

Analysis of Masses in the Chargino Sector of the Left-Right Supersymmetric Model

E. O. Aiyohuyin¹, Uduma Elem²

Department of Physics, University of Benin, Benin City, Edo State Nigeria.

Abstract

In this paper, we consider the chargino sector of the left-right supersymmetric model was investigated in the CP-violating phases. In this paper we consider the analysis of masses in the chargino sector of the left-right supersymmetric model in the CP conserving scenario. Chargino are the mixture of charged gauginos and the higgsinos fields. In the LRSUSY the Lagrangian is given by:

$$\mathcal{L}_{\text{chargino}} = -\frac{1}{2} (\psi^{+T} \ \psi^{-T}) \begin{pmatrix} \mathbf{0} & \mathbf{M}^T \\ \mathbf{M} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + H.C. \text{ Using the characteristic equations } (M_D^2)_{ii}^4 - a(M_D^2)_{ii}^3 + b(M_D^2)_{ii}^2 + c(M_D^2)_{ii} + d = 0 \text{ the masses of the Chargino at } \mu=0, \tilde{M}_{\chi_4^\pm} \text{ is equal to } 311\text{Gev}, \tilde{M}_{\chi_3^\pm} = 65.17\text{Gev}, \tilde{M}_{\chi_2^\pm} = 18.63\text{Gev and } \tilde{M}_{\chi_1^\pm} = 0.$$

Keywords: Left-right supersymmetric model (LRSUSY), Supersymmetry (SUSY), Charge-Parity (CP) conserving, Chargino mass, Gauginos and Higgsinos.

1.0 Introduction

Supersymmetry is a quantum field theory which postulates a symmetry between fermions and bosons [1]. For each fermionic state there should exist a bosonic partner, which has the same mass, couplings, and internal quantum numbers except for the spin, which differs by 1/2 unit, and vice versa. However, since no such states have been observed, SUSY must be a broken symmetry. Even though there is no experimental evidence whatsoever for any supersymmetric particles, supersymmetry remains popular because it provides potential solutions to several problems in particle physics [2]. The basic idea of supersymmetry is that each elementary fermion has a corresponding supersymmetric boson partner, and each elementary boson has a corresponding supersymmetric fermion partner. The supersymmetric partners must be heavier than their known particle partners since they have not been observed to date. Thus supersymmetry must be a broken symmetry. Supersymmetry (SUSY) predicts the existence of a superpartner for all known particles.

One of the most serious problems of the SM which is inherited in the L-R model is the so called gauge hierarchy problem (GHP) [3]. It manifests itself when we try to calculate the masses of the Higgs particles. By doing so, using perturbation series beyond tree level, we get quadratic divergences which would push the masses to the order of Planck scale ($M_p \sim 10^{19}\text{GeV}$) unless the perturbation terms cancel to 26 decimal places [4,5]. Supersymmetry reduces the size of the quantum corrections by having automatic cancellations between fermionic and bosonic Higgs interactions. If supersymmetry is restored at the weak scale, then the Higgs mass is related to supersymmetry breaking which can be induced from small non-perturbative effects explaining the vastly different scales in the weak interactions and gravitational interactions.

Cancellation of the Higgs boson quadratic mass renormalization between fermion top quark loop and scalar stop squark tadpole is as shown below in the Feynman diagrams in a supersymmetric extension of the Standard Model.

Corresponding author: E. O. Aiyohuyin, E-mail: aiyohuyin@yahoo.com, Tel.: +2348023381237

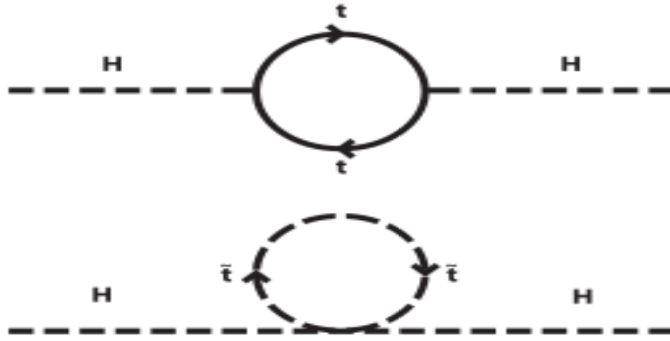


FIGURE 1: Feynman diagrams in a supersymmetric extension of the Standard Model.

