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A Derived Scheme for Integrated Formulation of TAU Method for Fourth-Order Ode with Third Degree Overdetermination

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Abstract

This paper is concerned with the solution of a class of fourth order initial value problems in ordinary differential equation by the integrated formulation of the tau method. The initial focus is the class with a maximum of third degree overdetermination. The matrix equations were constructed based on the degree of overdetermination and for purpose of automation. The automated Tau system was tested on some selected problems to validate the study numerical evidences, thus obtained, confirm the accuracy of the method.

Keywords: Tau method, Variant, Formulation, Approximant, perturbation term

1.0 Introduction

Lanczos proposed the tau method techniques in 1983 for the numerical solution of ordinary differential equation with some conditions given as

$$Ly_n(x)i = \sum_{r=0}^{m}\left(\sum_{k=0}^{N} P_{rk}x^k\right)y^{(r)}(x) = \sum_{r=0}^{n} f_r x^r \quad a \leq x \leq b \tag{1.1}$$

$$L^*Y(x_{rk}) = \sum_{r=0}^{m-1} a_{rk}y^{(r)}(x_{rk}) = \alpha_k \;\; k = 1(1)m \tag{1.2}$$

by seeking an approximate solution of the form:

$$Y_n(x) = \sum_{r=0}^{n} a_r x^r \tag{1.3}$$

$r < +x$ of $y(x)$ which is the exact solution of the corresponding perturbed system

$$L^*Y_n(x) = \sum_{r=0}^{n} f_r x^r + H_n(x) \tag{1.4}$$

$$LY_n(x_{nk}) = \alpha_k \;\; k = 1(1)m \tag{1.5}$$

where

L is the linear differential operator α_k, f_r, $P_{nk}, - N_M$: $r = 01(1)m$. $K = 0(1)N_r$, a and b are real constants, $y(r)$ denoted the derivatives of order r of $y(x)$. The perturba- tion term $H_n(x)$ in (1.4) is defined by

$$H_n(x) = \sum_{i=0}^{m+s-1}\tau_{i+1}\, T_{n-m+i+1}(x) = \sum_{i=0}^{m+s-1}\tau_{i+1} \sum_{i=0}^{n-m+i+1} C_r^{(n-m+i+1)}x^r \tag{1.6}$$

and $C_r^{(n)}$ s are the coefficient of power of x (that is x^r) in the nth degree chebyshev polynomial denoted and defined by

$$T_n(x) = \cos(n\cos^{-1}\left[\frac{2x-a-b}{b-a}\right]) = \sum_{r=0}^{n} C_r^{(n)} \tag{1.7}$$

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The r's are the free tau parameters to be determined alongside with a_r and S is the number of over-determination of (1.1), which is defined by

$$S = \max[N_r - r : \ 0 \le r \le m, N_r \ge r] \ge 0 \tag{1.8}$$

2.0 Literature Review

The Tau method was initially formulated as a tool for the approximation of special function of mathematical physics which could be expressed in terms of simple differential equations. It later developed into a powerful and accurate tool for the numerical solution of complex differential and functional equations. The main idea in it is to solve approximate problem.

Accurate approximate polynomial solution in a linear ordinary differential equation with polynomial coefficient can be obtained by the Tau method introduced in [1]. The method is related to the principle of economization of a differentiable function implicitly defined by a linear differential equation with polynomial coefficient. Techniques based on the Tau method have been reported in the literature with application to more general equations including non-linear ones [2-3], while techniques based on the direct Chebyshev replacement have been discussed in [4] and more recently in the work of [5]. Further details on the Tau method can be found in references [6-11]. Because of the limitation in some of the works [7], this study seeks to extend the scope to fourth order problems with third degree overdetermination.

2.1 The Integrated Formulation of the TAU Method

Description of the integrated Formulation

Let us consider the m-th order linear differential equation

$$Ly(x) : = \sum_{r=0}^{m} P_r\ (x)y^r(x) = \sum_{r=0}^{F} f_r x^r \tag{2.1}$$

$$L^*y(x_{rk}) = \sum_{r=0}^{m-1} \alpha_{rk}\ y^n(x_{rk}) = \alpha_k,\ k = 1\ (1)m \tag{2.2}$$

