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Optimal Location of Two E-Library Facilities Using The P-Median Model: A Case Study of Esan South East LGA of Edo State

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Abstract

In this paper, we attempt to use P-median model (2-median) to find suitable sites to locate two e-library facilities at Esan South East Local Government Area of Edo State. Two different methods were used to locate the facilities; the myopic algorithm and Lagrangian algorithm. Results of the Lagrangian algorithm which is the optimal results confirmed one of the results of the myopic algorithm that one of the facilities must be located at Ewohimi while the second facility be located at Onogholo. The optimal objective function value was 381031km. This gave an average demand-weighted distance of approximately 2.3km. It implies that, on average, each student (e-library user) would travel a distance of 2.3km to the nearby facility.

Keywords: Facility Location, Non-obnoxious facility, P-median Model, Myopic Algorithm, Lagrangian Relaxation Method.

1.0 Introduction

Facility location problems have occupied a central place in Operations Research since the early 1960's. They model design situations such as deciding placements of factories, libraries, warehouses, fire stations or hospitals and clustering analysis. Facility location problems arise in a wide set of practical applications in different fields of study: management, economics, production planning and many others. Facility is classified into three categories:

Non-obnoxious (desirable), semi-obnoxious and obnoxious (non-desirable).

A desirable facility includes supermarket, shops, banks, fire stations, schools, libraries, post offices, warehouses, etc., as the customer needs access, of some sort, to the facility providing the service, it is beneficial if these facilities are sited close to the customers that they will be serving.This implies that the customer has better access to the facility. Undesirable (obnoxious) facilities are those facilities that have adverse effects on people or the environment.

A facility is defined as obnoxious facility if its undesirable effect far outweighs its accessibility.In [1], an undesirable facility is defined as one that generates a disservice to the people nearby while producing an intended product or service. They generate some form of pollution, nuisance, potential health hazard, or danger to nearby residents; they also may harm nearby ecosystems. Some examples are nuclear power stations, military installations nuclear or chemical plants, incinerators, prisons, and pollution-producing industries. Although necessary to society, these facilities areundesirable and often dangerous to the surrounding inhabitants so lowering local house prices and quality of life [2].

The multi-period incremental service facility location problem was introduced by[3]where the goal was to set a number of new facilities over a finite time horizon so as to cover dynamically the demand of a given set of customers. [4]considered and presented formulations and solution approaches for the capacitated multiple allocation hub location problems. They presented a new mixed integer linear programming formulation for the problem. They also constructed an efficient heuristic algorithm, using shortest paths. [5] introduced a new model for the semi-obnoxious facility location problem. The new model is composed of a weighted minimum function to represent the transportation costs and a distance-based piecewise function to represent the obnoxious effects of the facility. [6] proposed a lagrangean relaxation which is a technique of quite general applicability it is studied in the particular context of the capacitated facility location problem with arbitrary additional constraints. For this class of problems they were able to obtain a reasonably complete algebraic and geometric understanding of how and why Lagrangean relaxation works. Extensive computational results are also reported.

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[7] introduced the term semi-desirable facility. They argued that the facilities cannot be classified as being purely desirable or purely obnoxious. Sometimes though a facility produces a negative or undesirable effect and this effect may be present even though a high degree of accessibility is required by the facility. For example, a stadium provides entertainment and so requires a large amount of access to enable supporters to attend a game. On the other hand, on match days, local non-football fans would have to contend with the noise and the traffic generated. This generation of noise is unpleasant for locals and therefore undesirable. The combination of the two makes this facility a semi-obnoxious. Another example is the garbage dump sites. Here, access is needed to deposit the waste produced by local population. Conversely, the disposal site may be offensive to look at, and also it emits offensive odour. These two contradicting points cause the disposal site to be defined as a semi-obnoxious facility. Other examples of semi-obnoxious facilities are ambulance and fire stations, airports, hospitals, power plants etc.
This paper aims to locate two sites for e-libraries as an example of non-obnoxious facility. Generally, libraries are useful and necessary for the communities and schools hence its location should not be placed very far from the people to make it easily accessible hence it is classified as non-obnoxious facility.

