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Heat Transfer Effects on Magnetogasdynamic Axi-Symmetric Boundary Layer Flow Over A Blunt Body

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Abstract

The effect of heat transfer on compressible axi-symmetric laminar boundary layer flow near the stagnation region of a blunt body in the presence of magnetic field has been considered for Mach number five and above. The governing first order boundary layer equations are derived by an order of magnitude analysis in which only 0(1) is retained and transformed to body-oriented coordinates. The Dorodnitsyn - Stewartsonsimilarity transformations are utilized to transform the resulting dimensionless partial differential equations to coupled non-linear ordinary differential equation. Solutions were obtained using the perturbation method of solution and the effects of the resulting parameters on the velocity and temperature profiles are shown graphically. The skin friction and heat transfer effects are also shown and discussed.

Keywords: Magneto-gasdynamic, blunt body, heat transfer, boundary layer

Nomenclature:

u = velocity component in the s – direction
v = velocity component in the n – direction
ts = coordinate along the body
n = coordinate normal to the body
M_∞ *= free stream mach number*
B_0 *= magnetic flux*
V_∞ *= free stream velocity*
R_e *= Reynolds number*
p = pressure
h = enthalpy
K_T *= thermal conductivity*
T_∞ *= free stream temperature*
M_m *= magnetic mach number*
Pr = Prandtl number
R_m *= magnetic Reynolds number*
k = longitudinal curvature parameter
ρ *= density*
σ *= electrical conductivity*
β *= pressure gradient*
μ *= dynamic viscosity*
τ *= shear stress*
γ *= specific heat ratio*
g_w *= surface temperature parameter*

1.0 Introduction

The computation of the hypersonic (Ma > 5) laminar boundary layer flow over axi-symmetric blunt bodies in the presence of magnetic field has been a topic of scientific research since late 1950's when it was first investigated in the study of applied magnetic fields on re-entry vehicles [1,2]. Hypersonic flow study is complicated due to the physical conditions involved, such as, high temperature chemically reacting, thin shock layer. The flow is characterized by high speed, which is slowed by the viscous effects within the boundary layer, and lost kinetic energy which is transformed into kinetic energy.

These extreme viscous effects can create very high temperature high enough that dissociation of gas takes place. To alleviate this problem of aerodynamic heating, it becomes necessary that the leading edge of the vehicle be sufficiently blunt in order to reduce the heat transfer rate. This theory of the heat transfer rates at the stagnation point is extended to the region away from the stagnation point in[3].

Lees [4] early in literature considered the laminar heat transfer over blunt-nosed hypersonic flight speed while the theory of stagnation point heat transfer in dissociated air was also investigated [5]. Crabtree et al [6] examined a system of predicting heat transfer to blunt-nosed bodies throughout the hypersonic speed range for boundary layers. The compressible axially

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symmetric laminar boundary layer flow in the stagnation region of a blunt body was also examined in the presence of magnetic field[7]. Also the effect of heat transfer on steady laminar boundary layer flow of a viscous compressible fluid was considered for flow past a horizontal circular cylinder[8].
Recently Nozaki[9], conducted a research on the reduction of skin friction and heat transfer over a hypersonic cruising vehicles by mass injection and most recently a general Reynolds analogy on the relation between skin friction and heat transfer along windward sides of blunt nosed bodies in hypersonic flows was analysed [10].
In this paper, an attempt is made to investigate the effect of heat transfer on steady laminar boundary layer flow of a viscous compressible fluid past an axi-symmetric blunt body in the presence of magnetic fields. Due to hypervelocity of the hypersonic flow, secondary effects are expected to come into play. Hence an order of magnitude analysis is utilized to 0(1) boundary layer equations which are considered, neglecting the 0(ε) equations[11]. The computation of the problem is achieved using the perturbation method of solution, though earlier investigators had applied other numerical methods.

2.0 Mathematical Analysis

The problem is steady, laminar, two-dimensional, compressible, viscous boundary layer flow over an axi-symmetric, impermeable blunt body. The fluid is treated as a continuum and is assumed to be a perfect gas, hence the effects of dissociation and diffusion are neglected. It is electrically conducting and conductivity,
σ << 1. The effect of magnetic Reynolds number is small as is applicable in aerospace applications[12]. The flow being over axi-symmetric blunt bodies, the two-dimensional problem, with x along the surface and y normal to the surface is transformed to body oriented coordinates s and n respectively, where the velocity vector $\overline{V} = (u , v)$ as shown in figure 1, [11].

