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Computation of the Theoretical Gravity Values and their Implications on Gravity Anomalies in Some Locations in Taraba State

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Abstract

The 1930, 1967 and 1980 International Gravity Equations were used in computing the theoretical gravity values for 21 stations in Taraba State for the purpose of comparison and choice of the most suitable equation that can likely give accurate and consistent gravity anomalies over the area under consideration. The mean gravity values obtained in ms^{-2} are 9.7816699, 9.781337 and 9.781316 using the 1930, 1967 and 1980 gravity equations respectively showing that the 1930 equations yielded the highest gravity values. Constant difference of about 17 mGal exists between the gravity values of 1930 and 1967 gravity equations and between 1930 and 1980 gravity equations which revealed discrepancy in the gravity values of 1967 and 1980 gravity equations. The application of the 1930 gravity equation in computing gravity anomalies over lowland areas of the State will produce negative anomalies while that of 1980 will be inconsistent. Thus the 1967 gravity equation would be most appropriate since slight negative gravity anomalies are expected over areas with flat topography while both 1930 and 1967 gravity equations can be used for high land areas and hilly terrain. The plot of the theoretical gravity against latitude indicates that the two parameters are linearly positively correlated.

Keywords: International gravity equations, latitude, theoretical gravity, gravity anomalies, hilly terrain

1.0 Introduction

The field of force in physics is often important than the absolute magnitude of the force. Lowrie [1] defined field as a force exerted on a material unit. Gravity field in the vicinity of an attracting mass is the force it exerts on a unit mass. Telford *et al* [2] defined gravitational acceleration g, (also known as acceleration due to gravity) as the resultant effect of the gravitational attraction and centrifugal repulsion between an object and the earth. In other words, Gravitational acceleration or simply acceleration due to gravity (g), is the time rate of change of a body's speed under influence of the gravitational force [2]. If an object such as a ball is dropped from a cliff, it falls under the influence of gravity in such a way that it's speed increased constantly with time. That is the object accelerates as it fall with constant acceleration. At sea level, the rate of acceleration is about 9.8 ms^{-2}. Measurement of gravity over the years has revealed that, gravity varies from point to point on the earth's surface.

Geophysical interpretation from gravity surveys (measurement of the gravity field at a series of different locations over an area of interest), are based on the mutual attraction experienced between two masses as expressed by Newton in his work "The mathematical principles of natural philosophy," Newton's law of universal gravity states that the mutual attractive force between two point masses m_1 and m_2 is proportional to the product of the masses and inversely proportional to the square of the distance between them.

Gravity is known to vary from latitude to latitude and for a given point or location, it varies with time. The two major factors that affects gravitation acceleration are: those that give rise to temporal variation and those that give rise to spatial variation [3]. **T**he mean value of gravity at the surface of the earth is approximately 9.8 ms^{-2} or 980,000 mGal [1]. It is important to note that the ms^{-2} unit of acceleration is unpractical for use in geophysics. The "Gal" is the preferred unit of gravity in

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geophysical studies. The small change in acceleration due to gravity caused by geological structure are measured thousand of this unit i.e. milliGal (mGal).

The deviation of acceleration due to gravity from the theoretical or normal value is the object of gravity surveys [4]. The earth's rotation and flattening caused gravity to increase by roughly 53000 mGal or 0.5 ms^{-2} from equator to pole. It was observed that anomalies (deviation from normal value) indicate lateral density variation which may be due to geological phenomena, geodynamic or tectonics activities in the earth [5]. The difference between observed and theoretical values is called gravitational anomalies. The departure of earth's surface from an equipotential spheroid, earth spin and density variation within the earth crust are the factors identified to be responsible for the change in g across the world [6].

This work is to study the absolute gravity field of different location in Taraba State, stimulated by the natural distribution of physical features such as mountains, valleys, ranges, gorge, cliff etc and tectonic activities e.g. earthquake, layside a vocation that had some times occurred in some parts of the selected localities.