In the Standard Model, the interactions (approximately) conserve lepton and baryon number, which are defined by:

$$L = N_\ell - N_{\bar{\ell}}, \quad B = \frac{1}{3} (N_q - N_{\bar{q}}) \quad (1.0)$$

with the N denoting the number of leptons, anti-leptons, quarks and anti-quarks respectively.

These conservation laws, for example, forbid the decay of the proton. When adding supersymmetry to the model, B and L are not conserved anymore, which would allow the proton to decay rapidly. However, measurements indicate that the lifetime of the proton is at least 2.1×10^{29} years [6], consistent with the approximate conservation of B and L in the Standard Model.

All particles in a supersymmetric theory obey a continuous global symmetry called R-symmetry. Certain conditions can break the continuity but nevertheless a discrete R-symmetry almost always applies which in turn gives rise to a conserved quantum number called R-parity, defined as:

$$R \equiv (-1)^{2j + 3B + L} \quad (1.1)$$

All 'ordinary' particles have $R = +1$ and all supersymmetric partners have $R = -1$. R is a multiplicative quantum number which, combined with the above observation, gives rise to two fundamental consequences on any supersymmetric model:

Firstly, all supersymmetric particles must be produced in pairs since at any present experimental situation incoming states consist only of ordinary particles, therefore the final state has to contain an even number of supersymmetric particles.

Secondly, there must be a lightest and stable SUSY particle since any decaying super-particle would not be able to decay only to non-supersymmetric particles due to R-parity conservation.

Also a unification of the strong, weak and electromagnetic forces could be facilitated by supersymmetry [6]. The energy scale dependence of these interactions is such that the coupling constants seem to approach each other at some higher energy scale. However, the coupling constants do not converge in a single point. It is argued that the almost convergence is unlikely to be a coincidence, therefore something is needed to improve the convergence. The extra particles introduced by supersymmetry can provide this improvement.

2.0 Left-Right Symmetry

According to the standard model all interactions except the weak, respect all space-time symmetries. It would appear therefore natural to try and extend the SM to make it left-right symmetric. But symmetry, although intuitive, is not the only concern. There are three main reasons [5, 6, 7] for trying to incorporate right handed bi-doublets in order to extend the SM in the $SU(2)_L \times SU(2)_R \times U(1)$ group; the so-called *Left-Right Model* (L-R).

The first reason is trying to understand parity violation, one approach would be to assume that nature is intrinsically left-right symmetric and the observed asymmetry takes place after some breakdown at low energy i.e., that is the vacuum is non-invariant under parity. This is why left-right symmetries involve some sort of parity breakdown mechanism that takes effect at some energy scale M_p .

Another reason, closely associated with the first, is the neutrino mass which has initiated many controversial experiments [8] and debates in the last decade. Astro-physical problems involving 'the missing mass of the universe, galaxy formation, etc., would be easily resolved if the neutrino indeed has a mass in the eV range [9]. If $m_\nu \neq 0$ then the left-right symmetric model is the most natural framework to incorporate it. It has been shown [10] that such a particle in a left-right symmetric model is a Majorana particle with mass:

$$m_{\nu_e} \sim O\left(\frac{1}{M_{W_R}}\right)$$

Lastly, another reason is the lack of any physical interpretation of the $U(1)_Y$ symmetry

[11], mainly due to the multiplicity of values of Y in the SM: $Y(\nu_L) = 1, Y(e_R^-) = -2, Y(\mu_L) = 1/3$, etc. In the left-right model $U(1)$ becomes the $B-L$ generator which is the only anomaly-free quantum number left ungauged. With this inclusion the weak gauge group becomes $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ and a similar to the Gell-Mann-Nishijima formula holds:

$$Q = I_L^3 + I_R^3 + \frac{B-L}{2} \tag{2.0}$$

Then $Y_{B-L} = -1$ for all leptons and $+1/3$ for quarks of all generations and helicities.