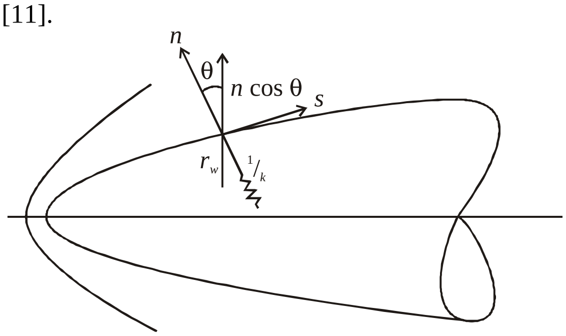

Figure 1: A body oriented coordinate system

Utilizing the transformation scale factors: $h_S = 1 + kn$, $h_n = 1$, $h_\phi = r^n$, a general continuum model under the magnetogasdynamics approximation, in axi-symmetric geometry, with the magnetic field assumed to originate from the magnetized body are,[11]and [13]. Only the normal component of the magnetic field is significant as the boundary layer.

$$\frac{\partial}{\partial n}\left(r^\sigma \rho u\right) + \frac{\partial}{\partial n}\left(h_S r^\sigma \rho v\right) = 0 \tag{1}$$

$$\rho\left(\frac{u}{h_S}\frac{\partial u}{\partial s} + \frac{v\partial u}{\partial n} + \frac{kuv}{h_S}\right) = -\frac{1}{h_S}\frac{\partial p}{\partial s} + \frac{1}{h_S}\frac{\partial}{\partial s}\left(\mu\frac{\partial u}{\partial s}\right) - \sigma B_0^2 u \tag{2}$$

$$\rho\left(\frac{u}{h_S}\frac{\partial u}{\partial s} + \frac{u\partial v}{\partial n} + \frac{ku^2}{h_S}\right) = -\frac{\partial p}{\partial n} + \frac{\partial}{\partial n}\left(\mu\frac{\partial v}{\partial n}\right) \tag{3}$$

$$\rho Cp\left(\frac{u}{h_S}\frac{\partial T}{\partial s} + v\frac{\partial T}{\partial n}\right) = \frac{u}{h_S}\frac{\partial p}{\partial s} + v\frac{\partial p}{\partial n} + K_T\frac{\partial}{\partial n}\left(\frac{\partial T}{\partial n}\right) + \mu\left(\frac{\partial u}{\partial n}\right)^2 + \sigma B_0^2 u^2 \tag{4}$$

With

$$p = \rho RT \tag{5}$$

Where $r = r_w + n\cos\theta$

With the no-slip continuum flow boundary conditions[14]:

$$\left.\begin{array}{l} \text{At} \quad n=0, \quad u=v=p=o\,, \quad T=T_w \\ \text{As} \quad n\to\infty \quad u=V_\infty\,, \quad v=p=o, \quad T=T_\infty \end{array}\right\} \tag{6}$$

Following the non-dimensional variable of and also introducing the boundary layer scaling; $u=\bar{u}$, $v=\varepsilon\bar{v}$ and $n=\varepsilon\bar{n}$ of[11] the dimensionless boundary layer equations are obtained as (retaining only terms of 0(1) and 0(ε)): Type equation here.

$$\frac{\partial}{\partial s}\left\{r_w^{\sigma_1}\rho\left(1+\frac{\varepsilon n\cos\theta}{r_w}\right)\right\}+\frac{\partial}{\partial n}\left\{r_w^{\sigma_1}\rho v\left(1+\frac{\varepsilon(k+\sigma\cos\theta)n}{r_w}\right)\right\}=0 \tag{7}$$

$$\rho\frac{du}{dt}=-\frac{1}{\gamma M_\infty^2}\frac{\partial p}{\partial s}+\frac{\partial}{\partial n}\left(\mu\frac{\partial u}{\partial n}\right)-\frac{R_m}{M_m^2}u+\varepsilon\left\{\left(2k+\frac{\sigma\cos\theta}{r_w}\right)\frac{\partial u}{\partial n}\right.$$
$$\left.+\frac{kn}{\gamma M_\infty^2\rho}\frac{\partial P}{\partial s}+k\rho u\left(n\frac{\partial u}{\partial s}-v\right)-k\frac{\partial}{\partial n}(\mu u)\right\} \tag{8}$$