2.0 Method

The location problem was modeled as P-median problem using the following steps:

1. Data on road distances between suburbs, and the total populations of each of the 8 major suburbs of Esan South East were collected from the 2006 population and housing census and used.
2. Dijkstra's algorithm was used to find the distance matrix, $d(i,j)$ for all pairs shortest path.
3. Myopic algorithm was used to estimate the demand-weighted distance which was then used as the upper bound (UB) for the Lagrangian algorithm.
4. Lagrangian algorithm was used in optimal location to find the two sites for the e-library facilities.

3.0 Mathematical Formulation and Solution

P-median problem. According to[8]
The P-median model formulation is based on the following notations:
Inputs:
I=1… r demand points, where r is the number of demand points in the space of interest.
J=1… s facility locations for facilities, where s is the total number of potential facility location.
h_i = customer i demand.
d_{ij} = distance between customer i and candidate facility j.
P= number of facilities to be located.
Decision variables:

$$X_j = \begin{cases} 1 & \text{if we locate a candidate site j} \\ 0 & \text{otherwise} \end{cases}$$

$$Y_{ij} = \begin{cases} 1 & \text{if customer i is served by facility j} \\ 0 & \text{otherwise} \end{cases}$$

Where X and Y are indicator functions with X as location variable and Y allocation variable.
The objective function minimizing the total weighted distance is given as:

$$\text{Minimize} \sum_{i=1}^{r}\sum_{j=1}^{s} h_i d_{ij} Y_{ij} \quad \text{(1)}$$

Subject to

$$\sum_{j=1}^{s} Y_{ij} = 1 \ \forall \ i \quad \text{(2)}$$

$$\sum_{j=1}^{s} X_j = P \quad \text{(3)}$$

$$Y_{ij} \le X_j \ \forall \ i,j \quad \text{(4)}$$

$X_j \in [0,1]\ \forall\ j$ …………………………………………………………………… (5)

$Y_{ij} \in [0,1]\ \forall\ i,j$……………………………………………………………… (6)

The objective function (1) minimizes the total demand-weighted distance between each demand node. The constraints insure that the various properties of the problem are enforced. Specifically; Constraint (2) requires that, each demand node i be assigned to exactly one facility j. Constraint (3) requires that exactly P facilities are located. Constraint (4) links the location variables, and the allocation variables. Constraints (5) and (6) insure that the location variables (X) and the allocation variable (Y) are binary.
The median formulation given above assumes that facilities are located on the nodes of the network [9]. Because of the binary constraints (5) and (6), the P-median formulation above cannot be solved with standard linear programming technique. From the time when [10] realized that P-median problems could be solved on a general graph as well as a tree, a number of heuristic algorithms have been proposed. These types of heuristic algorithm can be classified into what [11] called construction algorithm and improvement algorithm.

4.0 Myopic Algorithm for the P-median Problem

Step 1: Initialize $k = 0$ ($k$will count the number of facilities we have located so far) and $X_k = \emptyset$, the empty set (X_k will give the location of the k facilities that we have located at each stage of the algorithm).
Step 2: Increment k, the counter on the number of facilities located.

Step 3: Compute $Z_j^k = \sum_{i=1}^{r} h_i d(i, j \cup X_{k-1})$ for each node j which is not in the set X_{k-1}. Note that Z_j^k gives the value of the P-median objective function if we locate the K^{th} facility at node j, given that the first $k-1$ facilities are at the locations given in the set X_{k-1} (and node $j$is not part of that set).

Step 4: Find the node $j^*(k)$ that minimizes Z_j^k that is, $j^*(k)$ = arg min{ Z_j^k }. Note that Z_j^k gives the best location for the K^{th} facility, given the location of the first $k-1$ facilities. Add node $j^*(k)$ to the set X_{k-1} to obtain the set X_{k-1} that is, set $X_k = X_{k-1} \cup j^*(k)$.
Step 5: If $k = $ P(i.e., we have located P facilities), STOP; the set $XP$is the solution to the myopic algorithm. If $k < P$, go to Step 2.

