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Modelling the Impact of an Environmental Perturbation on the Incidence of Interaction Types

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Abstract

It is rare to find previous contributions that have evaluated the impact of an environmental perturbation on the incidence of each type of interaction between two legumes in Nigeria. Due to the present environmental upsets on crop production, it is imperative to consider the merit of tackling this proposed problem using the approach of a numerical simulation. It is clear from this present analysis that a longer duration of growth associated with late harvesting scenario has a relatively better potential to predict more volumes of mutualism than a shorter duration of growth that is associated with early harvesting. Therefore, in the loss of a pure competition between cowpea and groundnut, it is more likely to expect a relatively good evidence for biodiversity scenario. This is the key contribution of this study which we have not seen elsewhere.

1.0 Introduction

In the discipline of crop science, the notions of early harvesting and late harvesting in combination with some degree of an environmental perturbation have the potential to affect the level of crop production [1]. However, it has remain a long standing open problem to model the impact of an environmental perturbation on the type of interaction due to early and late harvesting. It is against this background that we have proposed the implementation of a proven numerical simulation to tackle this formidable mathematical problem under the simplifying assumption of the same random-noise intensity of 0.2 in the event of the competition between cowpea and groundnut. Other related works exist [2-3].

2.0 Mathematical Formulation

The interaction between cowpea and groundnut [2] is defined by the following system of continuous non-linear first order ordinary differential equations

$$\frac{dc(t)}{dt} = \alpha_1 c(t) - \beta_1 c^2(t) - \gamma_1 c(t) g(t) \quad (1)$$

$$\frac{dg(t)}{dt} = \alpha_2 g(t) - \beta_2 g^2(t) - \gamma_2 c(t) g(t) \quad (2)$$

Here, the notations c(t) and g(t) define the biomass of cowpea and groundnut respectively at time t in the unit of days; the α parameter values define the intrinsic growth rates; the β parameter values define the intra-competition coefficients; the γ

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parameter values define the inter-competition coefficients. This system of model equations evolve on the basis of an initial condition c(0) > 0 and g(0) > 0 which are biologically meaningful parameter values. For the purpose of this study, we have used the following precise parameter values: $\alpha_1 = 0.0225,\ \alpha_2 = 0.0446,\ \beta_1 = 0.006902,\ \beta_2 = 0.0133,$ $\gamma_1 = 0.0018,\ \gamma_2 = 0.0018,$ c(0)=0.02, g(0) =0.08.

2.0 Method of Solution

The core method of analysis is based on the implementation of a numerical simulation using the MATLAB ODE 45 integration function under the simplifying assumption of shorter and longer durations of growth. The key results of this method are presented and discussed next.

3.0 Results and Discussion

In this section, we have utilized the method of a numerical simulation to calculate the instances of the type of interaction due to the random noise intensity value of 0.2 for two competing legumes. These results are presented in Table 1 for the duration of growth ranging from sixty two (62) days to seventy-four (74) days and in Table 2 for the duration of growth ranging from one hundred and two (102) days to one hundred and fourteen (114) days.

Table 1: Numerical simulation of early harvesting of legumes due to environmental perturbation driven random noise intensity of 0.2 on the interacting system (3 instances of mutualism, 3 instances of predation, 1 instance of pure competition)

Example	Duration of growth in the unit of days	C_b^*	C_{bi}^*	G_b^*	G_{bi}^*
1	62	0.2381	0.2235	0.2995	0.2894
2	64	0.4529	0.4476	0.5433	0.5253
3	66	0.6616	0.6570	0.8193	0.7735
4	68	0.8555	0.9204	1.0748	1.0408
5	70	1.0821	1.1654	1.3326	1.3071
6	72	1.3108	1.4202	1.5984	1.6037
7	74	1.5360	1.6503	1.8946	1.8487

*C_b represents the biomass of cowpea interacting together with the groundnut legume; C_{bi} represents the biomass of cowpea legume growing in isolation of the main cowpea legume; G_b represents the biomass of groundnut interacting together with the cowpea legume; G_{bi} represents the biomass of groundnut legume growing in isolation of the main groundnut legume. The unit of each biomass is in grams per area of legumes cover.

Table 2: Numerical simulation of late harvesting of legumes due to environmental perturbation driven random noise intensity of 0.2 on the interacting system (5 instances of mutualism, 2 instances of predation, zero instance of pure competition)

Example	Duration of growth in the unit of days	C_b	C_{bi}	G_b	G_{bi}
1	102	0.2361	0.2235	0.2914	0.2977
2	104	0.4658	0.4369	0.5433	0.5226
3	106	0.7047	0.6786	0.8153	0.7570
4	108	0.8951	0.8836	1.0870	1.0280
5	110	1.1798	1.1602	1.3657	1.2958
6	112	1.4226	1.3915	1.6298	1.5647
7	114	1.6811	1.6920	1.9343	1.8554

What do we learn from the present analysis and how does this contribution compare and extend the foundation research of similar ecosystem? In the immediate post Nigerian civil war study on the preliminary trials on the intercropping of maize with different tropical legumes in the Western part of Nigeria, the role of early and late harvesting of maize was mentioned as one of the key characteristics of their work. In their work, there was no analysis of these two scenarios of harvesting due to an environmental perturbation realistically because as at then, it was highly unlikely to observe severe environmental perturbations as we witness in the recent environmental history. Our present analysis has succinctly considered a sound numerical simulation for the interaction between two legumes on the simplifying assumption of the same random noise intensity value of 0.2. Our study has predicted 3 instances of mutualism, 3 instances of predation and 1 instance of pure competition for the shorter duration of growth whereas the same numerical simulation prediction of 5 instances of mutualism, 2 instances of predation and zero instance of competition for the longer duration of growth. On the basis of this comparison, we can clearly report the cutting-edge contribution of this present analysis over the foundation study and the most recent work of [2] that only considered the impact of a variation of the intra-competition coefficient of cowpea on the stability of a mutualistic interaction with a similar doubling time.

4.0 Conclusion

In terms of crop production, the response analysis of this study clearly shows that a longer duration of growth tends to enhance a relatively more favourable condition for mutualism than the shorter duration of growth. Since more instances of mutualism can be attractive for biodiversity gain than biodiversity loss, some mitigation measures should be put into place to promote a longer duration of growth. This present analysis is consistent with the idea that early maturing crops impact more positively on food security than late maturing ones. However, the implication of using a system of second order ordinary differential equations to numerically model the impacts of early and late harvesting on the type of interaction between cowpea and groundnut will be the subject of a future publication.

5.0 References

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