$$\frac{\partial p}{\partial n}=\gamma M_\infty^2\rho k u^2 \tag{9}$$

$$\rho\frac{dT}{dt}=\frac{(\gamma-1)}{\gamma}\frac{dp}{dt}+\left[\mu\left(\frac{\partial u}{\partial n}\right)^2+\frac{R_m}{M_m^2}u^2\right](\gamma-1)M_\infty^2$$
$$+\frac{1}{\Pr}\frac{\partial}{\partial n}\left(\mu\frac{\partial T}{\partial n}\right)+\varepsilon\left\{knv\left(\rho\frac{\partial T}{\partial s}-\frac{\gamma-1}{\gamma}\frac{\partial p}{\partial s}\right)\right\}$$
$$+\frac{r_w^\sigma}{\Pr}\left(k+\frac{\sigma\cos\theta}{r_w}\right)\mu\frac{\partial T}{\partial u}-2(\gamma-1)M_\infty^2 k\mu u\frac{\partial u}{\partial n} \tag{10}$$

and $$p=\rho T \tag{11}$$

where $\varepsilon=\frac{1}{\sqrt{\text{Re}}}$ and $\frac{d}{dt}=u\frac{\partial}{\partial s}+v\frac{\partial}{\partial n}$ for steady flow.

The Reynolds number is large in the region near the body surface (boundary layer).

The perturbation scheme of[15] is employed with $\varepsilon=\frac{1}{\sqrt{\text{Re}}}$ as the perturbation parameter, and with the inner asymptotic expansion. That is $u(s,n,\varepsilon)\sim u_1(s,\bar{n})+\varepsilon u_2(s,\bar{n})$ to obtain the first order boundary layer equations. The second order equations are neglected since interest is not on secondary effects.

The asymptotic expansions with

$$\mu(T)=\mu(t_1)+\varepsilon\mu^1(t_1)t_2+\ldots\ldots$$

yield the boundary layer first order equations:

$$\frac{\partial}{\partial s}\left(r_w^{\sigma_1}p_1u_1\right)+\frac{\partial}{\partial n}\left(r_w^{\sigma_1}p_1v_1\right)=0 \tag{12}$$

$$\rho_1\left(u_1\frac{\partial u_1}{\partial s}+v_1\frac{\partial u_1}{\partial n}\right)=-\frac{1}{\gamma M_\infty^2}\frac{\partial p_1}{\partial s}+\frac{\partial}{\partial n}\left(\mu_1\frac{\partial u_1}{\partial n}\right)-\frac{R_m}{M_m^2}u_1 \tag{13}$$

$$\frac{\partial p_1}{\partial n}=0 \tag{14}$$

$$\rho_1\left(u_1\frac{\partial t_1}{\partial s}+v_1\frac{\partial t_1}{\partial n}\right)=\frac{\gamma-1}{\gamma}\left(u_1\frac{\partial p_1}{\partial s}+v_1\frac{\partial p_1}{\partial n}\right)$$

$$+\ (\gamma-1)M_\infty^2\left[\mu_1\left(\frac{\partial u_1}{\partial n}\right)^2+\frac{R_m}{M_m^2}u_1^2\right]+\frac{1}{\Pr}\frac{\partial}{\partial n}\left(\mu_1\frac{\partial t_1}{\partial n}\right) \qquad (15)$$

$$p_1 = \rho t_1 \qquad (16)$$

With boundary conditions

$$\left.\begin{aligned} &u_1(s,o) = V_1(s,o) = 0\ , \quad t_1(s,o) = \frac{T_w}{T_\infty} \\ &u_1(s,\infty) = t_1(s,\infty) = \rho_1(s,\infty) = 1\ , \ \ p_1 = \frac{1}{\gamma M_\infty^2} \end{aligned}\right\} \qquad (17)$$