5.0 Termination of the Lagrangian Algorithm

The Lagrangian algorithm is terminated when one/more of the following conditions are satisfied:

i. When a number of specified iterations is done.
ii. The lower bound equals the upper bound (i.e. $L^n = UB$), or L^n close enough to UB .
iii. When the maximum value of sum of square violation Q is gotten as many times the number ofn facilities P to be located.
iv. a^nbecomes very small. When a^n is very small, the changes are not likely to help solve the problem. See [12].
v. When there is no violation of the relaxed constraints i.e., $Q = \sum_{i=1}^{r}\left\{\sum_{j=1}^{s} Y_{ij}^n - 1\right\}^2 = 0$

6.0 Formulation of the Lagrangian Algorithm

To formulate the above P-median problem using Lagrangian relaxation, we relaxed constraint (3.4), the following problem is then obtained

$$\underset{\lambda}{MAX}\ \underset{X,Y}{MIN}\ \sum_{i=1}^{r}\sum_{j=1}^{s}(h_i d_{ij} - \lambda_i)Y_{ij} + \sum_{i=1}^{r}\lambda_i \quad (7)$$

Subject to:

$$\sum_{j=1}^{s} X_j = P \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (8)$$

$$Y_{ij} \le X_j \ \forall\ i,j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (9)$$

$$X_j \in [0,1]\ \forall\ j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (10)$$

$$Y_{ij} \in [0,1]\ \forall\ i,j \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

Solving the above problem, for fixed values of the Lagrange multipliers, λ_i,we begin by computing the value of setting each value of the X_j to 1, which is given by:

$$V_j = \sum_{j=1}^{s} \min\left(0, h_i d_{ij} - \lambda_i\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12)$$

For each candidate location j. We then find P smallest values of V and set the corresponding values of $X = 1$ and all other values of X= 0. The allocation variable Y_{ij} are then set to:

$$Y_{ij} = \begin{cases} 1, if \ X_j = 1 \ and \ h_i d_{ij} - \lambda_i < 0 \\ 0 \text{ Otherwise} \end{cases}$$

Based on subgradient optimization, a new variable t is introduced and defined as follows: $t^n = \dfrac{a^n\left(UB - L^n\right)}{\sum_{i=1}^{r}\left\{\sum_{j=1}^{s} Y_{ij}^n - 1\right\}^2}$ ……………………………………………………..… (13)

See [12]

Where

t^n = the stepsize at the n^{th} iterations of the Lagrangian procedure

a^n = a constant on the n^{th} iteration, with a^1 generally set to 2

UB = The best (smallest) upper bound on the P-median objective function

L^n = the objective function of the Lagrangian function on the n^{th} iteration

Y_{ij}^n = the optimal value of the allocation variable, Y_{ij} on the n^{th} iteration.

The Lagrange multipliers are then updated according to the following equation: $\lambda_i^{n+1} = \max\left\{0, \lambda_i^n - t^n\left(\sum_{j=1}^{s} Y_{ij}^n - 1\right)\right\}$ ……………………………………………. (14)

Where i is the index of demand points.

Table 1: Data of 2006 population of various Esan South East suburbs

NODE i	LOCATION	POPULATION
A	EWOHIMI	31895
B	EWATTO	15436
C	OHORDUA	14338
D	EMU	16839
E	UBIAJA	35307
F	ONOGHOLO	14885
G	ILUSHI	18342
H	UGBORHA	19267
Total = 166309		

Source: National Population Commission, Nigeria

Table2: Distances of roads connecting the suburbs (nearest kilometers)
NA represent where direct distance is not applicable.

	A	B	C	D	E	F	G	H
A	-	1	NA	NA	NA	NA	NA	NA
B	1	-	4	4	NA	NA	NA	NA
C	NA	4	-	3	NA	NA	NA	NA
D	NA	4	3	-	2	NA	NA	NA
E	NA	NA	NA	2	-	1	3	8
F	NA	NA	NA	NA	1	-	NA	3
G	NA	NA	NA	NA	3	NA	-	3
H	NA	NA	NA	NA	8	3	3	-

7.0 Lagrangian relaxation for the P-median

At this point, we use the formulated lagrangian algorithm to solve the 2-median problems. We begin by formulating the P-median as follows:

