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Asymptotic Analysis of an Improved Quadratic Model Structure Subjected to Static Loading

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Abstract

An asymptotic analysis on the buckling of a quadratic model structure subjected to static loading is discussed. The governing equations for the pre-buckling and buckling modes become a system of differential equations that are fully coupled and nonlinear, so that a closed form and easy solution to the problem is not possible. In this paper, we discuss the possibilities of using regular perturbation method in asymptotic expansions of the variables to get an approximate analytical solution to the problem and finally, discuss the results using some graphical plots.

1.0 Introduction

Bucklingis a form of instability that occurs suddenly with large changes in deformation but little change in loading [1]. For this reason, it is a dangerous phenomenon that must be avoided in structural design because it can lead to catastrophic failure. Buckling of elastic structures can occur under static or dynamic loading conditions. A structure buckles statically when the load duration is long compared to the response time of the structure and a structure buckles dynamically when the load duration is shorter than the response time of the structure. Many serious research works have been done in both dynamic and static buckling in recent times.

Kolakowski[2] studied static and dynamic interactive buckling of composite columns, whileChitra and Priyardasini[3] considered dynamic buckling of composite cylindrical shellssubjected to Axial Impulseusing finite element method. In the same token, the dynamic stability of suddenly loaded laminated cylindrical shells and the effect of static preloading upon the dynamic critical load were studied by Simitses [4] whileTabiei et al. [5]studied the numerical simulation of cylindrical laminated shells underimpulsive lateral pressureand Tanovet al. [6] likewise discussed the effect of static preloading on dynamic buckling of laminatedcylinders under sudden pressure.Jabareenand Izhak[7]on the other hand discussed the dynamic buckling of a beam on a nonlinear elastic foundation under step loading. Jankowski[8] discussed the static buckling of composite column-beams.The following review papers also outline recent research works in buckling: Patil et al. [9], Qatu[10] andTouati et al. [11]. Other investigations includeSimitses [12] and Sahu and Datta[13].

Another form of loading proposed by Simitses[4] is known as the quasi-static loading and is such that the static pressure is applied gradually, at small enough rate so as to keep the inertial effects negligible, thus emulating a static analysis (see for example, Jeong[14],Russell[15] and Zareiforoush et al. [16]), only to be trapped by a dynamic load after the initial static load. We remark that buckling phenomena have been investigated analytically, numerically and experimentally for decades.However, most buckling problems in the literatures are investigated using numerical methods. Examples of such investigations includeTouati et al. [12] andEglitis et al.[17]. We must note thatLuand Wang [18] discussed the asymptotic solutions for buckling delamination induced crack propagation inthe thin film- compliant substrate system while Lewandowski[19] studied the analysis of strongly non-linear freevibrations of beams using perturbation method.Similarly, Reboux[20] studied the asymptotic analysis of the buckling of ahighly shear-resistant vesicle whileEirik et al. [21] considered a semi-analytical model for global buckling and postbuckling analysis of stiffened panels. Much earlier on, Amazigo et al. [22]had studied asymptotic analysis of the buckling of imperfect columns on a nonlinear elastic foundation whileQiang et al. [23] considered the asymptotic solution of a dynamic buckling problem in elastic columns.Ette [24], on the other hand,used asymptotic expansions to examine the dynamic buckling of a spherical shell under an axial impulse whileEtte [25]similarly used analytical methods to study a two-parameter dynamic buckling of a lightly damped spherical cap trapped by a step load.Ette and Onwuchekwa [26] also studied the static buckling of an externally pressurized finite circular cylindrical shell using asymptotic methods.

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In this paper, regular perturbation method in asymptotic expansions of the variables (Bender and Orszag [27]) is used to analyze the buckling of an improved quadratic model structure under a static loading. Finally, numerical calculations are obtained with the help of Q-Basic codes and thereafter,beneficial conclusions are made.This simple quadratic elastic model structure was initially studied by Budiansky[28] while Danielson [29] made a refinement or an improvement on this initial model structure. In the same token, Ette [30] similarly studied the same structure. We strongly believe that this improved simple model structure provides a generalization of some of the structures encountered in real physical structural materials.

