Journal of the Nigerian Association of Mathematical Physics
***Volume32,(November, 2015*), pp 459 – 464**
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Real-Time Qualitative Behaviour of a Dynamical System of Corrosion Passivation Rate Data

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Abstract

The utilization of aluminium and zinc as building materials and the need to protect these materials from corrosion especially in a fast developing country like Nigeria has been a long-term sustainable development initiative. We have used a powerful numerical simulation technique to determine the influence of the growth rate on the qualitative behaviour of the dynamical system of interaction. The fundamental changes in the stability of the co-existence steady-state due to the variations of the growth rate parameter values when other model parameter values are fixed have been obtained. We strongly believe that this contribution will provide a short-term control of corrosion with the expectation of minimizing the familiar environmental perturbations which might affect the application of this research output in the building industry on the premise that a combination of zinc and aluminium metal spraying and hot dip galvanizing do provide a long-term cathodic protection on exposed edges. It is interesting to mention that while a bifurcation of the steady-state has occurred between a = -0.3819 and a = -0.4074, another bifurcation of the same steady-state has occurred between d = -0.2437 and d = -0.2681. These results have not been published elsewhere; they are clearly presented and discussed in this study.

1.0 Introduction

On the basis of the model of interaction between experimentally determined corrosion passivation rate time series data [1], the effect of the growth rates on the type of qualitative behaviour of the dynamical system is yet to be carefully studied. If successfully analysed, the qualitative behaviour of the continuous nonlinear first order ordinary differential equations of the Lotka-Volterra type that evolves over time has the potential of providing insight into the rationale of stabilizing its corresponding Al-Sn Duplex system. Due to the likely instability of the corrosion passivation rate interacting system, the existence of the environmental perturbation which characterise such system can introduce some sort of random noise elements that can influence either the recovery of the type of stability or the loss of a dominant type of stability. The severity of the random noise intensity on the characterization of the qualitative behaviour of the real-time qualitative behaviour of the dynamical system is yet to be systematically analysed and discussed by the previous authors [1]. Although, earlier regression equations have been derived to examine the relationship between the said corrosion passivation rate data in different concentrations, the present approach is based on the idea of mathematical modelling and numerical simulation which were not considered in the contribution of [1]. Our present technique is expected to extend the work of [1] so as to present a coherent knowledge-base of this complex Duplex system for the purpose of interpolation and extrapolation of the corrosion passivation rate time series data. The notion of the possible interaction between other Duplex systems and other related analyses havereceived detailed reports [2- 9].

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In the simplification of the interaction that exist between two time series data of corrosion passivation rate with the intent of diminishing the growth of either population, this Duplex interacting system has been analytically verified to have four steady-states namely the trivial, two semi-trivial and a unique positive co-existence steady-state. It is only the trivial steady-state that is stable having two negative eigenvalues while the other steady-states are clearly unstable having either two eigenvalues of opposite signs or two positive eigenvalues. It is interesting to mention in this context that the unique positive co-existence steady-state is unstable indicating the survival of the two types of corrosion passivation rate time series data.

Since the environmental perturbation that could occur in the interaction between two Duplex systems is inevitable, it would be a good applied scientific practice beyond this pioneering study to examine the extent of how this phenomenon will influence the type of stability or instability or degeneracy of steady-state as this level of analysis has the potential to provide real life corrosion passivation rate stabilization of the unstable co-existence steady-state and the control of severe corrosion of aluminium and zinc that can have a devastating implication for the reduction of building materials in a fast developing country like Nigeria where such materials are in high demand.

2.0 Mathematical Formulation

The formulation of this approximated model of interaction between aluminium and zinc in two different concentrations is based on the data of Ekuma et al. [1]. The mathematical structure takes the following Lotka-Volterra formalism where a = -0.5092, b = -0.157, c = -0.12, d = -0.4875, e = -0.08 and f = -0.15; the independent variable t is in the unit of hours

$$\frac{dC_a}{dt} = C_a(a - bC_a - cC_Z)\text{.....................}(1)$$

$$\frac{dC_z}{dt} = C_z(d - eC_a - fC_z)\text{.....................}(2)$$

The model parameter values denoted by a and d are known as the intrinsic growth rates whereas the parameter values b and f are known as the intra-specific coefficients. The parameter values c and e are called the inter-specific coefficients whereas the two dependent variables represent the aluminium and zinc corrosion passivation rate measured concentrations.