Minimize

$$0Y_{AA} + 31895Y_{AB} + 159475Y_{AC} + 159475Y_{AD} + 223265Y_{AE} + 255160Y_{AF} + 318950Y_{AG} + 350045Y_{AH} + 15436Y_{BA} + 0Y_{BB} + 61744Y_{BC} + 61744Y_{BD} + 92616_{BE} + 108052Y_{BF} + 138924_{BG} + 154360Y_{BH} + 71690Y_{CA} + 57352Y_{CB} + 0Y_{CC} + 43014Y_{CD} + 71690Y_{CE} + 86028Y_{CF} + 114704Y_{CG} + 129042Y_{CH} + 84195Y_{DA} + 67356Y_{DB} + 50517Y_{DC} + 0Y_{DD} + 33678Y_{DE} + 50517Y_{DF} + 84195Y_{DG} + 101034Y_{DH} + 247149Y_{EA} + 211842Y_{EB} + 176535Y_{EC} + 70614Y_{ED} + 0Y_{EE} + 35307Y_{EF} + 105921Y_{EG} + 141228Y_{EH} + 119080Y_{FA} + 104195Y_{FB} + 89310Y_{FC} + 44655Y_{FD} + 14855Y_{FE} + 0Y_{FF} + 59540Y_{FG} + 44655Y_{FH} + 183420Y_{GA} + 165078Y_{GB} + 146736Y_{GC} + 91710Y_{GD} + 55026Y_{GE} + 73368Y_{GF} + 0Y_{GG} + 55026Y_{GH} + 211937Y_{HA} + 192670Y_{HB} + 173403Y_{HC} + 115602Y_{HD} + 154136Y_{HE} + 57801Y_{HF} + 57801Y_{HG} + 0Y_{HH} \quad \ldots\ldots \quad (15)$$

Subject to:

$$\left.\begin{aligned}
&Y_{AA} + Y_{AB} + Y_{AC} + Y_{AD} + Y_{AE} + Y_{AF} + Y_{AG} + Y_{AH} = 1\\
&Y_{BA} + Y_{BB} + Y_{BC} + Y_{BD} + Y_{BE} + Y_{BF} + Y_{BG} + Y_{BH} = 1\\
&Y_{CA} + Y_{CB} + Y_{CC} + Y_{CD} + Y_{CE} + Y_{CF} + Y_{CG} + Y_{CH} = 1\\
&Y_{DA} + Y_{DB} + Y_{DC} + Y_{DD} + Y_{DE} + Y_{DF} + Y_{DG} + Y_{DH} = 1\\
&Y_{EA} + Y_{EB} + Y_{EC} + Y_{ED} + Y_{EE} + Y_{EF} + Y_{EG} + Y_{EH} = 1 \quad \ldots\ldots\\
&Y_{FA} + Y_{FB} + Y_{FC} + Y_{FD} + Y_{FE} + Y_{FF} + Y_{FG} + Y_{FH} = 1\\
&Y_{GA} + Y_{GB} + Y_{GC} + Y_{GD} + Y_{GE} + Y_{GF} + Y_{GG} + Y_{GH} = 1\\
&Y_{HA} + Y_{HB} + Y_{HC} + Y_{HD} + Y_{HE} + Y_{HF} + Y_{HG} + Y_{HH} = 1
\end{aligned}\right\} \quad (16)$$

$$X_A + X_B + X_C + X_D + X_E + X_F + X_G + X_H = 2 \quad \ldots\ldots \quad (17)$$

$$\left.\begin{array}{r}Y_{AA}, Y_{AB}, Y_{AC}, Y_{AD}, Y_{AE}, Y_{AF}, Y_{AG}, Y_{AH} \leq X_A \\ Y_{BA}, Y_{BB}, Y_{BC}, Y_{BD}, Y_{BE}, Y_{BF}, Y_{BG}, Y_{BH} \leq X_B \\ Y_{CA}, Y_{CB}, Y_{CC}, Y_{CD}, Y_{CE}, Y_{CF}, Y_{CG}, Y_{CH} \leq X_C \\ Y_{DA}, Y_{DB}, Y_{DC}, Y_{DD}, Y_{DE}, Y_{DF}, Y_{DG}, Y_{DH} \leq X_D \\ Y_{EA}, Y_{EB}, Y_{EC}, Y_{ED}, Y_{EE}, Y_{EF}, Y_{EG}, Y_{EH} \leq X_E \quad \ldots\ldots\ldots\ldots \\ Y_{FA}, Y_{FB}, Y_{FC}, Y_{FD}, Y_{FE}, Y_{FF}, Y_{FG}, Y_{FH} \leq X_F \\ Y_{GA}, Y_{GB}, Y_{GC}, Y_{GD}, Y_{GE}, Y_{GF}, Y_{GG}, Y_{GH} \leq X_G \\ Y_{HA}, Y_{HB}, Y_{HC}, Y_{HD}, Y_{HE}, Y_{HF}, Y_{HG}, Y_{HH} \leq X_H \end{array}\right\} \quad (18)$$