Description of the Model

In the Left-Right symmetric model the right-handed and left-handed fermion components both transform as doublets under a right-handed group $SU(2)_R$ or $SU(2)_L$. For one generation we have:

$$Q_{L/R} \equiv \begin{pmatrix} u \\ d \end{pmatrix}_{L/R}, \quad L_{L/R} \equiv \begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L/R} \tag{2.1}$$

With quantum numbers

$$Q_L: \left(\frac{1}{2}, 0, \frac{1}{3}\right), \quad Q_R: \left(0, \frac{1}{2}, \frac{1}{3}\right) \\ L_L: \left(\frac{1}{2}, 0, -1\right), \quad L_R: \left(0, \frac{1}{2}, -1\right)$$

The third number in parenthesis is $(B - L)$ and, as we saw above, corresponds to the $U(1)$ generator.

The fermion-gauge boson interaction part of the Lagrangian is given by:

$$\mathcal{L}_g = \frac{ig_L}{2} [\bar{Q}_L \gamma_\mu \sigma Q_L + \bar{L}_L \gamma_\mu \sigma L_L] W_L^\mu \\ + \frac{ig_R}{2} [\bar{Q}_R \gamma_\mu \sigma Q_R + \bar{L}_R \gamma_\mu \sigma L_R] W_R^\mu \\ + \frac{ig'}{2} \left[\frac{1}{3} \bar{Q} \gamma_\mu Q - \bar{L} \gamma_\mu L \right] B^\mu \tag{2.2}$$

Similarly, the fermion kinetic energy terms are:

$$\mathcal{L}_{kin} = -\bar{Q}_L \gamma_\mu \left(\partial^\mu - \frac{ig_L}{2} \sigma W_L^\mu - \frac{ig'}{6} B^\mu \right) L_L + R. h. p \\ -\bar{L}_L \gamma_\mu \left(\partial^\mu - \frac{ig_L}{2} \sigma W_L^\mu - \frac{ig'}{6} B^\mu \right) L_L + R. h. p \tag{2.3}$$

Notice that if we want the Lagrangian to be invariant under a left-right interchange, it immediately follows that: $g_L = g_R \equiv g$

One of the possible minimal Higgs sets required to break the symmetry 'down to'

$U(1)_{em}$ is the following: two triplets

$$\Delta_{L,R} \equiv \sigma \delta_{L,R} = \begin{pmatrix} \delta^+ / \sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+ / \sqrt{2} \end{pmatrix}_{L,R} \tag{2.4}$$

With the quantum number $(1, 0, 2)$ and $(0, 1, 2)$ respectively and bi-doublet

$$\Phi \equiv \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{2.5}$$

with $(1/2, 1/2, 0)$. Using the above fields we can break the symmetry in the following three stages:

$$SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P \\ \xrightarrow{M_P} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ \xrightarrow{M_{W_R}} SU(2)_L \times U(1)_Y \\ \xrightarrow{M_{W_L}} U(1)_{em}$$

With our particular choice of Higgs multiplets the parity P and $SU(2)$ symmetry can be broken at the same energy scale:

$$M_P = M_{W_R}$$

The vev's of the Higgs fields are chosen to be:

$$\langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}$$

and

$$\langle \phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' e^{i\alpha} \end{pmatrix},$$

where $e^{i\alpha}$ is the CP violating phase.

With this background information we can go on and derive the charged and neutral gauge boson masses in a similar way as we did for the SM. Also, certain arguments [12] can show that $\kappa \ll v_R$ and $v_L \ll \kappa$ which is essential in understanding why certain physical parameters like, for example, the mass of the neutrino are small compared to others.

3.0 Theory of the Left-Right Supersymmetric Model (Lrsusy)

The left-right supersymmetric model is described by the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where $B - L$ is a quantum number (baryon number minus lepton number) [13, 14]. The triplet vector boson $(W^\pm, W^0)_{L,R}$ and the superpartners (λ^\pm, λ^0) are assigned the gauge groups $SU(2)_{L,R}$; the singlet gauge boson V and its superpartner λ_V is assigned to the gauge group $U(1)_{B-L}$; g_L, g_R and g_v are the gauge coupling constant corresponding to the groups $SU(2)_L, SU(2)_R$ and $U(1)_{B-L}$ respectively.