3.0 Method of Analysis

In the theory of stability, the qualitative characteristics of the dynamical system are often based on the assumption that the two interaction functions as in this context are both continuous and partially differentiable.

First, the partial derivative of the interaction function

$F(C_a, C_z) = aC_a - b(C_a)^2 - cC_aC_z$………………. (3)

with respect to C_a and C_z were calculated and represented by J_{11} and J_{12} being the Jacobian elements of the first row-first column and the first row-second column of a two row-two column matrix otherwise called a 2 by 2 matrix.

Second, the partial derivative of the interaction function

$G(C_a, C_z) = dC_z - f(C_z)^2 - eC_aC_z$………………… (4)

with respect to C_a and C_z were calculated and represented by the Jacobian elements J_{21} and J_{22}. Third, the four Jacobian elements J_{11}, J_{12}, J_{21} and J_{22} were evaluated at the co-existence steady-state which we have analytically determined using the popular Cramer's Rule of linear algebra for finding the unknowns of a system of simultaneous linear equations and a 2 by 2 matrix was set up.

Fourth, a Matlab computational method was used to study the qualitative characteristic of the steady-state when the model parameter values a and d were varied one-at-a-time while other model parameter values were fixed. This method is computationally efficient with minimized approximate errors than the well- known analytical method that can attract lots of approximation errors if carelessly conducted. Caution must be taken in order to interpret the computational results consistently with the theory of steady-state and stability. If the sign of any of the dependent variables is negative, it clearly shows that the corresponding steady-state should be considered as degenerate even though the signs of the eigenvalues may be mathematically tractable, such observation will have no scientific meaning. It should be noted that when the signs of the eigenvalues are either both positive or of opposite signs, then the steady-state is said to be unstable. The same steady-state is said to be stable if the two eigenvalues are both negative.

4.0 Results and Discussion

A successful application of our proposed method of analysis has produced the following empirical results:

Table 1: Influence of the growth rate parameter values a = -0.0509 to a = -0.4930 on the qualitative behaviour of the dynamical system (QBDS) for different combinations of a and eigenvalues

Example	a	CPR* (DS1)	CPR* (DS2)	λ_1	λ_2	QBDS
1	-0.5092	1.2817	2.5664	0.0931	0.493	Unstable
2	-0.0509	-3.646	5.1945	-0.4209	0.6277	Degenerate
3	-0.0764	-3.3723	5.0485	-0.3866	0.6144	Degenerate
4	-0.108	-3.0985	4.9025	-0.3524	0.6013	Degenerate
5	-0.1273	-2.8247	4.7565	-0.3185	0.5885	Degenerate
6	-0.1528	-2.5510	4.6105	-0.2849	0.5759	Degenerate
7	-0.1782	-2.2772	4.4645	-0.2516	0.5637	Degenerate
8	-0.2037	-2.0034	4.3185	-0.2187	0.5519	Degenerate
9	-0.2291	-1.7297	4.1725	-0.1862	0.5406	Degenerate
10	-0.2546	-1.4559	4.0265	-0.1544	0.5298	Degenerate
11	-0.2801	-1.1822	3.8805	-0.1232	0.5196	Degenerate
12	-0.3055	-0.9084	3.7345	-0.0927	0.5103	Degenerate
13	-0.3310	-0.6346	3.5885	-0.0633	0.5019	Degenerate
14	-0.3564	-0.3609	3.4425	-0.035	0.4947	Degenerate
15	-0.3819	-0.0871	3.2965	-0.0082	0.4890	Degenerate
16	-0.4074	0.1867	3.1504	0.0169	0.4850	Unstable
17	-0.4328	0.4604	3.0044	0.040	0.4830	Unstable
18	-0.4583	0.7342	2.8584	0.0606	0.4835	Unstable
19	-0.4837	1.008	2.7124	0.0784	0.4868	Unstable
20	-0.4888	1.0627	2.6832	0.0815	0.4878	Unstable
21	-0.4930	1.1175	2.6540	0.0846	0.4889	Unstable