$X_A, X_B, X_C, X_D, X_E, X_F, X_G, X_H \in [0, 1]$ ……………………………. (19)

$Y_{AA}, Y_{AB}, Y_{AC}, Y_{AD}, Y_{AE}, Y_{AF}, Y_{AG}, Y_{AH}, Y_{BA}, Y_{BB}, Y_{BC}, Y_{BD}, Y_{BE}, Y_{BF},$
$Y_{BG}, Y_{BH}, Y_{CA}, Y_{CB}, Y_{CC}, Y_{CD}, Y_{CE}, Y_{CF}, Y_{CG}, Y_{CH}, Y_{DA}, Y_{DB},$
$Y_{DC}, Y_{DD}, Y_{DE}, Y_{DF}, Y_{DG}, Y_{DH}, Y_{EA}, Y_{EB}, Y_{EC}, Y_{ED}, Y_{EE}, Y_{EF}, Y_{EG},$
$Y_{EH}, Y_{FA}, Y_{FB}, Y_{FC}, Y_{FD}, Y_{FE}, Y_{FF}, Y_{FG}, Y_{FH}, Y_{GA}, Y_{GB}, Y_{GC},$
$Y_{GD}, Y_{GE}, Y_{GF}, Y_{GG}, Y_{GH}, Y_{HA}, Y_{HB}, Y_{HC}, Y_{HD}, Y_{HE}, Y_{HF}, Y_{HG}, Y_{HH} \in [0,1]$ …………………… (20)

At this stage, we want to relax the constraint (16). This process is in two steps; we first multiply the constraints through by the Lagrange multipliers be λ_i, and then bring theminto the objective function. The end result, as shown below in equation (21), is the Lagrangian objective function.

$$\underset{\lambda}{MAX}\ \underset{X,Y}{MIN}$$

$(0-\lambda_A)Y_{AA} + (31895-\lambda_A)Y_{AB} + (159475-\lambda_A)Y_{AC} + (159475-\lambda_A)Y_{AD} + (223265-\lambda_A)Y_{AE} + (255160-\lambda_A)Y_{AF} + (318950-\lambda_A)Y_{AG} + (350045-\lambda_A)Y_{AH} + (15436-\lambda_B)Y_{BA} + (0-\lambda_B)Y_{BB} + (61744-\lambda_B)Y_{BC} + (61744-\lambda_B)Y_{BD} + (92616-\lambda_B)Y_{BE} + (108052-\lambda_B)Y_{BF} + (138924-\lambda_B)Y_{BG} + (154360-\lambda_B)Y_{BH} + (71690-\lambda_C)Y_{CA} + (57352-\lambda_C)Y_{CB} + (0-\lambda_C)Y_{CC} + (43014-\lambda_C)Y_{CD} + (71690-\lambda_C)Y_{CE} + (86028-\lambda_C)Y_{CF} + (114704-\lambda_C)Y_{CG} + (129042-\lambda_C)Y_{CH} + (84195-\lambda_D)Y_{DA} + (67356-\lambda_D)Y_{DB} + (50517-\lambda_D)Y_{DC} + (0-\lambda_D)Y_{DD} + (33678-\lambda_D)Y_{DE} + (50517-\lambda_D)Y_{DF} + (84195-\lambda_D)Y_{DG} + (101034-\lambda_D)Y_{DH} + (247149-\lambda_E)Y_{EA} + (211842-\lambda_E)Y_{EB} + (176535-\lambda_E)Y_{EC} + (70614-\lambda_E)Y_{ED} + (0-\lambda_E)Y_{EE} + (35307-\lambda_E)Y_{EF} + (105921-\lambda_E)Y_{EG} + (141228-\lambda_E)Y_{EH} + (119080-\lambda_F)Y_{FA} + (104195-\lambda_F)Y_{FB} + (89310-\lambda_F)Y_{FC} + (44655-\lambda_F)Y_{FD} + (14885-\lambda_F)Y_{FE} + (0-\lambda_F)Y_{FF} + (59540-\lambda_F)Y_{FG} + (44655-\lambda_F)Y_{FH} + (183420-\lambda_G)Y_{GA} + (165078-\lambda_G)Y_{GB} + (146736-\lambda_G)Y_{GC} + (91710-\lambda_G)Y_{GD} + (55026-\lambda_G)Y_{GE} + (73368-\lambda_G)Y_{GF} + (0-\lambda_G)Y_{GG} + (55026-\lambda_G)Y_{GH} + (211937-\lambda_H)Y_{HA} + (192670-\lambda_H)Y_{HB} + (173403-\lambda_H)Y_{HC} + (115602-\lambda_H)Y_{HD} + (154136-\lambda_H)Y_{HE} + (57801-\lambda_H)Y_{HF} + (57801-\lambda_H)Y_{HG} + (0-\lambda_H)Y_{HH} + \lambda_A + \lambda_B + \lambda_C + \lambda_D + \lambda_E + \lambda_F + \lambda_G + \lambda_H$ ……………….. (21)