2.0 Formulation of the Problem

The real essence of the simple model structure under investigation is ably captured by the spring arrangements as in Figure 1, which was first studied by Budiansky and Hutchison [31].

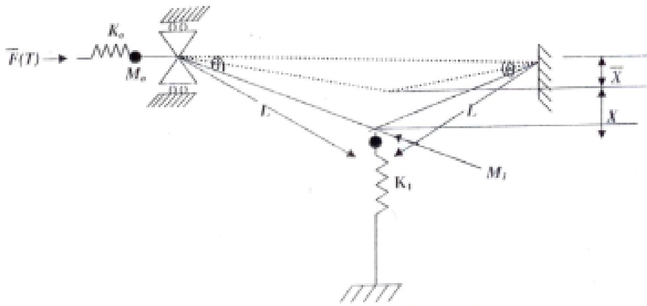

Figure 1: A simple Quadratic – Elastic Model Structure

Danielson [29] made an additional improvement on it by introducing an additional mass, M_0 and a spring with spring constant K_0, all with the aim of producing a pre-static buckling displacement, $\boldsymbol{\xi}_0(T)$. Except for the spring with spring constant K_0 and the mass M_0, the rest of the spring arrangement is as initially propounded by Hutchinson and Budiansky. Danielson obtained the following system of equations of motion.

$$\frac{1}{{\omega_o}^2}\frac{d^2\xi_o}{dT^2} + \xi_o - \frac{K_o}{\lambda_c}\xi_1(\xi_1 + 2\bar{\xi}) = \frac{\lambda}{\lambda_c}\bar{F}(T) \quad (2.1)$$

$$\frac{1}{{\omega_1}^2}\frac{d^2\xi_1}{dT^2} + \xi_1(1-\xi_0) - \alpha\xi_1^2 + \frac{K_o}{\lambda_c}\xi_1(\xi_1 + \bar{\xi})(\xi_1 + 2\bar{\xi})\bar{\xi}\xi_o \quad (2.2)$$

$$\lambda_c = \frac{K_1}{2}, \quad \omega_o = \left(\frac{K_0}{M_0}\right)^{\frac{1}{2}}, \quad \omega_1 = \left(\frac{K_1}{M_1}\right)^{\frac{1}{2}} \quad (2.3)$$

whereω_o and ω_1 are the circular frequencies of the pre-buckling mode $\boldsymbol{\xi}_0(T)$ and that of buckling mode $\boldsymbol{\xi}_1(T)$ respectively, and $\bar{\xi}$ is the imperfection amplitude; λ is the nondimensional load amplitude while $\bar{F}(T)$ is a factor of the load that depends on the time variable T, and λ_c is the classical buckling load while α is a constant.

In our study, we take $\bar{F}(T)$ to be a staticloadand our aim is to determine the static buckling load of the structure.

3.0 Perturbation Procedure

Before analyzing the static condition, we first consider (2.1) – (2.3) in full and now let $t = \omega_1 T$, and we get

$$\frac{d^2\xi_o}{dt^2} + \xi_o Q^2 - \frac{K_0}{\lambda_c}\xi_1 Q^2(\xi_1 + 2\bar{\xi}) = \frac{\lambda}{\lambda_c} f(t)Q^2 \quad (3.1)$$

$$\frac{d^2\xi_1}{dt^2} + \xi_1(1-\xi_0) - \alpha\xi_1^2 + \frac{K_o}{\lambda_c}\xi_1(\xi_1 + \bar{\xi})(\xi_1 + 2\bar{\xi}) = \bar{\xi}\xi_o \quad (3.2)$$

where,

$$f(t) = \bar{F}\left(\frac{t}{\omega_1}\right), 0 < \delta << 1, \ Q = \left(\frac{\omega_0}{\omega_1}\right), \ \xi_o(0) = \xi_1(0), \frac{d\xi_o(0)}{dt} = \frac{d\xi_1(0)}{dt} = 0 \quad (3.3)$$

Now equations $(3.1) - (3.3)$ are general equations regardless of the type of loading. In the problem at hand, we shall next determine the static deformation by which we intend to determine the static buckling load, λ_s.

