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Polynomial Expansions and Factorizations

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Abstract

The Binomial theorem as we know gives a general rule for finding the expansions of expressions involving the sum or difference of two numbers raised to stated powers. In this paper a method of obtaining the same expansions which is similar to differentiation and integration is presented. This method appears easier and simpler compared to the traditional Binomial method of obtaining the expansions. This paper also discussed a different method of obtaining factorization of polynomials completely without the use of factor and remainder theorems.

Keywords: Binomial theorem, polynomials, expansions, factorizations, differentiation, integration

1.0 Introduction

By the Binomial expansion,

$$(x+y)^n = x^n + nx^{n-1}y + \frac{n(n-1)x^{n-2}y^2}{2!} + \ldots + \frac{n(n-1)(n-2)(n-3)\ldots(n-r+1)x^{n-r}y^r}{r!} + \ldots + b^n \quad (1)$$

$$= nC_0x^n + nC_1x^{n-1}y + nC_2x^{n-2}y^2 + \ldots + nC_rx^{n-r}y^r + \ldots + nC_ny^n \quad (2)$$

where x, y are any real number and n is a natural number[1]. These are normally called the binomial series and the numbers which multiply “x” and “y“in this binomial expansion are called the binomial coefficients or the coefficients of the powersof “x” and “y “. This paper will discuss the method of obtaining the same expansionwithout recourse to combinatorial but with the application of differentiation and integration in a special form.

Recall also the factor theorem which states “If p(x) is a polynomial and p (a) = 0, then p(x) has a linear factor x-a [2]. This statement (the factor theorem) is also the special case of the remainder theorem where R = p (a) = 0. This theorem helps us to factor polynomials by trial. Given a polynomial p(x), all we need to do is to look for values of x $(x\varepsilon\Re)$ say x = “a “for which p (a) is zero, then divide p(x) by x-a to get the quotient Q(x). The process is repeated until all the factors are realized [1].Solving any algebraic equation f(x) = 0 means finding all complex numbers x for which f(x) = 0 where the x are the roots ofthe equation [3].

In this paper, a new method of factorizing polynomials completely is developed without recourse to factor/remainder theorem The next Section discusses the approach adopted in this paper while examples will be outlined in Section 3.

2.0 Methods

A. Let the expansion of the same expression (1) be

$$(x+y)^n = \delta(x)Q(y) + \delta_1^*(x)Q_*^{/}(y) + \delta_{11}^*(x)Q_*^{//}(y) + \ldots + \delta_n^*(x)Q_*^{\,n}(y) \quad (3)$$

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where $\delta(x) = x^n$ *and* $Q(y) = y^0$, δ^* *and* Q_* *are two operations similar to differentiation and integration* respectively but not exactly the same .The expansion process differentiates x and integrates y continuously until the required (n+1) terms in the expansion of $(x+y)^n$ for n = 1, 2, 3, . . . are obtained. One thing remarkable or special with this process is that the coefficients of the powers of "x"and"y"are obtained at the same time of differentiation and integration. The numbers are treated as variables in the process of differentiation. These will be illustrated with examples in Section 3.

B. The polynomial

$$p(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0 \qquad (4)$$

is completely factorizable if and only if there exists numbers such that:

(1) Theirproducts equal to $a_n^{\,n-1}.a_0$

(ii) Their sum equal a_{n-1} and

(iii) they have a suitable relationship with the coefficient of other lesser powers of x.

Put more concisely, the polynomial:

$$p(x) = a_3x^3 + a_2x^2 + a_1x + a_0 \qquad (5)$$

is completely factorizable if there exists three numbers say α , β, and γ such that

$$(i)\ \alpha\beta\gamma = a_3^{\,2}.\,a_0$$

$$(ii)\ \alpha + \beta + \gamma = a_2. \qquad (6)$$

This process avoids the trial method of searching for the linear factors and will be illustrated in Section 3 of this paper.

3.0 Examples

In this Section practical examples will be discussed to illustrate the approaches adopted in Section 2.