Since $\frac{\partial p_1}{\partial n} = 0$ the momentum and energy equations become:

$$\rho_1\left(u_1\frac{\partial u_1}{\partial s}+v_1\frac{\partial u_1}{\partial n}\right) = -\frac{1}{\gamma M_\infty^2}\frac{dp_1}{ds}+\frac{\partial}{\partial n}\left(\mu_1\frac{\partial u_1}{\partial n}\right)-\frac{R_m}{M_m^2}u_1 \qquad (18)$$

and

$$\rho_1\left(u_1\frac{\partial t_1}{\partial s}+v_1\frac{\partial t_1}{\partial n}\right) = \frac{(\gamma-1)}{\gamma}u_1\frac{dp_1}{ds}+(\gamma-1)M_\infty^2\left[\mu_1\left(\frac{\partial u_1}{\partial n}\right)^2+\frac{R_m}{M_m^2}u_1^2\right]$$

$$+\ \frac{1}{\Pr}\frac{\partial}{\partial n}\left(\mu_1\frac{\partial t_1}{\partial n}\right) \qquad (19)$$

3.0 Solution of Boundary Layer Equations

The system of steady laminar boundary layer equations are coupled, non-linear partial differential equations. The energy equation is simplified by introducing enthalpy,

$$h_0 = h + \frac{1}{2}u^2$$

so that $t = \frac{h}{c_p}$ becomes

$$t = X_\infty h_0 - \frac{(\gamma-1)}{2}M_\infty^2 u^2 \qquad (20)$$

where $X_\infty = 1 + \frac{\gamma-1}{2}M_\infty^2$

Then with the similarity transformation variable of Dorodnitsyn and Stewartson [11[:

$$\xi(s) = \int_0^s (\rho\mu u)_e\, r_w^{2\sigma_1}\, ds \qquad (21)$$

$$\eta(s,n) = \frac{r_w^{\sigma_1}\rho_e u_e}{(2\xi)^{1/2}}\int_0^n \frac{\ell}{\ell_e}dn \qquad (22)$$

We have $\frac{d\xi}{ds} = (\rho\mu u)_e r_w^{2r_1}$

$$\frac{\partial\eta}{\partial n} = \frac{r_w^{\sigma_1}\rho u_e}{(2\xi)^{1/2}}$$

$$\frac{\partial}{\partial n} = \frac{r_w^{\sigma_1}\rho u_e}{(2\xi)^{1/2}}\frac{\partial}{\partial\eta}$$

and $$\frac{\partial}{\partial s} = \frac{\partial \xi}{\partial s}\frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial s}\frac{\partial}{\partial \eta}$$

For similarity of solutions we have

$$\frac{u}{u_e} = f'(\eta) \tag{23}$$

$$\frac{h_0}{h_{0e}} = g(\eta) \tag{24}$$

Where f and g are depend only on η and hoe is a constant.
Now with the compressible flow stream function

$$\frac{\partial \varphi}{\partial s} = -r_w^{\sigma_1}\rho v \quad , \quad \frac{\partial \varphi}{\partial n} = r_w^{\sigma}\rho u \tag{25}$$

We have $$u = u_e f' = \frac{u_e}{(2\xi)^{1/2}}\frac{\partial \varphi}{\partial \eta}$$

The subscript e is for the boundary layer edge. The boundary layer equations become

$$\left(\frac{\rho\mu}{\rho_e\mu_e} f''\right)^1 + ff'' + \frac{2\xi}{u_e}\frac{du_e}{d\xi}\left(\frac{\rho_e}{\rho} - f'^2\right) - \frac{R_m}{M_m^2} f' = 0 \tag{26}$$

$$X_\infty h_o (\rho\mu g')^1 + \Pr X_\infty h_o fg' - (1-\Pr)(\gamma - 1)M_\infty^2 (\rho\mu f' f'')^1 = 0 \tag{27}$$

We use

$$\frac{\ell_e}{\ell} = \frac{T}{T_e} = \frac{h_o}{h_e} = \frac{h_o - \frac{1}{2}u^2}{h_{oe} - \frac{1}{2}u_e^2} = \frac{h_{oe}g - \frac{1}{2}u_e^2 f'^2}{h_{oe} - \frac{1}{2}u_e^2} \tag{28}$$

and the pressure gradient parameter is

$$\beta = \frac{2\xi}{u_e}\frac{du_e}{d\xi}\frac{T_{oe}}{T_e} \tag{29}$$

For similarity β = constant (30)

and $$\frac{\rho\mu}{\rho_e\mu_e} = C(\eta) \tag{31}$$

C is the Chapman-Rubesin constant and is assumed to be a constant.
If $\mu \sim T$ then C = 1.
Also

$$\frac{T_{oe}}{T_e} = \frac{h_{oe}}{h_{oe} - \frac{1}{2}u_e^2}$$

$$= 1 + \frac{\gamma - 1}{2}M_e^2 \tag{32}$$

Employing the above simplifications equations (26) and (27) become:

$$f''' + ff'' + \beta(g - f'^2) - \frac{R_m}{M_m^2} f' = 0 \tag{33}$$

and $$g'' + \Pr fg' - \lambda(1-\Pr)(ff'')^1 = 0 \tag{34}$$

where $$\lambda = \frac{(\gamma - 1)M_\infty^2}{1 + \frac{\gamma - 1}{2}M_\infty^2}$$