According to Lowrie [1], measurements of gravity are of two types. The first involved the determination of absolute magnitude of gravity at any location and the second consist measuring the changes in gravity from one place to another. Gravitational forces on a point on the earth's surface is in two-fold: first is the attraction of the earth according to Newton's universal law of gravity and secondly, the centrifugal force due to the rotation of the earth which is 0.3% of the attraction force at the equator but less elsewhere

The measurement of gravity can be done by two methods. First is the pendulum method, which is based on measuring the periodic time T with length L very accurately. The accuracy of this observation is increased by observing the pendulum during a long time. The second is the free "fall method" which is based on falling body in a vacuum tube [7]. In geophysical studies especially in gravity prospecting, it is necessary to measure accurately with a small change in gravity caused by underground structures. This requires an instrument with sensitivity of order of 0.1 mGal [1]. It is very difficult to design an instrument to measure absolute gravity that has this high precision and portable enough to be used easily in different places. Gravity surveying is usually carried out with portable instrument called gravimeter, which determine the variation of gravity relative to one place or more reference locations. The relative variation determined with gravimeter may be converted to absolute values by calibration with absolute measurement made at selected stations.

2.0 Gravity Measurement

Gravity measurement can be done in absolute and relative sense [7]. Absolute gravity measurement can be done using the following methods.

2.1 Pendulum Method

The classical method of measuring gravity is with a pendulum [7]. There are basically two types of pendulum: Simple and Compound pendulum. The Simple pendulum method is based on measuring the pendulum time T, of the pendulum with length L, very accurately. The accuracy of this observation is increased by time. The gravity value is found using the relation.

$$T = 2\pi\sqrt{\frac{l}{g}} \tag{1}$$

The length of the pendulum and the influence of the suspension point are needed to be known very precisely. The compound pendulum allows more precise exact measurements. It consists of a stiff metal or quartz rod, about 50 cm long to which is attached a moveable mass. Near each end of the rod is a fixed pivot which consists of quartz knife-edge resting on a quartz plane. The period T, of the pendulum is measured for oscillations about the pivots. The position of the moveable mass is adjusted until the period of about the two pivots are equal. The value of g can be calculated from equation (1).

2.2 Free Fall Method

Marc and Erik [7] noted that the pendulum method has been superseded by the free fall method, which is based on a falling prism in a vacuum tube. The basic relation for the free fall method is

$$Z = Z_0 + V_0 t + \frac{1}{2} g t^2 \tag{2}$$

Where Z is the fall distance, t is the time and Z_0 and V_0 are respectively distance and velocity at t= 0 (t = 0 is a time epoch shortly after the start of the fall). If we want to determine g with an accuracy of 1 microGal, the accuracy of the observation should be 0.25 nanoseconds, and accuracy of Z should be 0.3 nanometer [8]. This can only be achieved by measuring t with an atomic clock and with light interference. Burger [3] and Marc and Erick [7] identified that since gravity is not constant during a fall, there is a vertical gravity gradient dg/dZ of about 3 microGal per cm if a linear gradient is assumed. In this case the differential equation describing the free fall yields

$$\frac{d^2Z}{dt^2} = g(Z) = g_o + \frac{dg}{dZ}(Z - Z_o) \tag{3}$$

With g_0 as the gravity at the location of the start of the timing, the vertical gradient dg/dz can be solved from the equation. The term $(Z - Z_0)$ from equation (3) is

$$(Z - Z_o) = V_o t + \frac{1}{2} g t^2 \tag{4}$$

Hence

$$\frac{d^2Z}{dt^2} = g(Z) = g_o + \frac{dg}{dZ}\left(V_o t + \frac{1}{2}gt^2\right) \qquad (5)$$

Relative gravity measurements are based on a completely different principle [7]. The elongation of springs with a small weight attached to them is measured. Taking measurement at different locations yields differences in elongation. These differences can be converted to gravity difference on the condition that the instrument is calibrated on stations with known absolute gravity values. Absolute gravity measurements are necessary as a base for gravity measurement, for the calibration of relative gravimeters, and for determining the drift of relative gravimeter.