Subject to:
Constraints (17), (18), (19) and (20).

8.0 Lagrangian Algorithm

Steps:

1. Use the myopic algorithm to determine the upper bounds (UB)
2. Input λ_i, $a^n = 2$, $h_i d_{ij}$ and UB for $i, j = A, B, C, D, E, F, G$ and H
3. For each j, compute $U_{ij} = \begin{cases} h_i d_{ij} - \lambda_i & if\ h_i d_{ij} < \lambda_i \\ 0 \text{ if } h_i d_{ij} > \lambda_i \end{cases}$
4. Calculate $V_j = \sum_{i=1}^{r} U_{ij}$
5. Pick the two least values of V_j
6. For such j values, assign $X_{j1} = 1,\ X_{j2} = 1,$ and $Y_{ij} = 1,$ for $U_{ij} < 0$

7. Calculate sum of square violation, $Q = \sum_{i=1}^{r}\left\{\sum_{j=1}^{s} Y_{ij}^{n} - 1\right\}^{2}$

8. Calculate $L^{n} = \sum_{i=1}^{r}\sum_{j=1}^{s}\left(h_i d_{ij} - \lambda_i^{n}\right)Y_{ij} + \sum_{i=1}^{r}\lambda_i^{n}$

9. Otherwise test, if $L^{n} - L^{n-1} \leq 0$ then use $a^{n} = 0.5$, otherwise use $a^{n} = 2$

10. Calculate $t^{n} = \dfrac{a^{n}\left(UB - L^{n}\right)}{\sum_{i=1}^{r}\left\{\sum_{j=1}^{s} Y_{ij}^{n} - 1\right\}^{2}}$

11. Compute $\lambda_i^{n+1} = \max\left\{0, \lambda_i^{n} - t^{n}\left(\sum_{j=1}^{s} Y_{ij}^{n} - 1\right)\right\}$

12. Return to step 2

Table 3: Computational results of the various iterations.

