 1. Schematic diagram of System I

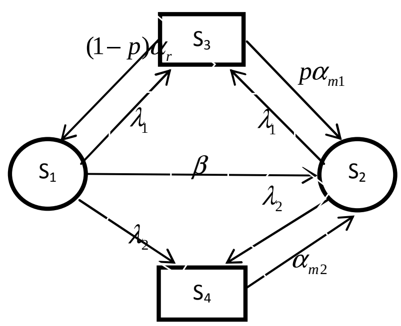

Figure 2: Schematic Diagram of System II

4.0 Mean Time to System Failure.

4.1 Mean Time to System Failure Calculations for System I.

Let $P_n(t)$ be the probability row vector at time t ($t \geq 0$), then the initial conditions for this problem are as follows:

$$P(0) = [P_1(0), P_2(0), P_3(0)] = [1, 0, 0] \tag{1}$$

We obtain the following differential equations from Figure 1.

$$\begin{aligned} \frac{dP_1(t)}{dt} &= -(\lambda_1 + \beta)P_1(t) + (1-p)\alpha_r P_3(t) \\ \frac{dP_2(t)}{dt} &= \beta P_1(t) - \lambda_1 P_2(t) + p\alpha_{m1} P_2(t) \\ \frac{dP_3(t)}{dt} &= \lambda_1 P_1(t) + \lambda_1 P_2(t) - (p\alpha_{m1} + (1-p)\alpha_r)P_3(t). \end{aligned} \tag{2}$$

The differential equation can be expressed in matrix form as

$$P' = AP, \tag{3}$$

Where,

$$A=\begin{bmatrix}-(\lambda_1+\beta) & 0 & (1-p)\alpha_r \\ \beta & -\lambda_1 & p\alpha_{m1} \\ \lambda_1 & \lambda_1 & -(p\alpha_{m1}+(1-p)\alpha_r\end{bmatrix}.$$

To evaluate the transient solution is too difficult. Therefore, to calculate the MTSF, we take the transpose of matrix A and delete the row and column of absorbing state i.e. state 3. The new matrix is called Q and the expected time to reach an absorbing state is given by

$$E[T_{P(0)\to P(absorbing)}]=MTSF_1=P(0)(-\mathrm{Q}^{-1})[1, \quad 1]^T, \tag{4}$$

where,

$$Q=\begin{bmatrix}(\lambda_1+\beta) & \beta \\ 0 & -\lambda_1\end{bmatrix}.$$

Therefore, the explicit expression for the mean time to system failure is given by

$$MTSF_1=\frac{\lambda_1+\beta}{\lambda_1(\lambda_1+\beta)}. \tag{5}$$

4.2 Mean time to system failure Calculations for System II.

Let $P_n(t)$ be the probability that the system is working at time t ($t\geq 0$). The initial conditions are

$$P(0)=[P_1(0),P_2(0),P_3(0),P_4(0)]=[1,0,0,0]. \tag{6}$$

We obtain the following differential equations from Figure 2,

$$\frac{dP_1(t)}{dt}=-(\lambda_1+\lambda_2+\beta)P_1(t)+(1-p)\alpha_r P_3(t)$$
$$\frac{dP_2(t)}{dt}=\beta P_1(t)-(\lambda_1+\lambda_2)P_2(t)+p\alpha_{m1}P_3(t)+\alpha_{m2}\,\mathrm{P}_4(t) \tag{7}$$
$$\frac{dP_3(t)}{dt}=\lambda_1 P_1(t)+\lambda_1 P_2(t)-(\mathrm{p}\,\alpha_{m1}+(1-p)\alpha_r)P_3(t)$$
$$\frac{dP_4(t)}{dt}=\lambda_2 P_1(t)+\lambda_2 P_2(t)-\alpha_{m2}P_4(t)$$

The differential equation can be expressed in matrix form as

$$P^{'}=BP, \tag{8}$$

where,

$$B=\begin{bmatrix}-(\lambda_1+\lambda_2+\beta) & 0 & (1-p)\alpha_r & 0 \\ \beta & -(\lambda_1+\lambda_2) & p\alpha_{m1} & \alpha_{m2} \\ \lambda_1 & \lambda_1 & -(\mathrm{p}\,\alpha_{m1}+(1-p)\alpha_r) & 0 \\ \lambda_2 & \lambda_2 & 0 & -\alpha_{m2}\end{bmatrix}.$$