2.3 Gravimeters

A gravimeter is an instrument used in measuring the local gravitational field of the earth. The first gravimeters were based on the application of Hooke's law [1]. The gravimeter is calibrated at a known location. If gravity is different at another location, the extension on the spring changes, and from this change, gravity can be computed. Gravimeters are today used for petroleum and mineral seismology, geophysical surveys and metrology. Gravimeters are light, robust and portable. It should be noted that after initially leveling the instrument, an accurate measurement of gravity difference can be made in a few minutes .The gravimeter have sensitivity of 0.01 mGal. This high precision makes it susceptible to small changes in its own properties.

3.0 Factors that Affect the Gravitational Acceleration

Although absolute gravity of a location can be calculated, it is maintained that two factors are basically responsible for the variation of gravity; those that give rise to temporary variation and those spatial variations in the gravitational acceleration [8].

3.1 Temporal Based Variation

These are changes in acceleration that is time dependent. In other worlds, these factors caused variations in acceleration that would be observed even if we didn't move our gravimeter. They include: Instrument Drift which refers to changes in the observed acceleration caused by changes in responses of the gravimeter over time, and Tidal effects which refers to changes in the observed acceleration caused by the gravitation attraction of the sun and moon.

3.2 Spatial Based Variation

These changes in observed acceleration are spaced dependent. That is, these change the gravitation acceleration from place to place, just like the geologic affects but they do relate to geology. They include Latitude variation, elevation variation and slab effects.

4.0 Area of Study

This study was carried out in seven Local Government Areas of Taraba State, North -east Nigeria. The State has a total land mass of 54,473km^2, it lies between latitude 6.3^0 and 9.1^0 N and longitude 6.3^0 and 11.5^0 E. Taraba State has a unique topography with Nguroje, a town in Sardauna LGA on an altitude of 1830 m above sea level as the highest peak in Nigeria [9]. This area is developed basement of complex rocks measured about 96 km^2 along its curved length and 40 km^2 wide, bounded by an escarpment which is about 900 m high.

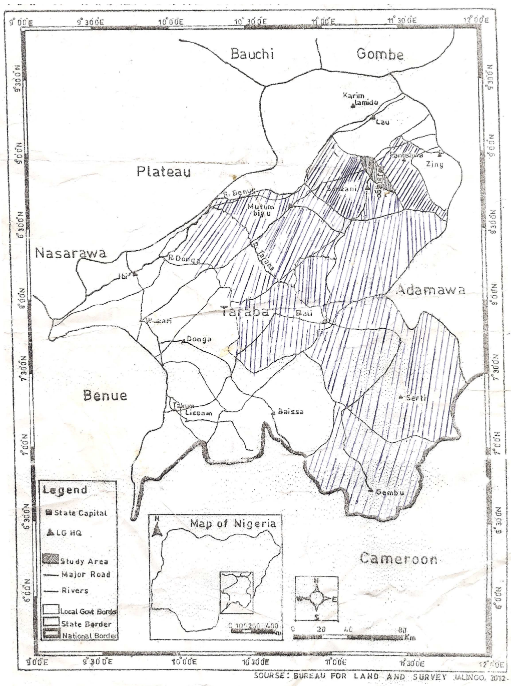

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Fig. 1. Map of Taraba State showing Study Areas (shaded portions)

5.0 Geology of the Area

Taraba state is underlain by the undifferentiated Basement Complex rocks which consist mainly of migmatites, gneisses, and the Older Granites. Tertiary to recent basalt also occurs in the areas. The undifferentiated Basement Complex particularly the migmatites, generally varies from coarsely mixed gneisses to diffused texture rocks of variable grain size [10]. This rock unit constitutes principally the undifferentiated igneous and metamorphic rock of Precambrian Age [11].