Chargino are the mixture of charged gauginos and the higgsinos fields [15]. In the LRSUSY the Lagrangian is given by:

$$\mathcal{L}_{chargino} = -\frac{1}{2} (\psi^{+T} \psi^{-T}) \begin{pmatrix} 0 & M^T \\ M & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + H.C. \tag{3.0}$$

with the chargino states given by

$$\psi^+ = (-i\lambda_L^+ -i\lambda_R^+ \tilde{\phi}_{1u}^+ \tilde{\phi}_{1d}^+ \bar{\Delta}_R^+)^T \tag{3.1}$$

$$\psi^- = (-i\lambda_L^- -i\lambda_R^- \tilde{\phi}_{2u}^- \tilde{\phi}_{2d}^- \bar{\delta}_R^+)^T \tag{3.2}$$

Where $i\lambda_{L,R}^\pm$ are the $SU(2)_{L,R}$ gauginos field, $\tilde{\phi}_{1u}^+, \tilde{\phi}_{2u}^+, \tilde{\phi}_{1d}^-$ and $\tilde{\phi}_{2d}^-$ are the charged higgsinos field associated with u and d-quarks respectively.

The mass matrix is read directly off the Lagrangian and is given by

$$M = \begin{pmatrix} M_L & 0 & 0 & g_L k_d \\ 0 & M_R & 0 & g_R k_d \\ g_L k_\mu & g_R k_\mu & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix} \tag{3.3}$$

M is asymmetric matrix we require two unitary matrices U and V to diagonalized M. That can be expressed as:

$$U^* M V^{-1} = M_D \tag{3.4}$$

The diagonalizing procedures results in the physical chargino states given by

$$\chi_i^+ = V_{ij} \psi_j^+, \chi_i^- = U_{ij} \psi_j^- \quad (i, j = 1, \dots, 4) \tag{3.5}$$

Whose masses are the positive square root

$$M_D^2 = V M^+ M V^{-1} = U^* M M^+ (U^*)^{-1} \tag{3.6}$$

Since the matrix V is unitary ($V = V^{-1}$), it is not difficult to from Eq.(3.6) that the following relation holds

$$V(M^+ M) - M_D^2 V = 0 \tag{3.7}$$

Equating the elements of the i th row of Eq. (3.7), it yields

$$\begin{aligned} [M_{11} - (M_D^2)_{ii}]V_{i1} + M_{21}V_{i2} + M_{31}V_{i3} + M_{41}V_{i4} &= 0, \\ M_{12}V_{i1} + [M_{22} - (M_D^2)_{ii}]V_{i2} + M_{32}V_{i3} + M_{42}V_{i4} &= 0, \\ M_{13}V_{i1} + M_{23}V_{i2} + [M_{33} - (M_D^2)_{ii}]V_{i3} + M_{43}V_{i4} &= 0, \\ M_{14}V_{i1} + M_{24}V_{i2} + M_{34}V_{i3} + [M_{44} - (M_D^2)_{ii}]V_{i4} &= 0. \end{aligned} \tag{3.8}$$

The i th system of homogeneous linear equations contains only the i th row of V and one of the masses. The chargino masses can be determined by solving the eigenvalue equation

$$\begin{vmatrix} M_{11} - (M_D^2)_{ii} & M_{21} & M_{31} & M_{41} \\ M_{12} & M_{22} - (M_D^2)_{ii} & M_{32} & M_{42} \\ M_{13} & M_{23} & M_{33} - (M_D^2)_{ii} & M_{43} \\ M_{14} & M_{24} & M_{34} & M_{44} - (M_D^2)_{ii} \end{vmatrix} = 0 \tag{3.9}$$