3.1 Static Deformation

On setting f(t) ≡ 1 and ignoring all time-dependent terms in (3.1) and (3.2), we get

$$\xi_o - \frac{K_0}{\lambda_c}\xi_1(\xi_1 + 2\bar{\xi}) = \frac{\lambda}{\lambda_c} \quad (3.4)$$

$$\xi_1(1-\xi_1) - \alpha{\xi_1}^2 + \frac{K_o}{\lambda_c}\xi_1(\xi_1 + \bar{\xi})(\xi_1 + 2\bar{\xi}) = \bar{\xi}\xi_0 \quad (3.5)$$

We shall let

$$Q = \left(\frac{\omega_0}{\omega_1}\right), \qquad 0 < \left(\frac{\omega_0}{\omega_1}\right) < 1, \qquad 0 < \lambda < \lambda_c$$

We now let $\epsilon = \left(\frac{\lambda}{\lambda_c}\right)\left(\frac{\omega_0}{\omega_1}\right)^2 = \frac{\lambda}{\lambda_c}Q^2$ (3.6)

where,

$0 < \epsilon << 1$

Let $\xi_0 = \sum_{i=1}^{\infty} P_0^{(i)}\epsilon^i, \ \xi_1 = \sum_{i=1}^{\infty} P_1^{(i)}\epsilon^i$ (3.7)

On substituting (3.7) into (3.4) and (3.5) and equating the coefficients of powers of ϵ, we get, first for ξ_0

$$\mathbf{O}(\epsilon): \ P_0^{(1)} - 2\frac{K_0}{\lambda_c}\bar{\xi}P_1^{(1)} = \frac{1}{Q^2} \tag{3.8}$$

$$\mathbf{O}(\epsilon^2): P_0^{(2)} - 2\frac{K_0}{\lambda_c}\left(P_1^{(0)^2} + 2\bar{\xi}P_0^{(2)}\right) = 0 \tag{3.9}$$

$$\mathbf{O}(\epsilon^3): \quad P_0^{(3)} - 2\frac{K_0}{\lambda_c}\left(P_1^{(0)^3} + 2\bar{\xi}P_0^{(3)}\right) = 0 \tag{3.10}$$

etc.

and for ξ_1, we get

$$\mathrm{O}(\epsilon): \ P_1^{(1)} + 2\frac{K_0}{\lambda_c}\bar{\xi}^2P_1^{(1)} = \bar{\xi}P_0^{(1)} \tag{3.11}$$

$$\mathbf{O}(\epsilon^2): \ P_1^{(2)} - P_1^{(1)}P_0^{(1)} - \alpha P_1^{(1)^2} + \frac{K_0}{\lambda_c}\left(3\bar{\xi}P_1^{(1)^2} + 2\bar{\xi}^2P_1^{(2)}\right) = \bar{\xi}P_0^{(2)} \tag{3.12}$$

$$\mathbf{O}(\epsilon^3): P_1^{(3)} - P_1^{(1)}P_1^{(2)} - P_1^{(2)}P_0^{(1)} - \alpha P_1^{(1)^3} + \frac{K_0}{\lambda_c}\left[P_1^{(1)^3} + 6\bar{\xi}P_1^{(1)}P_1^{(2)} + 2\bar{\xi}^2P_1^{(3)}\right] = \bar{\xi}P_0^{(3)} \tag{3.13}$$

etc.

From (3.8), we get

$$P_0^{(1)} = \frac{1}{Q^2}\left(1 + 2\frac{K_0}{\lambda_c}Q^2\bar{\xi}P_1^{(1)}\right) \tag{3.14a}$$

On substituting for $P_0^{(1)}$ in (3.11), we get, after some simplification,

$$P_1^{(1)} = \frac{\bar{\xi}}{Q^2} \tag{3.14b}$$

$$\Rightarrow P_0^{(1)} = \frac{1}{Q^2}\left(1 + 2\frac{K_0}{\lambda_c}\bar{\xi}^2\right) \tag{3.14c}$$