The Pan Africa Older Granites are equally widespread in the area. They occur either as basic or intermediate intrusive [12]. Different kinds of textures ranging from fine to medium to coarse grains can be noticed on the Older Granite [13]. Other localized occurrences of minor rock types include some doleritic and pegmatitic rocks mostly occurring as intrusive dykes and vein bodies. These occurrences are common to both the undifferentiated Basement Complex and Older Granite Rocks. The tertiary basalts on the other hand are found in the Mambila plateau mostly formed by trachytic lavas and extensive basalts which occur around Ngoreje [14].

6.0 Research Procedures

Three existing international gravity equations, namely g_{1930}, g_{1967} and g_{1980} given in equations (7), (8) and (9) were used for computation of values of g at selected locations within the Taraba State. Twenty one (21) locations were selected from seven Local Government Areas of the State. The general equation for the variation of gravity with latitude over the surface of the ellipsoidal earth is given as [1].

$$g = g_o(1+ASin^2\lambda + B\sin^2 2) \qquad (6)$$

Where g_o is the gravity value on the equator, A and B are constants which depends on the shape of the earth and λ is the latitude of the location.

In 1930, the International Association of Geodesy and Geophysics adopted a formula for the theoretical calculations of gravity values at locations in terms of latitude. This equation assigned numerical values to the constants g_o, A and B and is given as:

$$g_{1930} = 9.78049(1 + 0.0052884Sin^2\lambda - 0.000059Sin^2 2\lambda) \qquad (7)$$

The values of these constants were later adjusted in 1967 to give the best fit to the observed variation of gravity over the earth's surface and called Geodetic Reference System (GRS67). The equation is given as

$$\mathrm{g}_{1967} = 9.78031846\ (1 + 0.0053024\ \mathrm{Sin}^2\lambda\ -\ 0.0000058\ \mathrm{Sin}^2 2\lambda) \qquad (8)$$

Further adjustments were made by the Geodetic Reference System in 1980 (GRS80) giving the gravity formula of equation (9).

$$\mathrm{g}_{1980} = 9.78032677\ (1 + 0.00527889\ \mathrm{Sin}^2\lambda\ -\ 0.000023462\ \mathrm{Sin}^2 2\lambda) \qquad (9)$$

The value of the latitude (λ) was taken with the aid of a 12 channel Garmin Global Positioning System (GPS 12). The value of λ was then substituted into the three equations above to obtain three different values of g for each latitude. These values of g were subjected to appropriate statistical analyses in order to compare and obtain which gravity formula will give high precision value of gravity anomaly.

7.0 Precision Measures for Absolute Gravity Measurements

The standard deviation of n experiment in one series (set) is given as:

$$\sigma_g(set) = \sqrt{\frac{\sum_{i=0}^{n}(g_i-\overline{g_i})^2}{n-1}} \qquad (10)$$

where $\overline{g_i}$, the mean value of the set. At a station k, several sets measured consequently, for every set, the standard deviation, σ_g (set) can be computed. The mean value of these $\overline{\sigma}_{gi}$ (set) is called drop standard deviation or mean drop standard deviation.

$$\sigma_{gi}(stationK) = \frac{\sum_{i=0}^{m}(\sigma_g seti)}{m}$$

With m the number of sets measured at a station. The standard deviation of mean value per set (g, s) is called set standard deviation.

$$\sigma_{gi}(stationK) = \sqrt{\frac{\sum_{i=0}^{n}(\overline{g}_i-\overline{g})^2}{m-1}} \qquad (11)$$

With g the mean gravity value for all m-sets together. If the m sets at a station are not correlated, the standard deviation of mean can be computed by

$$\sigma_{gi}(stationK) = \frac{\sigma_g(stationK)}{\sqrt{m}} \qquad (12)$$

The standard deviation is a measure for the internal precision of measurement.

8.0 Results and Discussion

The results of the computed theoretical gravity values for the 21 localities in Taraba State are presented in Table (1). The result showed that the highest gravity value using the g_{1930} formula was obtained from Nukkai with latitude of 8.90 ^oN. The highest gravity value using the g_{1967} and g_{1980} was obtained from Mile six with the highest latitude of 8.93 ^oN. The standard deviation values showed the g_{1930} formula gave the most variable gravity values between locations.