Substituting the corresponding expressions for the M_{ij} we get

$$(M_D^2)_{ii}^4 - a(M_D^2)_{ii}^3 + b(M_D^2)_{ii}^2 + c(M_D^2)_{ii} + d = 0 \tag{3.10}$$

For our specific problem we have that,

$$\begin{aligned}
 a &\equiv (M_{22} + M_{33} + M_{44} + M_{11}), \\
 b &\equiv \left(\begin{array}{l} M_{33}M_{44} + M_{33}M_{11} + M_{44}M_{11} - M_{12}M_{21} - M_{14}M_{41} \\ -M_{24}M_{42} + M_{22}[M_{33} + M_{44} + M_{11}] \end{array} \right), \\
 c &\equiv \left(\begin{array}{l} M_{24}M_{33}M_{42} + M_{14}[M_{22}M_{41} + M_{33}M_{41} - M_{21}M_{42}] - \\ M_{22}M_{33}M_{44} + M_{12}[-M_{24}M_{41} + M_{21}[M_{33} + M_{44}]] - \\ M_{22}M_{33}M_{11} + M_{24}M_{42}M_{11} - M_{22}M_{44}M_{11} - M_{33}M_{44}M_{11} \end{array} \right), \\
 d &\equiv \left(\begin{array}{l} M_{14}[-M_{22}M_{41} + M_{21}M_{42}] + M_{12}[M_{24}M_{41} - M_{21}M_{44}] + \\ M_{11}[-M_{24}M_{42} + M_{22}M_{44}] \end{array} \right).
 \end{aligned}
 \tag{3.11}$$

Solving Eq. (3.10), the charginos exact masses analytic formulas are given by

$$\tilde{M}_{\chi_1^\pm} \leq \tilde{M}_{\chi_2^\pm} \leq \tilde{M}_{\chi_3^\pm} \leq \tilde{M}_{\chi_4^\pm}
 \tag{3.12}$$

Where \Re represents the value of the real value of the function and

$$\begin{aligned}
 \alpha &\equiv \sqrt{(\beta + v + \varpi)}, \quad \beta \equiv \left[\frac{a^2}{4} - \frac{2b}{3} \right], \quad \epsilon \equiv (\delta + \sqrt{\eta})^{\frac{1}{3}}, \quad \eta \equiv (-4\gamma^3 + \delta^2), \quad v \equiv \frac{(2\bar{3}\gamma)}{3\epsilon} \\
 \lambda &\equiv (a^3 - 4ab - 8c), \quad \gamma \equiv (b^2 + 3ac + 12d), \quad \delta \equiv (2b^3 + 9abc + 27c^2 + 27a^2d - 72bd), \quad \xi \equiv \left[\frac{a^2}{2} - \frac{4b}{3} - v \right], \\
 S &\equiv (\xi - \varpi - \lambda), \quad \varpi \equiv \frac{\epsilon}{32^{\frac{1}{3}}}
 \end{aligned}$$

4.0 Results

The results of this work are based on the CP-conserving scenarios. The masses of chargino were predicted [13] numerically. Applying the same scenario and using the analytic method results were gotten. The scenario is as shown below:

TABLE 1: Input parameters for scenarios $Scpc_1$ and $Scpc_2$

Scenario	M_R	M_L	M_{WL}	$\tan\theta_k$
$Scpc_1$	300		50	50.271
$Scpc_2$	200	150		80.456

All mass quantities are given in Gev. Where M_{WL} denoted the mass of the left-handed gauge boson. The mathematical tool used is MATLAB.

TABLE 2: Chargino masses as a function of μ for scenario input parameter of scenario $Scpc_1$

μ (Gev)	$\tilde{M}_{\chi_1^\pm}$ (Gev)	$\tilde{M}_{\chi_2^\pm}$ (Gev)	$\tilde{M}_{\chi_3^\pm}$ (Gev)	$\tilde{M}_{\chi_4^\pm}$ (Gev)
- 200	48.97	110.40	311.15	311.15
- 150	49.17	84.95	277.20	277.20
- 100	51.95	51.95	184.81	300.48
- 50	33.05	33.05	107.45	307.70
0	0	18.63	65.17	308.45
50	33.23	33.23	107.70	307.65
100	52.08	52.08	185.14	300.32
150	49.56	84.88	277.23	277.23
200	49.23	110.41	311.18	311.18