On substituting from (3.14b,c) in (3.9), we get, (after simplification)

$$P_0^{(2)} = \frac{\frac{K_0}{\lambda_c}\bar{\xi}^2}{Q^4\left(1 - 2\frac{K_0}{\lambda_c}\bar{\xi}\right)} \quad ; \qquad \left(1 \neq 2\frac{K_0}{\lambda_c}\bar{\xi}\right) \tag{3.15a}$$

On substituting for terms in (3.12) and simplifying, we get

$$P_1^{(1)} = \frac{1}{\left(1 + 2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)}\left[\frac{\frac{K_0}{\lambda_c}\bar{\xi}^3}{Q^4\left(1 - 2\frac{K_0}{\lambda_c}\bar{\xi}\right)} - \left(3\frac{K_0}{\lambda_c}\bar{\xi} - \alpha\right)\left(\frac{\bar{\xi}}{Q^2}\right)^2 + \frac{\bar{\xi}}{Q^4}\left(1 + 2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)\right] \tag{3.15b}$$

Next, we substitute for terms in (3.10) and after a slight simplification, we get

$$P_0^{(3)} = \frac{\frac{K_0}{\lambda_c}\bar{\xi}^3}{Q^4\left(1 - 2\frac{K_0}{\lambda_c}\bar{\xi}\right)} \tag{3.16a}$$

If we now substitute in (3.13) and simplify, we get

$$P_1^{(3)} = \frac{1}{\left(1+\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)}\left[\frac{\frac{K_0}{\lambda_c}\bar{\xi}^4}{Q^6\left(1-\,2\frac{K_0}{\lambda_c}\bar{\xi}\right)} + \left(\alpha-\frac{K_0}{\lambda_c}\right)\left(\frac{\bar{\xi}}{Q^2}\right)^3 - \left(\frac{\bar{\xi}}{Q^2}\right)\left\{\left\{\left(\frac{6\frac{K_0}{\lambda_c}\bar{\xi}}{\left(1+\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)}\right)\left\{\frac{\frac{K_0}{\lambda_c}\bar{\xi}^3}{Q^4\left(1-\,2\frac{K_0}{\lambda_c}\bar{\xi}\right)} + \left(\frac{\bar{\xi}}{Q^2}\right)^2\left(\alpha-3\frac{K_0}{\lambda_c}\bar{\xi}\right)\left(\frac{\bar{\xi}}{Q^4}\right)\left(1+\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)\right\}\right\}\right\} + \left(\frac{1}{Q^2}\right)\left\{\frac{\frac{K_0}{\lambda_c}\bar{\xi}^3}{\left(\left(1-\,2\frac{K_0}{\lambda_c}\bar{\xi}\right)\right.} + \left(\frac{\bar{\xi}}{Q^2}\right)^2\left(\alpha-3\frac{K_0}{\lambda_c}\bar{\xi}\right) - \left(\frac{\bar{\xi}}{Q^4}\right)\left(1-\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)\right\}\right] \quad (3.16b)$$

So far, we get

$$\xi_0 = \epsilon P_0^{(1)} + \epsilon^2 P_0^{(2)} + \epsilon^3 P_0^{(3)} + \ldots \quad (3.17a)$$

$$\xi_1 = \epsilon P_1^{(1)} + \epsilon^2 P_1^{(2)} + \epsilon^3 P_1^{(3)} + \ldots \quad (3.17b)$$

The net displacement, Z, at this stage is

$$Z = \xi_0 + \xi_1 = \epsilon C_1 + \epsilon^2 C_2 + \epsilon^3 C_3 \ldots \quad (3.17c)$$

where,

$$C_1 = P_0^{(1)} + P_1^{(1)}, C_2 = P_0^{(2)} + P_1^{(2)}, C_3 = P_0^{(3)} + P_1^{(3)} \quad (3.17d)$$

We can now determine the Static buckling load,λ_s, at this stage by using the equivalent form of (1.1), which now takes the form

$$\frac{d\lambda}{dZ} = 0. \quad (3.18a)$$

The process, as in Ette[24, 25], is the reversal of the series (3.17c) in the form

$$\epsilon = Zd_1 + Z^2 d_2 + Z^3 d_3 + \cdots \quad (3.18b)$$

By substituting in (3.18a) for Z in (3.17c), and equating the coefficients of powers of ϵ, we get

$$d_1 = \frac{1}{C_1},\ \ d_2 = \frac{-C_2}{C_1^3},\ \ d_3 = \frac{2C_2^2 - C_1C_3}{C_1^5} \quad (3.18c)$$