Variable	Iteration 1	Iteration 2	Iteration 3	Iteration 4	Iteration 5	Iteration 6
V_A	**-104504**	-288820	**-288534**	-194955	-17881	**-290364**
V_B	-90735	-241975	**-256413**	-208589	-48345	**-288243**
V_C	-69483	-239972	-205031	-199800	-39437	-157816
V_D	-92331	**-320621**	-256104	-277928	-40794	-132504
V_E	**-136441**	-314477	-195907	-289679	**-69219**	-140395
V_F	-96375	**-394239**	-249900	**-326072**	**-60859**	-162352
V_G	-62659	-291847	-243959	**-300868**	-9974	-72221
V_H	-80319	-295169	-232396	-275413	-22756	-126110
L^n	239085	49032	41945	117196	354906	36321
Q	2	7	8	6	7	8
t^n	141946	23714	21193	87945	7464	-
α^n	2	0.5	0.5	2	2	-
λ_A^{n+1}	60000	83714	62521	150466	157930	-
λ_B^{n+1}	60000	83714	62521	150466	143002	-
λ_C^{n+1}	201946	178232	157039	69094	76558	-
λ_D^{n+1}	60000	36286	57479	0	7464	-
λ_E^{n+1}	60000	36286	57479	57479	50015	-
λ_F^{n+1}	60000	36286	57479	57479	50015	-
λ_G^{n+1}	60000	60000	81193	0	464	-
λ_H^{n+1}	201946	178232	208425	120425	120480	-

The bolded cells in Table 3 above are the median points of each iterations.

9.0 Discussion of Results

The total population of the various towns is the demand allocated to the two facilities. The overall total demands (total population of Esan South East) is 166309 as given in Table 1. Using the above stated myopic algorithm, the first myopic median was gotten as 586814. Thus, the optimal total demand-weighted distance if only one facility were to be located is 586814km, resulting in an average distance of approximately 3.5km. This result suggests that, if only one facility were to be located, then it should be located at node D which is Emu and each individual has to cover an average distance of approximately 3.5km to reach the facility at Emu.

For the second myopic median the value was 381031km, which shows that the facility is to be located at node A, which is Ewohimi, resulting in an average distance of approximately 2.3km.

This result also means that, if the two facilities are located at nodes A and D (Ewohimi and Emu), then the average distance that each person would travel from any part of Esan South East LGA to the nearby facility is approximately equal to 2.3km.

The result obtained from the myopic algorithm, therefore, suggested that the two facilities must be located at nodes A and D, representing Ewohimi and Emu respectively.

It must be noted here that, the myopic algorithm also served as the stepping stone algorithm for the Lagrangian algorithm. It provided the upper bound (UB) value for the Lagrangian algorithm (UB = 381031km).

To begin the iteration of the Lagrangian algorithm, two important choices were made;

1. The initial values of Lagrangian multipliers, λ_i, (i = A, B, C, D, E, F, G, H) where chosen to be 60000, (i.e. $\lambda_A = \lambda_B = \cdots = \lambda_H = 60000$).
2. The constant $a^1 = 2$ see [12].

In Table 4.13, there was a decrease in the value of

L^n from the 1st to 3rd iterations, then increase in the 4th to 5th iterations and it also decrease in the 6th iteration. As a result, the value of

a^n decrease in the 2nd iteration, increase in the 4th iteration and again decreases in the 6th iteration. As could be seen, the relaxed constraint (16) have been violated from the 1st to 6th iterations with the 1st maximum value at the 3rd iteration and 2nd maximum value at the 6th iteration. The lagrangian algorithm has, therefore, confirmed that one of the facility be located at node A (Ewohimi) as suggested by the myopic algorithm.

The optimal solution is therefore, X_A and X_F. Thus, the two facilities must be located at Ewohimi (node A) and Onogholo (node F).

10.0 Conclusion

The main objective of the research was to use the P-median model, (P= 2) to determine suitable locations at Esan South East to establish two e-library facilities. For the above objective to be realized, the sites must be located such that, the average distance travel by e-library user (person) from any part of the LGA to the nearer of the two facilities be minimized (i.e. the average time taken is minimized).

Two different methods were used to locate the suitable sites, but the main one was the Lagrangian algorithm. The results obtained using the Lagrangian algorithm suggested that, the two facilities be located at Ewohimi (node A) and Onogholo (node F). The maximum of the lower bounds obtained was 381031km. This value gave the demand-weighted distance. It resulted in the average distance of approximately 2.3km. It implies that, on average, each person would travel a distance of approximately 2.3km to the nearby facility.

The myopic algorithm, which served as a stepping stone algorithm, also gave the same result as to where one of the facilities must be located as confirmed by the lagrangian algorithm. Thus, the myopic algorithm is a good approximation of the lagrangian algorithm.

The two facilities must, therefore, be located at Ewohimi and Onogholo.

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