Let

$\int\int\int \ldots . \int g(x)dx$ denote the indefinite integration I times applied to the function $g(x)$ and let

$$I_L = \int\int\int \overset{m}{\cdots} \int L(\cdot)dx \tag{2.3}$$

Thus the integral form of the linear differential equation of the form

$$Ly(x) : = \sum_{r=0}^{m} P_r(x)\,y^r(x) = f(x) \quad a \le x \le b \tag{2.4}$$

Now becomes

$$I_L(y(x)) = \int\int\int \overset{m}{\cdots} \int f(x)dx + C_{m-1}(x) \tag{2.5}$$

The tau approximant $y_n(x)$ of (2.4) satisfies the perturbed problem.

$$I_L(y_n(x)) = \int\int\int \overset{m}{\cdots} \int f(x)dx + C_m + H_n + m\ (x) \tag{2.6}$$

$$Iy_n(x_{rk}) = \alpha_k\ ;\ k = 1\ (1)m \tag{2.7}$$

where

$$H_n + m(x) = \sum_{r=0}^{m} T_{m+s-r} T_{n-m+r+1}(x) \tag{2.8}$$

3.0 A Class of Overdetermined Fourth Order Differen-Tial Equations

We consider here the integrated form of the tau methods for the class of problems:

$LY(x) : = (\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \alpha_4 x^4 + \alpha_5 x^5 + \alpha_6 x^6 + \alpha_7 x^7)y^{iv}(x) + (\beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 x^4 + \beta_5 x^5 + \beta_6)y''(x) + (\gamma_0 + \gamma_1 x + \gamma_2 x^2 + \gamma_3 x^3 + \gamma_4 x^4 + \gamma_5 x^5)\ y''(x) + (\lambda_0 + \lambda_1 x + \lambda_2 x^2 + \lambda_3 x^3 + \lambda_4 x^4)\ y'(x) + (\mu_0 + \mu_1 x + \mu_2 x^2 + \mu_3 x^3)y(x).$

$$= f(x) = \sum_{r=0}^{n} f_r x^r \tag{3.1}$$

$a \le x \le b$, $m = 4$, $s = 3$, $y(0) = \rho_0$, $y'(0) = \rho_1$, $y''(0) = \rho_2$, $y'''(0) = \rho_3$

THAT IS

$\int_0^x \int_0^u \int_0^t \int_0^s (\alpha_0 + \alpha_1 u + \alpha_2 u^2 + \alpha_3 u^3 + \alpha_4 u^4 + \alpha_5 u^5 + \alpha_6 u^6 + \alpha_7 u^7)\ y^{iv} dudsdtdu +$

$$\int_0^x \int_0^u \int_0^t \int_0^s \big(\beta_0 + \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \beta_4 u^4 + \beta_5 u^5 + \beta_6) y''(u) dudsdtdu +$$
$$\int_0^x \int_0^u \int_0^t \int_0^s \big(\gamma_0 + \gamma_1 u + \gamma_2 u^2 + \gamma_3 u^3 + \gamma_4 u^4 + \gamma_5 u^5)\, y''(u) dudsdtdu + (\lambda_0 + \lambda_1 u + \lambda_2 u^2$$
$$+ \lambda_3 u^3 + \lambda_4 u^4)\, y'(u) dudsdtdu + \int_0^x \int_0^u \int_0^t \int_0^s \big(\mu_0 + \mu_1 u + \mu_2 u^2 + \mu_3 u^3) y(u)$$
$$= \int_0^x \int_0^u \int_0^t \int_0^s f^{(V)} dudsdtdu + \tau_1 T_{n+7}(x) + \tau_1 T_{n+6}(x) + \tau_2 T_{n+5}(x) + \tau_3 T_{n+4}(x) + \tau_4 T_{n+3}(x) + \tau_5 T_{n+2}(x) + \tau_6 T_{n+1}(x) \tag{3.2}$$

After Simplifying and equating the Corresponding Coefficient Powers of x ,we have the recurrence relation;

$$\alpha_0 a_0 - \tau_1 C_0^{m+7} - \tau_2 C_0^{m+6} - \tau_3 C_0^{m+5} - \tau_4 C_0^{m+4} - \tau_5 C_0^{m+3} - \tau_6 C_0^{m+2} - \tau_7 C_0^{m+1} = \alpha_0 \rho_0 \tag{3.3}$$

$$\beta_0 a_0 + \alpha_0 a_1 - \tau_1 C_1^{m+7} - \tau_2 C_1^{m+6} - \tau_3 C_1^{m+5} - \tau_4 C_1^{m+4} - \tau_5 C_1^{m+3} - \tau_6 C_1^{m+2} - \tau_7 C_1^{m+1} =$$
$$\alpha_1 \rho_1 - 2\alpha_0 \rho_1 - 2\beta_0 a_0 \tag{3.4}$$