The corresponding boundary conditions are:

$$\left.\begin{array}{l} f(0) = f_w = \text{constant and } f'(0) = 0\,, \quad f'(\infty) = 1 \\ g(0) = g_w = \text{constant or } g'(0) = 0 \quad , \quad g(\infty) = 1 \end{array}\right\} \tag{35}$$

For the existence of similar solutions for hypersonic flow, it is assumed that $\lambda = 2$ as $M_\infty \to \infty$, $\Pr \neq 1$ and β is a constant [16].

Since the interaction parameter $\frac{R_m}{M_\infty^2} << 1$ for most aerodynamic flow, $\frac{R_m}{M_\infty^2}$ which is $0(\varepsilon)$, it is taken as the perturbation parameter in this problem. Thus employing the perturbation method of solution, we seek the asymptotic series solution of [17]:

$$\left.\begin{array}{l} f(\eta) = f_w + \varepsilon f_1(\eta) + \dots \\ g(\eta) = g_w + \varepsilon g_1(\eta) + \dots \end{array}\right\} \tag{36}$$

which satisfies the boundary conditions (35) and yields a set of linear differential equations which are solved to obtain the results

$$g(\eta) = 1 - (1 - g_w)e^{-\Pr f_w \eta} \tag{37}$$

and

$$f(\eta) = f_w - \frac{1}{f_w} + \eta + \frac{e^{-f_w\eta}}{f_w} + \beta(1 - g_w)\left\{ C\left(\frac{\eta}{f_w} - \frac{1}{f_w^2} + \frac{e^{-f_w\eta}}{f_w^2}\right) - \eta^2 - \frac{e^{-f_w\eta}}{\Pr(1-\Pr)f_w^3} + \frac{e^{-f_w \Pr\eta}}{\Pr^2(1-\Pr)f_w^3} - \frac{1}{\Pr^2 f_w^3}\right\} \tag{38}$$

where C = 3.4 is constant[11].

The skin friction and the rate of heat transfer at the surface of the body is determined from the relations

$$\tau_w = \left(\mu\frac{\partial u}{\partial n}\right)_w \qquad \text{and} \qquad q_w = -K\left(\frac{\partial T}{\partial n}\right)_w$$

Normalization yields

$$\tau_w = \frac{1}{\sqrt{2s}}\, f_w''$$

and $$q_w = -\frac{(1-g_w)g_\eta^1}{\sqrt{2s}\,\Pr g_w} = 0$$

Hence the skin friction coefficient is

$$C_f = \left(\sqrt{\frac{2}{\operatorname{Re} s}}\right) f_w'' \tag{39}$$

and the Stanton No

$$St = \frac{g_{\eta=0}^1}{\sqrt{2\operatorname{Re} s}} \tag{40}$$

4.0 Results and Discussion

The study presents an analytical model of heat transfer on hypersonic flow over blunt bodies in the presence of magnetic field. The analysis is carried out for a hot wall with surface temperature parameter $g_w > 1$, of Mach number 5. Pr = 0.72 for varying pressure gradient parameter $0 \le \beta \le 3.0$.