Table 1: Calculated theoretical values of g using 1930, 1967 and 1980 gravity empirical equations for different stations.

Stations	**Latitude (degree)**	**g_{1930} (m/s^2)**	**g_{1967} (m/s^2)**	**g_{1980} (m/s^2)**
Nguroje	6.95	9.781243983	9.781074491	9.781069469
Chappal waddi	7.03	9.781261356	9.781091914	9.781086587
Gembu	6.71	9.781193043	9.781023412	9.781019268
Mayo Sebbe	7.28	9.781316903	9.781147610	9.781141307
Serti	7.38	9.781339651	9.781170419	9.781163723
Garbabi	7.83	9.781445770	9.781276818	9.781268278
Gassol	8.53	9.781622993	9.781177314	9.781170489
Mutum biyu	8.63	9.781649512	9.781221665	9.781214062
Mararaba	8.48	9.781609846	9.781441343	9.781429958
Sunkani	8.70	9.781668259	9.781499916	9.781487498
Iware	8.83	9.781703456	9.781535212	9.781214062
Mallum	8.92	9.781728115	9.781559941	9.781546513
Bali	7.41	9.781346536	9.781177314	9.781170489
Suntai	7.60	9.781300750	9.781221665	9.781214062
Mai-hula	7.38	9.781330531	9.781161255	9.781195640
Male Six	8.93	9.781730871	9.781562707	9.781549205
Sabon Gari	8.91	9.781725368	9.781557184	9.781543772
Nukkai	8.90	9.785318116	9.781554428	9.781541071
Pupuli	8.81	9.781698002	9.781529748	9.781516813
Pantisawa	8.92	9.781728115	9.781540767	9.781546494
Lankaviri	8.90	9.781722612	9.781554428	9.781541071
Mean	8.14	9.781699	9.781337	9.781316
Median	8.53	9.781623	9.781277	9.781214
Minimum -	6.71-	9.781193 -	9.781023 –	9.781019 –
Maximum	8.93	9.785318	9.781563	9.781549
Standard Deviation	0.80	0.000852	0.000200	0.000193

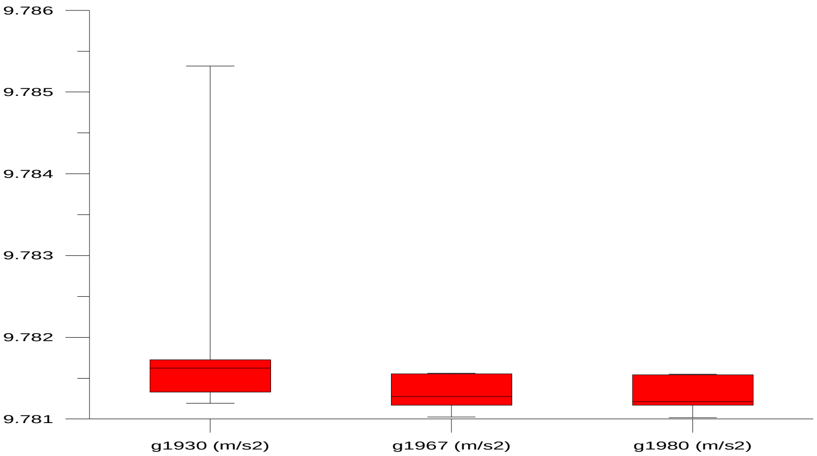

Fig. 2: Box Plot showing Variation of Gravity Values between the three International Gravity Formulas

The result of our measurement show that for any latitude, there exist a constant difference of approximately 17 mGal between the g_{1930} and g_{1967} or g_{1980} and a variable difference of approximately between 0.5 mGal and 1 mGal between g_{1967} and g_{1980}. It was also found that for given latitude, the theoretical g values decreases in the following order: $g_{1930} < g_{1967} < g_{1980}$. This result is more clearly shown in the box plot shown in Fig. 2. Similar result was also obtained by Yerima *et al* [6] in the neighboring Adamawa State.