1. Expandthe expressions $(a)\ (x+y)^4 \qquad (b)\ (3+2x)^4 \qquad (c)\ \left(x^2 - \frac{1}{2x}\right)^3$

By applying Equation (3) inSection 2 above we obtain:

$$(a)\ (x+y)^4 = x^4y^0 + 4x^3y^1 + 12x^2\frac{y^2}{2} + 24x\frac{y^3}{6} + 24\frac{y^4}{24}$$

$$= x^4 + 4x^3y + \frac{12x^2y^2}{2!} + \frac{24xy^3}{3!} + \frac{24y^4}{4!} \qquad (7)$$

$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4 \qquad (8)$$

$$(b)\ (3+2x)^4 = 3^4(2x)^0 + 4.3^3(2x)^1 + 12.3^2\frac{(2x)^2}{2} + 24.3^1\frac{(2x)^3}{2.3} + 24.3^0\frac{(2x)^4}{6.4}$$

$$= 81 + 216x + 216x^2 + 96x^3 + 16x^4. \qquad (9)$$

$$(c)\ \left(x^2 - \frac{1}{2x}\right)^3 = (x^2)^3\left(-\frac{1}{2x}\right)^0 + 3(x^2)^2\left(-\frac{1}{2x}\right)^1 + 6(x^2)^1\frac{\left(-\frac{1}{2x}\right)^2}{2} + 6(x^2)^0\frac{\left(-\frac{1}{2x}\right)^3}{6}.$$

$$= x^6 - \frac{3x^3}{2} + \frac{3}{4} - \frac{1}{8x^3} \qquad (10)$$

One feature of this method is that the numbers are treated as variable and differentiated as such as can be observed in example (b) above. Example (c) which involves rationales is also handled in the way of example (a).

1. Factorize the following completely.

$(a)\ 2x^3+11x^2+17x+6.$

$(b)\ 2x^3-3x^2-5x+6\ .$

By applying the method outlined in Section 2(b), we have:

(a) *Let*

$2x^3+11x^2+17x+6. \equiv a_3x^3+a_2x^2+a_1x+a_0$ *and let* $\alpha=1,\ \beta=4\ \&\ \gamma=1,$ *such that*:

$\alpha.\beta.\gamma = (1)(4)(6) = 2^2.6 = a_3{}^2.a_0$ *and* $\alpha+\beta+\gamma = 1+4+6 = 11 = a_2\ .$

$$= \left(2x^3+x^2\right)+\left(4x^2+2x\right)+\left(6x^2+3x\right)+(12x+6) \qquad (11)$$

$$= 2x+1(\ x^2+2x+3x+6)\ = 2x+1(\ x(x+2)+3(x+2))$$

$$= (2x+1)(x+2)(x+3) \qquad (12)$$

(b) *Let*

$2x^3-3x^2-5x+6 = a_3x^3-a_2x^2-a_1x+a_0$ *and let* α,β,γ *be* $-2,-4,\&\ 3$ *respectively such that*:

$\alpha.\beta.\gamma = (-2)(-4)(3) = 2^2.6 = a_3{}^2.a_0$ *and* $\alpha+\beta+\gamma = -2-4+3 = -3 = a_2$

$$2x^3-3x^2-5x+6 = 2x^3-2x^2-4x^2+3x^2+4x-3x-6x+6$$

$$= 2x^3-2x^2-4x^2+4x+3x^2-3x-6x+6 \qquad (13)$$

$$= 2x^2(x-1)-4x(x-1)+3x(x-1)-6(x-1)$$

$$= (x-1)[\,2x^2-4x+3x-6\,] = (x-1)[\,2x(x-2)+3(x-2)$$

$$= (x-1)(x-2)(2x+3) \qquad (14)$$

4.0 Discussion and Conclusion

The method presented in Section 2 is valid for both positive and rational indices. The method when rightly used gives the same result with the Binomial expansion method. The method obtains the Binomial coefficients through differentiation and integration.

The factorization method avoids obtaining the linear factors by trial and the argument can be easily followed without difficulty.

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