For clarity, we take only the first two terms on the right hand side of (3.18b). The maximization (3.18a) is easily executed from (3.18b) to get

$$Z_c = \frac{-d_1}{2d_2} = \frac{C_1^2}{2C_1} \quad (3.18d)$$

where $Z_c$is the value of Z at Static buckling. If we now evaluate (3.18b) at Static buckling
(i.e at Z = Z_c, we get

$$\epsilon = \frac{1}{4}\left(\frac{C_1}{C_2}\right) \quad (3.18e)$$

On simplification, this yields

$$\frac{\lambda_s}{\lambda_c} = Q_0 \quad (3.19a)$$

where,

$$Q_0 = \frac{\frac{Q^2}{4}\left(1+\bar{\xi}+\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)}{\left[\frac{\frac{K_0}{\lambda_c}\bar{\xi}^2}{\left(1-2\frac{K_0}{\lambda_c}\bar{\xi}\right)} + \frac{1}{\left(1+2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)}\left\{\frac{\frac{K_0}{\lambda_c}\bar{\xi}^2}{\left(1-2\frac{K_0}{\lambda_c}\bar{\xi}\right)} + \left(\alpha-3\frac{K_0}{\lambda_c}\bar{\xi}\right)\bar{\xi}^2 + \bar{\xi}\left(1+\,2\frac{K_0}{\lambda_c}\bar{\xi}^2\right)\right\}\right]} \quad (3.19b)$$

4.0 Results and Discussion

Equation (3.19b) is an algebraic equation that determines the static buckling load, $\frac{\lambda_s}{\lambda_c}$. Sample codes written in Q-Basic programming were able to determine the Static Buckling of the structure as we vary imperfection factors (I.F) for each Q, as seen in Table 1. Using Table 1, a graph of the Static buckling load against imperfection factors for each Q is shown in Figure 2. We observe from Figure 2 that imperfection is a key factor in determining the buckling load of the structure because as the imperfection factors increase, the static buckling load decreases and is in agreement with known results obtained by Eglitis et al. [17].

Table 2 presents the numerical values of the static buckling loads for different values of Q for each imperfection factor and it is observed that as the values of Q decreases and the imperfection factors increase, the static buckling loads decrease, which also agrees with results of Table 1. A graph of the static buckling loads against Q for each imperfection factor is shown in Figure 3.

Table 3 also presents the numerical values of the static buckling loads for different values of Q for each imperfection factor, but in this case, the imperfection factors are smaller than those in Table 2 and as expected, the static buckling loads here are larger than those in Table 2, that is, the smaller the imperfection in the structure, the bigger the static buckling loads. A graph of the static buckling loads against Q for each imperfection factor is shown in Figure 4.

Table 1: Static Buckling Load and Imperfection at different Values of Q

Imperfection factor (I.F), $\bar{\xi}$	**Static Buckling Load, λ_S (Q = π/5)**	**Static Buckling Load, λ_S(Q = π/7)**	**Static Buckling Load, λ_S(Q = π/9)**	**Static Buckling Load, λ_S(Q = π/12)**
0.002	49.87765	25.24370	15.27088	8.589869
0.004	24.78378	12.64479	7.649316	4.302740
0.006	16.55251	8.445157	5.108799	2.873699
0.008	12.43687	6.345345	3.838541	2.159179
0.010	9.967501	5.085460	3.076389	1.730469
0.030	3.382600	1.725817	1.044012	0.587257
0.050	2.065733	1.053945	0.6375719	0.3586342
0.070	1.501447	0.776044	0.4634095	0.2606679
0.090	1.188025	0.606135	0.3666745	0.2062544
0.100	1.078353	0.550180	0.3328249	0.1872140