$$\frac{1}{2}[(3\gamma_0 - 4\beta_1 - 7\alpha_2) a_0 - (\alpha_1 + \beta_0) - 2\alpha_0 a_0] - \tau_1 C_2^{m+7} - \tau_2 C_2^{m+6} - \tau_3 C_2^{m+5} - \tau_4 C_2^{m+4} - \tau_5 C_2^{m+3} - \tau_6 C_2^{m+2} - \tau_7 C_2^{m+1}$$
$$= \frac{\alpha_0\rho_1 + 2\alpha_2\beta_2 - \beta_0\gamma_0 - 3\gamma_0\rho_0 - 4\beta_1\rho_0 - 7\alpha_2\rho_0 + \alpha_1\rho_1 + 2\alpha_0\rho_2}{2} \tag{3.5}$$

$$\frac{1}{6}[(30\alpha_3 + 12\beta_2 + 5\gamma_1 + 2\lambda_0) a_0 - (8\alpha_2 + 5\beta_1 + 3\gamma_0) a_1 - 2(2\alpha_1 + \beta_0) a_2 + \alpha_0 a_3] \quad - \tau_1 C_3^{m+7} - \tau_2 C_3^{m+6} - \tau_3 C_3^{m+5} - \tau_4$$
$$C_3^{m+4} - \tau_5 C_3^{m+3} - \tau_6 C_3^{m+2} - \tau_7 C_3^{m+1}$$
$$= \frac{181\alpha_3\rho_0 + 3\alpha_1\rho_2 + 6\alpha_2\rho_1 - 3\beta_0\rho_2 + 10\beta_2\rho_0 + 6\gamma_1\rho_0 + 2\gamma_0\rho_1 + \lambda_0\rho_0 - 5\alpha_0\rho_3 + 3\rho_1 + 5\beta_1\rho_1}{6} \tag{3.6}$$

$$\frac{1}{24}[(116\alpha_4 + 30\beta_3 + 8\gamma_2 + \lambda_1 + \mu_0) a_0 + (42\alpha_3 + 14\beta_2 + 4\gamma_1 + 2\lambda_0) a_1 + (18\alpha_2 + 12\beta_1 + 6\gamma_0) a_2 - (18\alpha_2 + 6\beta_0) a_3 + 24\alpha_0 a_4] - \tau_1$$
$$C_4^{m+7} - \tau_2 C_4^{m+6} - \tau_3 C_4^{m+5} - \tau_4 C_4^{m+4} - \tau_5 C_4^{m+3} - \tau_6 C_4^{m+2} - \tau_7 C_4^{m+1} = 0 \tag{3.7}$$

$$\frac{1}{120}[(720\alpha_5 + 168\beta_4 + 368\gamma_3 + 6\lambda_2 + \mu_1) a_0 + (264\alpha_4 + 96\beta_3 + 24\gamma_4 + 3\lambda_1 + \mu_0) a_1 + 2(24\alpha_3 + 34\beta_2 + 11\gamma_1 + 2\lambda_0) a_2 + (60\alpha_2 +$$
$$426\beta_1 + 18\gamma_0) a_3 + 24(4\alpha_1 + \beta_0) a_4 + 120\alpha_0 a_5] - \tau_1 C_5^{m+7} - \tau_2 C_5^{m+6} - \tau_3 C_5^{m+5} - \tau_4 C_5^{m+4} - \tau_5 C_5^{m+3} - \tau_6 C_5^{m+2} - \tau_7 C_5^{m+1} = 0 \tag{3.8}$$

$$\frac{1}{360}[(1440\alpha_6 + 900\beta_5 + 48\gamma_4 + 24\lambda_3 + \mu_2) a_0 + (1240\alpha_5 + 300\beta_4 + 60\gamma_3 + 8\lambda_2 + \mu_1) a_1 + (432\alpha_1 - 198\beta_3 - 46\gamma_2 - 5\lambda_0 + \mu_0) a_2$$
$$+ (72\alpha_3 + 144\beta_2 + 42\gamma_1 + 6\lambda_0) a_3 + 120(11\alpha_2 - 8\beta_1 - 3\gamma_0) a_4 + 60(5\alpha_1 + \beta_0) a_5 + 360\alpha_0 a_6] - \tau_1 C_6^{m+7} - \tau_2 C_6^{m+6} - \tau_3 C_6^{m+5} - \tau_4$$
$$C_6^{m+4} - \tau_5 C_6^{m+3} - \tau_6 C_6^{m+2} - \tau_7 C_6^{m+1} = 0 \tag{3.9}$$