CPR(DS1) represents corrosion passivation rate of data set 1 DS1; CPR(DS1)represents corrosion passivation rate of data set 2 DS2; λ_1 and λ_2 are called eigenvalues.

From example 2 to example 15, degeneracy of the co-existence steady-state has occurred between a = -0.0509 to a = -0.3819. This observed characteristic can be attributed to the fact that the first steady-state solution co-ordinate carries a negative sign which contradicts a meaningful dynamical system of the interacting dependent variables. This is so because the unique positive steady-state solution of the defined model differential equations is $(\frac{af-cd}{bf-ce},\frac{bd-ae}{bf-ce})$. In contrast, the co-existence steady-state (1.2817, 2.5664) is unstable having two positive eigenvalues as shown above. The reality is that the degeneracy of the steady-state will be lost between a = -0.3819 and a = -0.4074 using the bisection method. The value of a = -0.5092specifies where the instability of the co-existence steady-state has occurred due to the theory of stability which states that a steady-state solution is said to be unstable if the derived eigenvalues carry either positive signs or opposite signs.

Table 2: Influence of the growth rate parameter values a = -0.4990 to a = -0.5601 on the qualitative behaviour of the dynamical system (QBDS) for different combinations of a and eigenvalues

Example	a	CPR (DS1)	CPR (DS2)	λ_1	λ_2	QBDS
22	-0.4990	1.1722	2.6248	0.0876	0.4902	Unstable
23	-0.5041	1.2270	2.5956	0.0904	0.4916	Unstable
24	-0.5143	1.3365	2.5372	0.0956	0.4948	Unstable
25	-0.5194	1.3912	2.5080	0.0980	0.4966	Unstable
26	-0.5245	1.4460	2.4788	0.1003	0.4985	Unstable
27	-0.5347	1.5007	2.4496	0.1024	0.5006	Unstable
28	-0.5296	1.5555	2.4204	0.1045	0.5028	Unstable
29	-0.5398	1.6102	2.3912	0.1063	0.5052	Unstable
30	-0.5448	1.6650	2.3620	0.1081	0.5076	Unstable
31	-0.5499	1.7197	2.3328	0.1097	0.5102	Unstable
32	-0.5550	1.7745	2.3036	0.1112	0.5130	Unstable
33	-0.5601	1.8292	2.2744	0.1125	0.5158	Unstable

Combining the thirty two (32) results for the two intervals of the growth rate a except the first example, fourteen (14) instances indicate the degeneracy characteristic of the co-existence steady-state compared to eighteen (18) instances in which the steady-state is unstable.

The next set of results concerns the influence of the second growth rate d on the qualitative behaviour of the dynamical system. These results are presented in Table 3 and Table 4.

Table 3: Influence of the growth rate parameter values d = -0.0488 to d = -0.4778 on the qualitative behaviour of the dynamical system (QBDS) for different combinations of d and eigenvalues