TABLE 3: Chargino masses as a function of μ for scenario input parameter of scenario $Scpc_2$

μ (Gev)	$\tilde{M}_{\chi_1^\pm}$ (Gev)	$\tilde{M}_{\chi_2^\pm}$ (Gev)	$\tilde{M}_{\chi_3^\pm}$ (Gev)	$\tilde{M}_{\chi_4^\pm}$ (Gev)
- 400	135.13	135.13	465.51	465.51
- 300	119.46	119.46	377.76	377.76
- 200	98.04	98.04	296.74	296.74
0	0	27.40	146.29	237.13
200	98.13	98.13	269.77	269.77
300	119.54	119.54	377.79	377.79
400	135.20	135.20	465.53	465.53

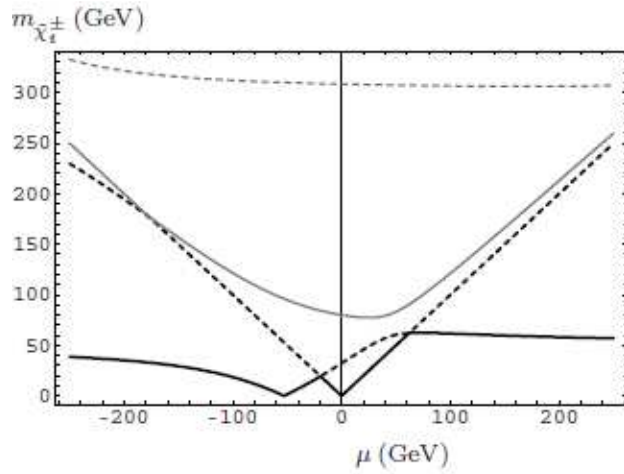


Figure 3: Numerical result for $Scpc_1$
 Source: Nivaldo Alvarez-Moraga (Determining fundamental parameters from the chargino sector in Left-Right Supersymmetric models (2013))

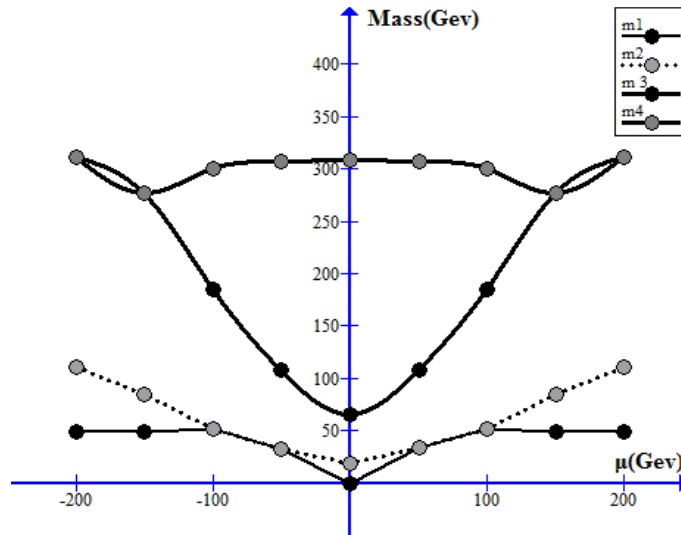


Figure 4: chargino masses $\tilde{M}_{\tilde{\chi}_i^\pm}$, $i = 1, \dots, 4$, as functions of μ for scenario input parameters of scenario $Scpc_1$.