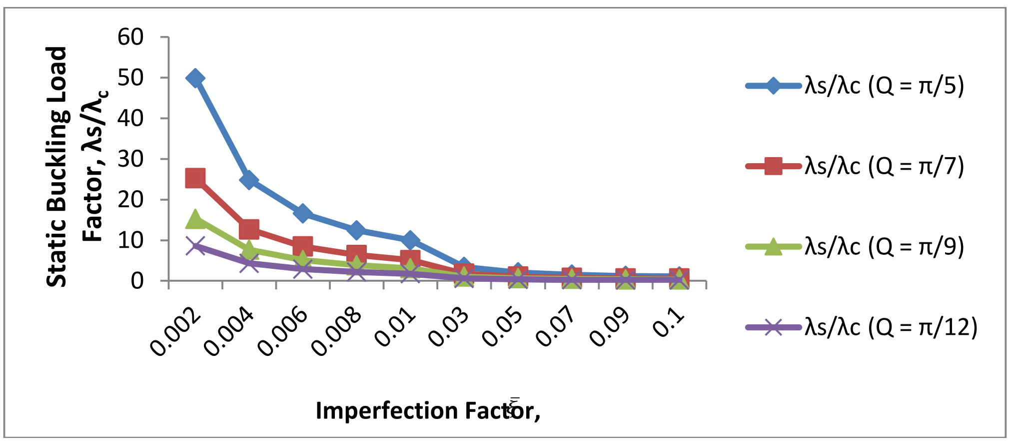

Figure 2: Static Buckling Load against Imperfection for various values of Q.

We observe that values of Q decrease and imperfection factors decrease, the static buckling load grows out of bounds as can be clearly seen from Table 1. We expect the static buckling load to lie between 0 and 1, confirming the fact that initial imperfection in a structure plays a key role in its stability, (see Eglitis et al. [17] and Teng and Rotter [32]).

Table 2: Static Buckling Load against Q at different Values of Imperfection

Q = ω_0/ω_1	**λ_S ($\bar{\xi}$ = 0.1)**	**λ_S ($\bar{\xi}$ = 0.2)**	**λ_S ($\bar{\xi}$ = 0.3)**	**λ_S ($\bar{\xi}$ = 0.4)**	**λ_S ($\bar{\xi}$ = 0.5)**
π/4	1.6849260	0.9145876	0.6589348	0.5321248	0.4569878
π/5	1.0783530	0.5835360	0.4217183	0.3405598	0.2924722
π/6	0.7488561	0.4064834	0.2928599	0.2364999	0.2031057
π/7	0.5501800	0.2986409	0.2151624	0.1737550	0.1492205
π/8	0.4212316	0.2286469	0.1647337	0.1330312	0.1142469
π/9	0.3328249	0.1800659	0.1301600	0.1051111	0.0902692
π/10	0.2695882	0.1463340	0.1054296	0.0851399	0.0731181
π/11	0.2228002	0.1209937	0.0871319	0.0703636	0.0604281
π/12	0.1872140	0.1016208	0.0732150	0.0591249	0.0507754
π/13	0.1595197	0.0865882	0.0623844	0.0503787	0.0432651
π/14	0.1375450	0.0746602	0.0537906	0.0434388	0.0373051

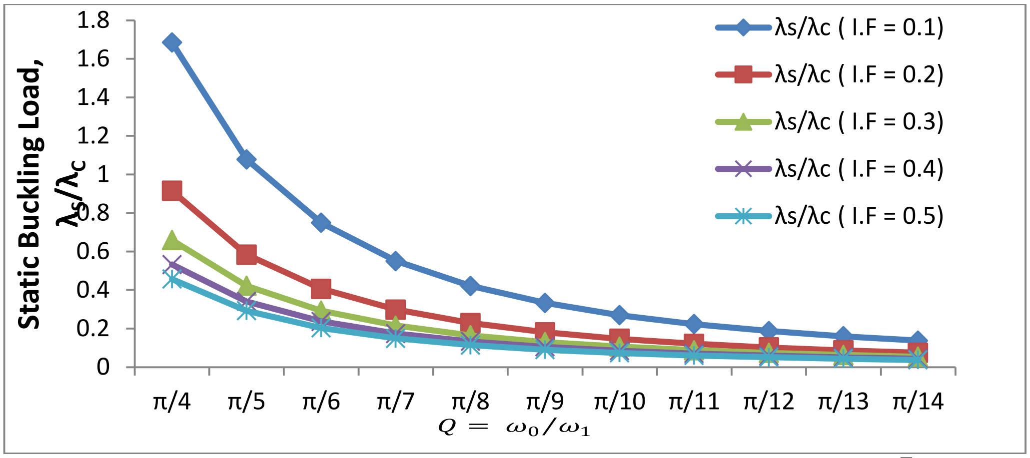

Figure 3: Static Buckling Load against Q for some fixed values of imperfection, $\bar{\xi}$**.**