Example	d	CPR (DS1)	CPR (DS2)	λ_1	λ_2	QBDS
1	-0.4875	1.2817	2.5664	0.0931	0.493	Unstable
2	-0.0488	5.0559	-2.3715	0.683	-0.2449	Degenerate
3	-0.0731	4.8462	-2.0972	0.6608	-0.2145	Degenerate
4	-0.0975	4.6366	-1.8228	0.6390	-0.1845	Degenerate
5	-0.1219	4.4269	-1.5485	0.6176	-0.1548	Degenerate
6	-0.1462	4.2172	-1.2742	0.5966	-0.1256	Degenerate
7	-0.1706	4.0075	-0.9998	0.5762	-0.0970	Degenerate
8	-0.1950	3.7978	-0.7255	0.5565	-0.0691	Degenerate
9	-0.2194	3.5882	-0.4512	0.5377	-0.042	Degenerate
10	-0.2437	3.3785	-0.1769	0.5199	-0.0160	Degenerate
11	-0.2681	3.1688	0.0975	0.5036	0.0086	Unstable
12	-0.2925	2.9591	0.3718	0.4890	0.0314	Unstable
13	-0.3169	2.7495	0.6461	0.4766	0.0520	Unstable
14	-0.3413	2.5398	0.9208	0.4670	0.0698	Unstable
15	-0.3656	2.3301	1.1948	0.4608	0.0843	Unstable
16	-0.3900	2.1204	1.4691	0.4585	0.0948	Unstable
17	-0.4144	1.9108	1.7434	0.4606	0.1009	Unstable
18	-0.4387	1.7011	2.0178	0.1025	0.4673	Unstable
19	-0.4631	1.4914	2.2921	0.0997	0.4783	Unstable
20	-0.4680	1.4495	2.3470	0.0987	0.4809	Unstable
21	-0.4729	1.4075	2.4018	0.0975	0.4838	Unstable
22	-0.4778	1.3656	2.4567	0.0961	0.4868	Unstable

Table 4: Influence of the growth rate parameter values d = -0.4826 to d = -0.5363 on the qualitative behaviour of the dynamical system (QBDS) for different combinations of d and eigenvalues

Example	d	CPR (DS1)	CPR (DS2)	λ_1	λ_2	QBDS
23	-0.4826	1.3237	2.5116	0.0947	0.4899	Unstable
24	-0.4924	1.2398	2.6213	0.0913	0.4965	Unstable
25	-0.4972	1.1978	2.6761	0.0894	0.5001	Unstable
26	-0.5021	1.1559	2.7310	0.0874	0.5037	Unstable
27	-0.5070	1.1140	2.7859	0.0853	0.5075	Unstable
28	-0.5119	1.0720	2.8407	0.0831	0.5113	Unstable
29	-0.5168	1.0301	2.8956	0.0807	0.5153	Unstable
30	-0.5216	0.9882	2.9505	0.0783	0.5194	Unstable
31	-0.5265	0.9462	3.0053	0.0758	0.5236	Unstable
32	-0.5314	0.9043	3.0602	0.0731	0.5279	Unstable
33	-0.5363	0.8624	3.1151	0.0704	0.5322	Unstable

In this scenario, out of the thirty two (32) results for the two intervals of the growth rate d except the first example, nine (9) instances indicate the degeneracy characteristic of the co-existence steady-state compared to twenty-three (23) instances in which the steady-state is unstable. It is interesting to observe that the degeneracy of the steady-state will be lost between d = -0.2437 and d = -0.2681.

On the basis of this systematic analysis, we have found that the variation of the growth rate d is relatively associated more with the instability of the steady-state than the variation of the growth rate a while the variation of the growth rate a is relatively associated more with the degeneracy of the steady-state than the variation of the growth rate d. Over sixty-six (66) repeated simulations, it is more likely to have a greater incidence of the instability of the steady-state than the degeneracy of the steady-state irrespective of the variations of the growth rates of this Duplex interacting system. Since the model parameters for the interacting corrosion passivation rate data were estimated using the three popular penalty functions error estimates, our concern is to examine in detail the extent of the incidence of degeneracy of the steady-state in the event of an environmental perturbation which cannot be avoided in the Duplex interacting system. This idea will be the subject of a near future investigation.

5.0 Conclusion

The precise parameter values that define the deterministic interaction between aluminium and zinc alloys have been estimated using the application of the penalty function numerical scheme on the basis of some primary data. The qualitative behaviour of the type of stability for this dynamical system has been systematically determined with the expectation of extending this present contribution into tackling other aspects of this pioneering research that we have not considered in this present study.

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