To evaluate the transient solution is too difficult. Therefore, to calculate the MTSF, we take the transpose of matrix B and delete the row and column of absorbing states i.e. states 3 and 4. The new matrix is called R. The expected time to reach an absorbing state is given by

$$E[T_{P(0)\to P(arbsorbing)}]=MTSF_2=P(0)(-R^{-1})[1, \quad 1, \quad 1]^T, \tag{9}$$

where,

$$R=\begin{bmatrix}-(\lambda_1+\lambda_2+\beta) & \beta \\ 0 & -(\lambda_1+\lambda_2)\end{bmatrix}.$$

Therefore, the explicit expression for the mean time to system failure is given by

$$MTSF_2 = \frac{\lambda_1 + \lambda_2 + \beta}{(\lambda_1 + \lambda_2)(\lambda_1 + \lambda_2 + \beta)}. \qquad (10)$$

5.0 Availability Analysis

5.1 Availability Calculations for System I

For the availability of System I, we use the same initial conditions (1) and differential equations (2). The differential equations (2) can be expressed in matrix form as

$$\begin{bmatrix} \frac{dP_1(t)}{dt} \\ \frac{dP_2(t)}{dt} \\ \frac{dP_3(t)}{dt} \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\beta) & 0 & (1-p)\alpha_r \\ \beta & -\lambda_1 & p\alpha_{m1} \\ \lambda_1 & \lambda_1 & -(p\alpha_{m1}+(1-p)\alpha_r \end{bmatrix} = \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix}.$$

The steady- state probability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero which allows us to calculate the steady -state probabilities .The states S_1 and S_2 are the only working states of the system. The steady-state availability is sum of the probability of operational states. Thus,

$$AV_1(\infty) = P_1(\infty) + P_2(\infty), \qquad (11)$$

and

$$AP = 0,$$

or in matrix form

$$\begin{bmatrix} -\left(\lambda_1+\beta\right) & 0 & (1-p)\alpha_r \\ \beta & -\lambda_1 & p\alpha_{m1} \\ \lambda_1 & \lambda_1 & -(p\alpha_{m1}+(1-p)\alpha_r \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \qquad (12)$$

Using the following normalizing condition,

$$P_1(\infty) + P_2(\infty) + P_3(\infty) = 1, \qquad (13)$$

wesubstitute (13) in any one of the redundant rows in (12) to obtain

$$\begin{bmatrix} -\left(\lambda_1+\beta\right) & 0 & (1-p)\alpha_r \\ \beta & -\lambda_1 & p\alpha_{m1} \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \qquad (14)$$

The solution of (14) provides the steady-state probabilities and the explicit expression for availability is given by

$$AV_1(\infty) = \frac{\alpha_r(\lambda_1 - \lambda_1 p + \beta - \beta p) + \alpha_{m1}(\lambda_1 p + \beta p)}{(\lambda_1 + \beta)(\lambda_1 + \alpha_r + \alpha_{m1} p - \alpha_r p)}. \qquad (15)$$

5.2 Availability Calculations for System II.

For the availability of System II, we use the same initial conditions (6) and the differential equations (7). The differential equations (7) can be expressed in matrix form as

$$\begin{bmatrix} \frac{dP_1(t)}{dt} \\ \frac{dP_2(t)}{dt} \\ \frac{dP_3(t)}{dt} \\ \frac{dP_1(t)}{dt} \end{bmatrix} = \begin{bmatrix} -(\lambda_1+\lambda_2+\beta) & 0 & (1-p)\alpha_r & 0 \\ \beta & -(\lambda_1+\lambda_2) & p\alpha_{m1} & \alpha_{m2} \\ \lambda_1 & \lambda_1 & -(\mathrm{p}\,\alpha_{m1}+(1-p)\alpha_r) & 0 \\ \lambda_2 & \lambda_2 & 0 & -\alpha_{m2} \end{bmatrix} \begin{bmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}.$$

The steady- state probability can be obtained using the following procedure. In the steady-state, the derivatives of the state probabilities become zero which allows us to calculate the steady -state probabilities. The states S_1 and S_2 are the only operational states of the system. The steady-state availability is the sum of the probability of operational states. Thus,

$$AV_2(\infty) = P_1(\infty) + P_2(\infty), \tag{16}$$

and

$$BP = 0,$$

or in matrix form

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\beta) & 0 & (1-p)\alpha_r & 0 \\ \beta & -(\lambda_1+\lambda_2) & p\alpha_{m1} & \alpha_{m2} \\ \lambda_1 & \lambda_1 & -(\mathrm{p}\,\alpha_{m1}+(1-p)\alpha_r) & 0 \\ \lambda_2 & \lambda_2 & 0 & -\alpha_{m2} \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{17}$$