The result also agrees with the results obtained in Figure 2. As we decrease the values of Q and increase the imperfection factors, the static buckling loads decrease as can be clearly seen from Table 2.

Table 3: Static Buckling Load and Q at different Values of Imperfection

$Q = \omega_0/\omega_1$	λ_S ($\bar{\xi}$ = 0.01)	λ_S ($\bar{\xi}$ = 0.02)	λ_S ($\bar{\xi}$ = 0.03)	λ_S ($\bar{\xi}$ = 0.04)	λ_S ($\bar{\xi}$ = 0.05)
π/4	15.57422	7.857487	5.285314	3.999282	3.227708
π/5	9.967501	5.028791	3.382600	2.559540	2.065733
π/6	6.921875	3.492216	2.349028	1.777458	1.434537
π/7	5.085460	2.565710	1.725817	1.305888	1.053945
π/8	3.893555	1.964372	1.321328	0.9998204	0.806927
π/9	3.076389	1.552096	1.044012	0.7899815	0.6375719
π/10	2.491875	1.257198	0.8456501	0.6398851	0.5164332
π/11	2.059401	1.039006	0.6988844	0.5288306	0.4268043
π/12	1.730469	0.873054	0.5872570	0.4443646	0.3556342
π/13	1.474483	0.743904	0.5003847	0.3786302	0.3055818
π/14	1.271365	0.641428	0.4314542	0.3264720	0.2634864

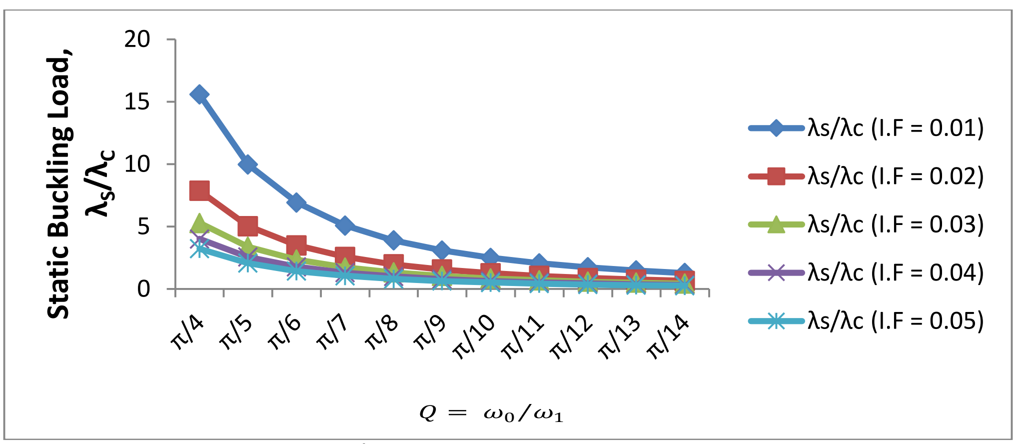

Figure 4: Static Buckling Load, $\frac{\lambda_s}{\lambda_c}$, **against Q for some fixed values of imperfection factor,** $\bar{\xi}$**.**

The result also agrees with the results obtained in Figure 2. As we decrease the values of Q and decrease the imperfection factors, the static buckling loads increase as can be clearly seen from Table 2.

5.0 Conclusion

The perturbation methods in asymptotic expansion of the variables proved to be both efficient and reliable in solving coupled nonlinear differential equations and it can be used as a general tool for performing buckling analyses of elastic material structure and it is our contention that this same procedure can be extended in solving actual elastic structures such as shells, columns, plates, etc. irrespective of the kind of loading history imposed on the structures.

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