The numerical result for $Scpc_2$:

Source: Nivaldo Alvarez-Moraga (Determining fundamental parameters from the chargino sector in Left-Right Supersymmetric models (2013))

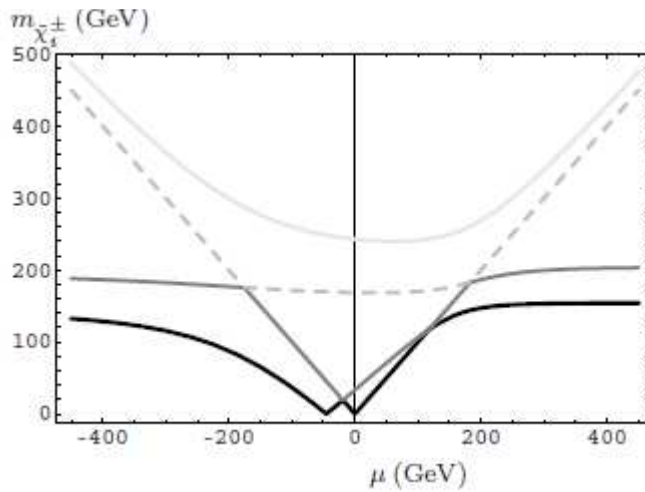


Figure 5: Numerical result for $Scpc_2$
The result of this work was compare with the one done in [13].

5.0 Discussions

The result of this work follow the same trend with the one done numerical. The graph of this work is more parabolic and symmetric about the original which is what is expected from the characteristic equation.

At $\mu = 0$, $\tilde{M}_{\chi_4^\pm}$ is equal to 311Gev in Figure 2 and equal to 325Gev in Figure 3, the result Figure 2 is 4.3% lower than the result in Figure 3. Also at $\mu = 0$, $\tilde{M}_{\chi_1^\pm} = 0$ in Figure 2 but it shifted toward the left and is about zero in Figure 3. $\tilde{M}_{\chi_1^\pm}$ and $\tilde{M}_{\chi_2^\pm}$ intercept at $\mu = -100$ at the left and $\mu = 100$ at the right in figure 2 while from Figure 3 $\tilde{M}_{\chi_1^\pm}$ and $\tilde{M}_{\chi_1^\pm}$ intercept at $\mu = -20$ at the left and $\mu = 60$ at the right in Figure 3.

6.0 Conclusion

This work is concerned with the analysis of masses in the left-right supersymmetric model (LRSUSY), in the CP conserving phase in the chargino sector.

In this work, we have obtained the chargino masses in terms of the analytic expressions. We proved the agreement between our result with the numerical solution previously published.

The non-symmetric chargino mass matrix was diagonalized by constructing two digonalizing unitary matrices. The masses obtained by solving the associated characteristic polynomial and plotted as a function of the Higgsino parameter μ .

The graph of this work is more parabolic and symmetric about the original which is what is expected from the characteristic equation.

7.0 References

- [1] T. Teubner, The Standard Model, Lecture presented at the School for Experimental High Energy Physics Student Somerville College, Oxford, September 2009.
- [2] C. J. Solano Salinas, K. Hurtado and C. Romero*Introduction to Elementary Particle Physics,2009 American Institute of Physics 978-0-7354-0659-9, 2009.
- [3] V. Barger, Collider Physics Addison-Wesley 1987
- [4] Per Johansson,Search for Supersymmetry with DELPHI, and preparation for ATLAS, Department of Physics Stockholm University(2005)
- [5] R. Francis, MSc. Thesis, Concordia University 1989

- [6] S. Antonopoulos, MSc. Project, Concordia University 1997.
- [7] R.N. Mohapatra, Unification and Supersymmetry, Springer-Verlag 1992.
- [8] R.N. Mohapatra, NATO ASI Series Vol. 122, 219 1983.
- [9] V. Lyubimov et al., Phys. Lett. 94B 266 (1980).
- [10] R Cowsik, J. McClelland, Phys. Rev. Lett. 29,669 (1972); S.S. Gershtein, Y.B. Zhdovitch, JETP Letters 4, 120 (1966).
- [11] G. Senjanovic, R.N. Mohapatra, Phys. Rev. D12, 1502 (1975).
- [12] M. Kaku, Quantum Field Theory, Oxford University Press 1993 .
- [13] Nivaldo Alvarez-Moraga, Determining fundamental parameters from the chargino sector in Left Right Supersymmetric models (2013).
- [14] Artoix de la Cruz de Ona, MSc. Thesis, Concordia University, (2005).
- [15] Chargino, Wikipedia.