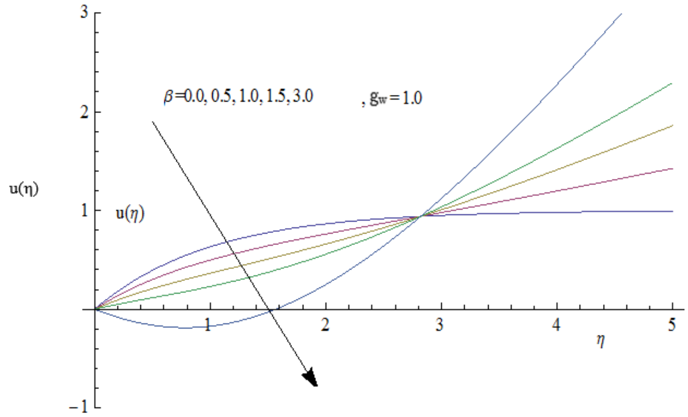

Fig 2: Effect of pressure gradient parameter, β on velocity

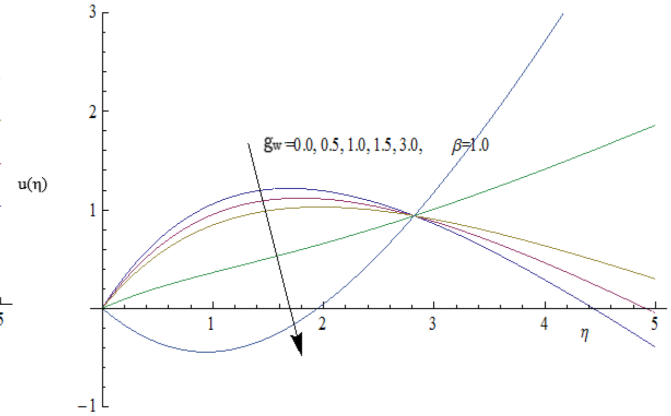

Fig 3: Variation of temperature parameter, g_w on velocity

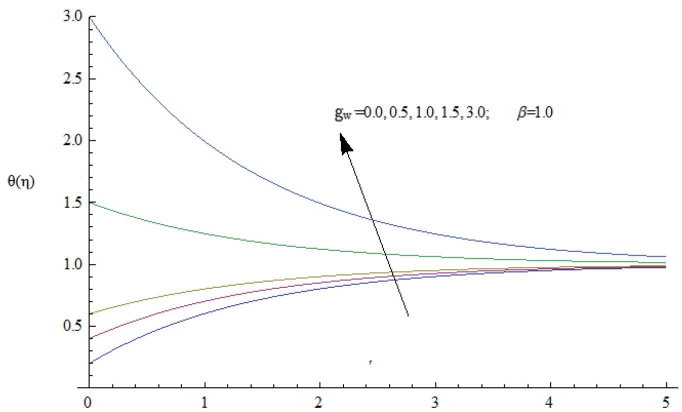

Fig 4: Effect of wall temperature parameter, g_w on enthalpy η

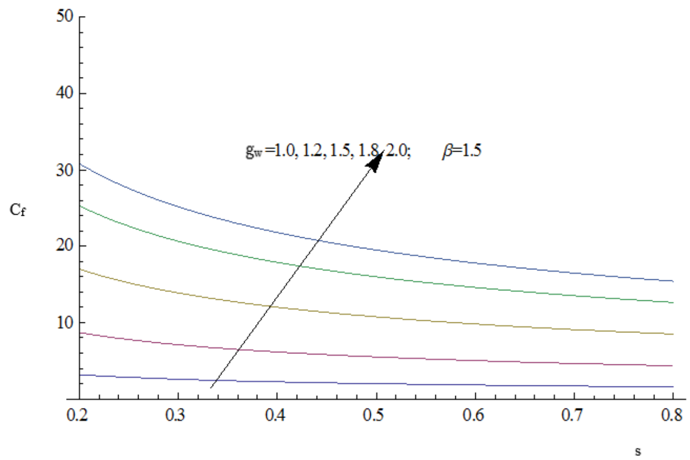

Fig 5: Effect of wall temperature parameter, g_w on the skin friction coefficient

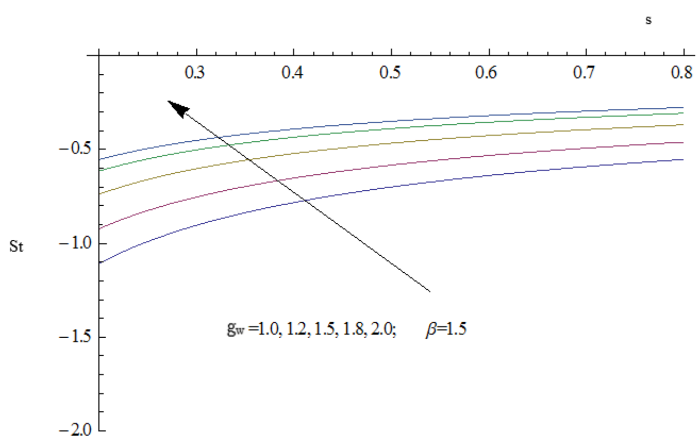

Fig 6: Effect of wall temperature parameter, g_w on the heat transfer coefficient

Figure 2 shows the variation β for velocity, and indicates that as β increases, the velocity decreases. There is an overshoot for $\beta > 1.5$ which is in agreement with existing literature [11].