The trend observed in this work implied that calculating the Bouguer gravity anomalies using the 1930 equation will give strong negative anomalies and moderate positive anomaly due to its high values compared with the g_{1967} and g_{1980} values. Conversely, 1967 and 1980 equations will give moderate negative Bouguer anomalies and strong positive anomalies. However, the 1980 equations appear to be ambiguous because of the many decimal places contained in the constants. This work therefore suggest that since Taraba State has relatively undulated topography with points that have elevations ranging from 150 to 1829 meter above sea level, negative gravity anomalies over lowland areas of Gashaka ,Gassol, Bali, Jalingo, Ardo-kola and Yorro Local Government Areas are expected and positive in Highland area of Sardauna Local Government Area. Hence applying 1930 gravity empirical equation in computing anomalies in these lowland areas will be misleading. It is therefore seen that 1967 equation can be used for lowland area while the 1930 equation can be used for highland areas for accuracy and consistency.

Figures 3 to 5 showed a plot of gravity values against latitude for the different locations using the three gravity equations. The curves show that both parameters are positively correlated. The graph plotted using the 1930 equation shoed the highest correlation coefficient with $R^2 = 0.987$ followed by the 1967 and 1980.

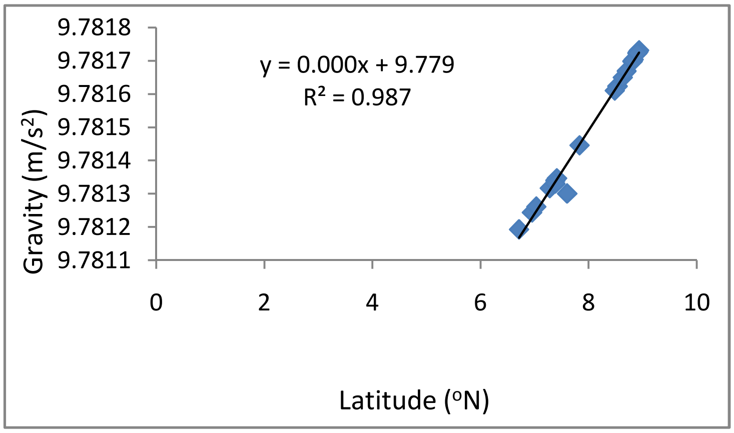

Figure 3: g_{1930} Gravity Values against Latitude

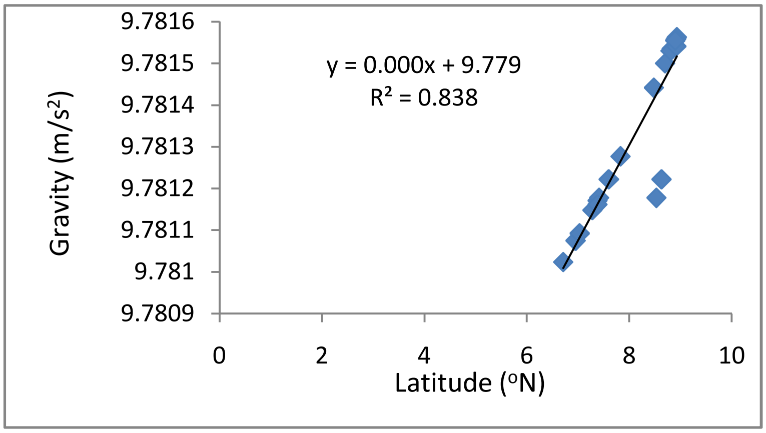

Figure 4: g_{1967} Gravity Values against Latitude

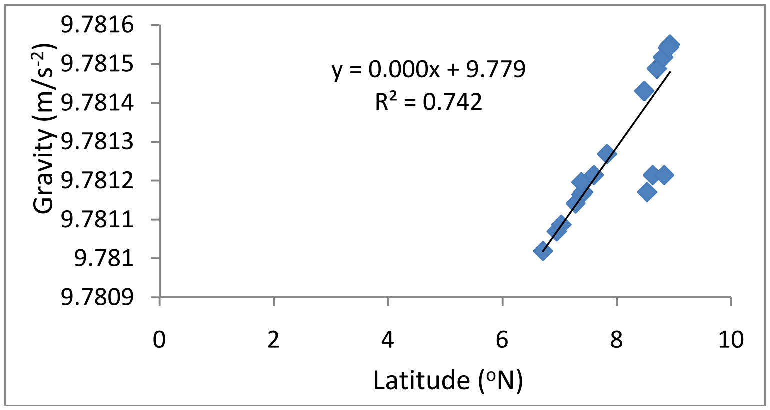

Figure 5: g_{1980} Gravity Values against Latitude

9.0 Conclusion

The computed gravity values of selected locations in Taraba State presented in Table 1 indicates that for a given latitude, the theoretical g values decreased in the order: $g_{1930} < g_{1967} < g_{1980}$. Results also showed that from one location and another, there is a constant difference of about 17 mGal between gravity values of g_{1930} and g_{1967} or g_{1980}. The 1967 equation is recommended for gravity surveys in low land areas of the study area while the 1930 equation is to be preferred in the high land areas during gravity surveys. The variation of gravity with latitude showed positive correlation indicating that gravity is expected to increase with latitude.