Using the following normalizing condition

$$P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) = 1, \tag{18}$$

We substitute (18) in any one of the redundant rows in (17) to obtain

$$\begin{bmatrix} -(\lambda_1+\lambda_2+\beta) & 0 & (1-p)\alpha_r & 0 \\ \beta & -(\lambda_1+\lambda_2) & p\alpha_{m1} & \alpha_{m2} \\ \lambda_1 & \lambda_1 & -(\mathrm{p}\,\alpha_{m1}+(1-p)\alpha_r) & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_1(\infty) \\ P_2(\infty) \\ P_3(\infty) \\ P_4(\infty) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{19}$$

The solution of (19) provides the steady-state probabilities and the explicit expression for availability is given by

$$AV_2(\infty) = \frac{N}{D}, \tag{20}$$

where,

$$N = \lambda_1\alpha_{m2}\alpha_r - \lambda_1\alpha_{m2}\alpha_r p + \lambda_2\alpha_{m2}\alpha_r + \alpha_{m2}\alpha_r\beta + \lambda_1\alpha_{m1}\alpha_{m2} p + \lambda_2\alpha_{m1}\alpha_{m2} p - \lambda_2\alpha_{m2}\alpha_{rp} + \alpha_{m1}\alpha_{m2}\beta p$$

$$D = (\lambda_1+\lambda_2+\beta)(\lambda_1\alpha_{m2} + \lambda_2\alpha_r + \alpha_{m2}\alpha_r + \lambda_2\alpha_{m1} p - \lambda_2\alpha_r p + \alpha_{m1}\alpha_{m2} p - \alpha_{m2}\alpha_r p).$$

6.0 Results and Discussions

In this section, we use Matlab to compare the results for Mean time to system failure and Steady state Availability for the two systems using the following set of parameter values $\lambda_1 = 0.2$, $\lambda_2 = 0.4$, $\alpha_{m1} = 0.1$, $\alpha_{m2} = 0.14$, $\alpha_r = 0.18$, $\beta = 0.04$, $p = 0.7$.

Table 1: Mean time to system failurecomparison of the two systems with respect to λ_1

λ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$MTSF_1$	10.0000	5.0000	3.3333	2.5000	2.0000	1.6667	1.4286	1.2500	1.1111
$MTSF_2$	2.0000	1.6667	1.4286	1.2500	1.1111	1.0000	0.9091	0.8333	0.7692

Table 2: Steady State Availability comparison of the two systemswith respect to λ_1

λ_1	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$AV_1(\infty)$	0.5569	0.3909	0.3008	0.2443	0.2057	0.1776	0.1563	0.1395	0.1260
$AV_2(\infty)$	0.2162	0.1853	0.1622	0.1442	0.1298	0.1180	0.1082	0.0999	0.0928

Table 3: Steady State Availability comparison of the two systemswith respect to α_{m1}

α_m	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$AV_1(\infty)$	0.3909	0.4950	0.5681	0.6222	0.6639	0.6971	0.7240	0.7463	0.7652
$AV_2(\infty)$	0.1853	0.2059	0.2175	0.2250	0.2302	0.2340	0.2369	0.2393	0.2411

Table 4: Steady State Availability comparison of the two systemswith respect to α_r

α_r	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$AV_1(\infty)$	0.3300	0.3909	0.4417	0.4846	0.5214	0.5533	0.5813	0.6059	0.6278
$AV_2(\infty)$	0.1707	0.1853	0.1958	0.2037	0.2098	0.2147	0.2188	0.2221	0.2250

From Table 1, it is clear that the Mean time to system failure for the two systems decreases with increase in the value of λ_1 which reflects the effect of failure rate on mean time to system failure. Similarly, from Table 2, the Steady state Availability of the two systems decreases with the increase in the value of λ_1 which reflects the effect of failure rate on system availability. From Table 3 and 4,Steady state Availability of the two systems increases with the increase in the value of α_{m1} and α_r which reflects the effect of repair and replacement on system availability. Thus, $MTSF_1 > MTSF_2$ and $AV_1(\infty) > AV_2(\infty)$. In summary, the optimal system using Mean time to system failure and Steady-state availability is system I.

7.0 Conclusion

Inthis paper, two different systems subject to different types of failure are considered. Explicitexpressions for Mean time to system failure and Steady-state Availability are derived. Comparisons between the two systems using assumed numerical parameter values are performed. From the simulations results, the optimal system using mean time to system failure and Steady-state availability is system I.

8.0 References

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