$$[\frac{[\alpha_1 k - \alpha_1 + \beta_0]}{k} a_{k-1} \frac{[\alpha_2(k-1) - (2\alpha_2 + \beta_1)(k-1) - 3(2\alpha_2 + \beta_1 + \gamma_0)]}{k(k-1)} a_{k-2} +$$
$$\frac{[\alpha_3(k-2)(k-1)k - (3\alpha_3 + \beta_2)(k-2)(k-1) - 3(6\alpha_3 + 2\beta_2 + \gamma_1)(k-2) - 2(6\alpha_3 + 2\beta_2 + \gamma_1 - \lambda_0)]}{(k-2)(k-1)k} a_{k-3}$$
$$[\alpha_4(k-3)(k-2)(k-1)k - (\alpha_4 + \beta_3)(k-3)(k-2)(k-1) - 3(12\alpha_4 + \beta_3 + \gamma_2)$$
$$\frac{(k-3)(k-2) - 2(24\alpha_4 + 6\beta_3 + 2\gamma_2 - \lambda_1)(k-3) + (24\alpha_4 + 6\beta_3 + 2\gamma_2 + \lambda_1 + \mu_0)]}{(k-3)(k-2)(k-1)k} a_{k-4}$$
$$[\alpha_5(k-3)(k-2)(k-1)k - (5\alpha_5 + \beta_4)(k-3)(k-2)(k-1) - 3(20\alpha_5 + 4\beta_4 + \gamma_3)$$

$$\frac{(k-3)(k-2)-2(60\alpha_5+12\beta_4+3\gamma_3-\lambda_2)(k-3)-(120\alpha_5+24\beta_4+6\gamma_3+2\lambda_2+\mu_1)]}{(k-3)(k-2)(k-1)k}a_{k-5}$$

$[\alpha_6(k-3)(k-2)(k-1)k-(6\alpha_6+\beta_5)(k-3)(k-2)(k-1)-3(30\alpha_6+5\beta_5+\gamma_4)$

$$\frac{(k-3)(k-2)-2(120\alpha_6+120\beta_5+4\gamma_4-\lambda_3)(k-3)+(360\alpha_6+60\beta_5+12\gamma_4+3\lambda_3+\mu_2)]}{(k-3)(k-2)(k-1)k}a_{k-6}$$

$[\alpha_7(k-3)(k-2)(k-1)k-(7\alpha_7+\beta_6)(k-3)(k-2)(k-1)-3(42\alpha_7+6\beta_6+\gamma_5)$

$$\frac{(k-3)(k-2)-2(210\alpha_7+30\beta_6+5\gamma_5-\lambda_4)(k-3)+(840\alpha_7+120\beta_6+20\gamma_5+4\lambda_4+\mu_3)]}{(k-3)(k-2)(k-1)k}a_{k-7}$$

$\tau_1 C_K^{m+7}-\tau_2 C_K^{m+6}-\tau_3 C_K^{m+5}-\tau_4 C_K^{m+4}-\tau_5 C_K^{m+3}-\tau_6 C_K^{m+2}-\tau_7 C_K^{m+1}]$