10.0 References

[1] Lowrie, W. (1997), Fundamental of Geophysics 1st Edition. UK: Cambridge University Press. Pp 49-62

[2] Telford, W. M., Geldart, L.P. and Sheriff, R. E. (1990), Applied Geophysics, 2nd Edition. UK: Cambridge University Press. Pp5

[3] Burger, H. R. (1992), Exploration Geophysics of the shallow subsurface, Prentice hall PTR

[4] Alkasim, A., J.B. Yerima and P. Timetere (2005), The Computation of Absolute Gravity Fields of Some Locations in Adamawa State using an Empirical Approach. *The Nigerian Academic Forum, Vol. 8(7), 87-104.*

[5] Ugwueke, B.U.D and Nkereuwem, O.T. (1995), Theoretical Gravity Values in Borno Province Using the 1930, 1967 and 1980 International Gravity Formulae, Implications on the Associated Bouguer Anomalies. *Borno Journal of Geology, Vol.2 (1), 15-22*

[6] Yerima, J. B., Timtere, P. and Abubakar, A. (2005), Preliminary Studies of Theoretical Gravity Values and their Implications on the Associated Bouguer Anomalies in Adamawa State. *The Nigerian Academic Forum. Vol 8 (2),50-64*

[7] Marc, C. and Erik, M. (2002). The First Absolute Gravity Measurement in the Netherlands Period1991-1999, Gonert Strang Vanttees Publications on Geodesy 50

[8] Robinson, E.S. and C. Coruh (2003). Basic Exploration Geophysics, New York: John Wiley & Sons.

[9] Danjuma, S. (2009). The Economic Profile of Mambila Plateau, Qalmatech Publications, Jalingo Pp. 5

[10] Macleod, W.N., Turner, D.C. and Wright, E. P. (1971): the Geology of the Jos Plateau. Geological Survey of Nigeria, bull. No. 32, 18

[11] Grant, N. K. (1971). A Compilation of the Radiometric Age from Nigeria. *Journ. of Mining and Geology*, 6: 37-54.

[12] Turner, D.C (1964): Notes on Field Work of the Basement Rocks of 1:250, 000 Sheets 7 and 8. Geol. Survey of Nigeria report No. 5503

[13] McCurry, P. (1976): A Generalised view of the Geneology of the Precambrian to Lower Paleozoiuc Rocks, Northern Nigeria: In Kogbe, C.A (ed), Geology of Nigeria, Elizabeth Press, Lagos. Pp. 13-387

[14] du Preez, J. W. and Barber, W. (1965). The Distinction of Chemical Quality of Ground Water in Northern Nigeria, Bull. 36, Geological Survey of Nigeria, 93