The effect of g_w on velocity and enthalpy profile in figures 3 and 4 respectively show that as g_w increases, the velocity decreases, while the enthalpy increases. There is an adverse drop in velocity for $g_w > 1.5$ which is in agreement with [11].

Figures 4 and 5, gives the effect of g_w on the skin friction coefficient and Stanton number respectively and show increase in both as the surface temperature parameter increases.

5.0 Conclusion

Computational study of heat transfer effect on hypersonic axi symmetric laminar boundary layer flow near stagnation point of a blunt body in the presence of magnetic field has been considered for $Ma \geq 5$. A semi analytic approach, the perturbation method was employed with the magnetic interaction parameter, $\frac{R_m}{M_\infty^2} << 1$ as the perturbation parameter. The results obtained are in agreement with that obtained numerically.

The skin friction and heat transfer coefficients are found to increase as the wall temperature parameter, $g_{w,}$ increases. This is an indication that they are higher as the stagnation point is approached. Hence their effects are greater at that point.

6.0 References

[1]. Bush, W.B., (1958). Magnetohydrodynamic hypersonic flow past a blunt body. Journ. of the Aerospace Scs. Vol. 23, No. 11.

[2]. Meyer, R.X., (1959). Magnetohydrodynamics and aerodynamic heating. American Rockel Society Journal. Vol. 29, No. 3, pp.187-192.

[3]. Kemp, N.H., Rose, P.H. and Delta, R.W., (1959). Laminar heat transfer around blunt bodies in dissociated air. J. Aerospace Sc. Vol. 26, No. 7, pp.421-430.

[4]. Lees, L., (1956). Laminar heat transfer over blunt nosed bodies at hypersonic flight speeds. Journal of Jet Propulsion. Vol. 26, No. 4, pp.259-269.

[5]. Fay, J.A. and Ridell, F.R., (1958). Theory of stagnation point heat transfer in dissociated air. J. Aero. Sc. Vol. 25, No. 2, pp.73-85.

[6]. Crabtree, L.F., Domnett, R.L. and Woodley, J.G., (1965). Estimation of heat transfer to flat plates, cones and blunt bodies. R&M No. 3637.

[7]. Nath, G., (1971). Compressible axially symmetric laminar boundary layer flow in the stagnation region of a blunt body in the presence of magnetic field. Acta Mechanica 12, pp.267-273.

[8]. Hossain, M.B., Dhaka, B., Pop, I., Chu, R. and Na, T.Y., (1998). Effect of heat transfer on compressible boundary layer flow over a circular cylinder. Acta Mechanica 131, pp.267-272.

[9]. Nozaki, Y., (2007). Reducing skin friction and heat transfer over a hypersonic cruising vehicle by mass injection. Thesis of University of Florida.

[10]. Chen, X-X., Wang, Z-H. and Yu, Y-L., (2015). General. Reynold analogy for blunt-nosed bodies in hypersonic flow. Physics of fluid dynamics.

[11]. Emanuel, G., (1993). Analytical fluid dynamics. CRC Press Inc.

[12]. Hughes, W.F. and Young, F.J., (1989). The electromagnetic dynamics of fluids. Robert E. Krieger Publ.

[13]. Poggie, J. and Gaitonde, D.V.(2001). Computational studies of magnetic control in hypersonic flow. AIAA, 0196, pp.1-12.

[14]. Rossow, V.J., (1958). Magnetohydrodynamic analysis of heat transfer near a stagnation point. Journal of the Aeronautical Sci., Vol. 25, No. 5, pp.334-335.

[15]. Van Dyke, M., (1964). Perturbation method in fluid mechanics. Academic Press.

[16]. Cho, Y. and Aessopos, A., (2004). Similarity transformation methods in the analysis of the two dimensional steady compressible laminar boundary layer. Term paper 226. Compressible fluid dynamics.

[17]. Bestman, A.R., (1990). The boundary layer flow past a semi-infinite heated porous plate for a two component plasma. Astrophysics and space science. 173, pp.93-100.