$$=\frac{f_{k-7}}{(k)(k-1)(k-2)(k-3)(k-4)(k-5)(k-6)} \qquad (3.10)$$

$\frac{1}{(n-2)(n-1)(n)(n+1)}[[\alpha_7(n-2)[n^3-4n^2-120n-294]+840\alpha_7)-(\beta_6(n-2)[n^2+17n+62]-120\beta_6)-(\gamma_5(n-2)(n-1)]-20\gamma_5)-2\lambda_4(n-4)+\mu_3)a_{n-6}]+(\alpha_6(n-2)(n^3-6n^2-85n-150)+360\alpha_6)-(\beta_5(n-2)[n^2+14n+225]-60\beta_5)-\gamma_4(n-2)[3n+5]-12\gamma_4)-2\lambda_3(n-17)+\mu_2)]a_{n-5}+[(\alpha_5(n-2)[n^3-5n^2-56n-60]-120\alpha_5)-(\beta_4(n-2)(n^2+11n+12)-24\beta_4)-\gamma_3(n-2)(3n+3)+6\gamma_3)+\lambda_2 n+\mu_1)]a_{n-4}+[\alpha_4(n-2)[-35n^2+20n^2+15n)+24\alpha_4)-(\beta_3(n-2)[n^2-10n-3]+6\beta_3)+(\gamma_2(n-2)(-3n-1)+2\gamma_2)+(\lambda_1(5-2n)+\mu_0)]a_{n-3}+[(\alpha_3(n-2)n^3-3n^2-19n+6)]+\beta_2[2n^2+10n)+\gamma_1(1-3n)-2\lambda_0)]a_{n-2}+(\alpha_2(n-2)(n^2-7n+6)]-\beta_1[n+3)-3\gamma_0]a_{n-1}+(\alpha_1(n-2)(n^2-n)(n+\beta_0)]a_n-\tau_1 C_{n+1}^{m+7}-\tau_2 C_{n+1}^{m+6}-\tau_3 C_{n+1}^{m+5}-\tau_4 C_{n+1}^{m+4}-\tau_5 C_{n+1}^{m+3}-\tau_6 C_{n+1}^{m+2}-\tau_7 C_{n+1}^{m+1}]x^{n+1}=\frac{f_{n-6}}{(n+1)(n)(n-1)(n-2)(n-3)(n-4)(n-5)}$ (3.11)

$\frac{1}{(n-1)(n)(n+1)(n+2)}[[\alpha_7(n-1)[n^3-4n^2-131n-420]+840\alpha_7)-(\beta_6(n-1)n^2+19n+60]-120\beta_6)-(\gamma_5(n-1)(3n+10)]-20\gamma_5)-2\lambda_4(n-3)+\mu_3)a_{n-5}]+(\alpha_6(n-1)(n^3-3n^2-86n-240)+360\alpha_6)-(\beta_5(n-1)[n^2+16n+240]-60\beta_5)-\gamma_4(n-1)[3n+8]-12\gamma_4)-2\lambda_3(n+6)+\mu_2)]a_{n-4}+[(\alpha_5(n-1)[n^3-2n^2-63n-120]-120\alpha_5)-(\beta_4(n-1)(n^2+13n+24)-24\beta_4)-\gamma_3(n-1)(3n+6)+6\gamma_3)-2\lambda_2 n-\mu_1)]a_{n-3}+[\alpha_4(n-1)[n^3-39n^2+38n-48)+24\alpha_4)-(\beta_3(n-1)[n^2-2n)+6\beta_3)-(\gamma_2(n-1)(3n+4)-2\gamma_2)+(\lambda_1(2n+3)+\mu_0)]a_{n-2}+[(\alpha_3(n-1)(n^3-19n)-12\alpha_3)-\beta_2(n^2-7n)-4\beta_2)(\gamma_1(3n+2)-2\lambda_0)]a_{n-1}+(\alpha_2(n-1)(n^2-7n)-\beta_1[n+4)-3\gamma_0]a_n-\tau_1 C_{n+2}^{m+7}-\tau_2 C_{n+2}^{m+6}-\tau_3 C_{n+2}^{m+5}-\tau_4 C_{n+2}^{m+4}-\tau_5 C_{n+2}^{m+3}-\tau_6 C_{n+2}^{m+2}=$

$$\frac{f_{n-5}}{(n+2)(n+1)(n)(n-1)(n-2)(n-3)(n-4)} \qquad (3.12)$$

$\frac{1}{(n)(n+1)(n+2)(n+3)}[[\alpha_7 n[n^3-n^2-136n-554]+840\alpha_7)-(\beta_6 n)[n^2+21n+80]-120\beta_6)-(\gamma_5 n(3n-7)]-20\gamma_5)-2\lambda_4(n-2)+\mu_3)a_{n-4}]+[(\alpha_6 n(n^3-85n-336)+360\alpha_6)-(\beta_5 n(n^2+18n+25]-60\beta_5)-(\gamma_4 n(3n-5)-12\gamma_4)-2\lambda_3(n-15)+\mu_2)]a_{n-3}+[(\alpha_5 n[n^3+2n^2-64n-184]-120\alpha_5)-(\beta_4 n(n^2+15n+38)+24\beta_4)-\gamma_3 n(3n+9)+6\gamma_3)-2\lambda_2(n+1)-\mu_1)]a_{n-2}+[\alpha_4 n[n^3-2n^2-37n-86)+24\alpha_4)-(\beta_3 n[n^2+12n+23)-(\gamma_2 n(3n+7)-2\gamma_2)+(\lambda_1(2n-1)+\mu_0)]a_{n-1}+[(\alpha_3 n(n^3+3n^2-16n-30)-\beta_2 n(n+9)-12\beta_2)(\gamma_1(3n+5)-2\lambda_0)]a_n-\tau_1 C_{n+3}^{m+7}-\tau_2 C_{n+3}^{m+6}-\tau_3 C_{n+3}^{m+5}-\tau_4 C_{n+3}^{m+4}-\tau_5 C_{n+3}^{m+3}=$

$$\frac{f_{n-4}}{(n+3)(n+2)(n+1)(n)(n-1)(n-2)(n-3)} \qquad (3.13)$$

$\frac{1}{(n+1)(n+2)(n+3)(n+4)}[[\alpha_7(n+1)[n^3+2n^2-135n-688]+840\alpha_7)-(\beta_6(n+1)(n^2+23n+102)-120\beta_6)-(\gamma_5(n+1)(3n+16)]-20\gamma_5)-(2\lambda_4(n-1)+\mu_3)a_{n-3}]+(\alpha_6(n+1)(n^3-9n^2-94n-282)+360\alpha_6)-(\beta_5(n+1)(n^2+120n+276]-60\beta_5)-\gamma_4(n+1)[3n+14]-12\gamma_4)-2\lambda_3(n-14)+\mu_2)]a_{n-2}+[(\alpha_5(n+1)[n^3+4n^2-59n-234]-120\alpha_5)-(\beta_4(n+1)(n^2+17n+54)+24\beta_4)-\gamma_3(n+1)(3n+36)+6\gamma_3)-2\lambda_2(n+2)-\mu_1)]a_{n-1}+[\alpha_4(n+1)[n^3+5n^2-30n-120)+24\alpha_4)-$

$(\beta_3(n+1)[n^2-4n-24)-6\beta_3)\ -(\gamma_2(n+1)\,(3n+2)-\ 2\gamma_2)-\ (\lambda_1(2n+1)+\mu_0)]a_n-\tau_1 C_{n+4}^{m+7}-\tau_2 C_{n+4}^{m+6}-\tau_3 C_{n+4}^{m+5}-\tau_4 C_{n+4}^{m+4}$

$$= \frac{f_{n-3}}{(n+4)(n+3)(n+2)(n+1)(n)(n-1)(n-2)} \qquad (3.14)$$

$\frac{1}{(n+2)(n+3)(n+4)(n+5)}[[\alpha_7(n+2)[n^3+5n^2-128n-822]+840\alpha_7)-(\beta_6(n+2)(n^2+25n+12)-120\beta_6)-(\gamma_5(n+2)(3n+19)]-20\gamma_5)-(2\lambda_4 n+\mu_3)a_{n-2}]+[(\alpha_6(n+2)(n^3+6n^2-85n+520)+360\alpha_6)-(\beta_5(n+2)(n^2+22n+297]-60\beta_5)-\gamma_4(n+2)(3n-5)-12\gamma_4)-2\lambda_3(n-13)+\mu_2)]a_{n-1}+[(\alpha_5(n+2)[n^3+7n^2-48n-\ 300]-120\alpha_5)-(\beta_4(n+2)(n^2+19n+72)+24\beta_4)-\gamma_3(n+2)(3n+15)+6\gamma_3)-2\lambda_2(n+3)-\mu_1)]a_n-\tau_1 C_{n+5}^{m+7}-\tau_2 C_{n+5}^{m+6}-\tau_3 C_{n+5}^{m+5}=$

$$\frac{f_{n-2}}{(n+5)(n+4)(n+3)(n+2)(n+1)(n)(n-1)} \qquad (3.15)$$

$\frac{1}{(n+3)(n+4)(n+5)(n+6)}[[\alpha_7(n+3)[n^3+8n^2-11n-944]+840\alpha_7)-(\beta_6(n+3)(n^2+27n+152)-120\beta_6)-(\gamma_5(n+3)(n+14)]-20\gamma_5)-(2\lambda_4(n+1)+\mu_3)a_{n-1}+[(\alpha_6(n+3)(n^3+9n^2+70n-600)+360\alpha_6)-(\beta_5(n+3)(n^2+24n+320]-60\beta_5)-\gamma_4(n+3)(3n+20)-12\gamma_4)-2\lambda_3(n-12)+\mu_2)]a_n-\tau_1 C_{n+6}^{m+7}-\tau_2 C_{n+6}^{m+6}$

$$= \frac{f_{n-1}}{(n+6)(n+5)(n+4)(n+3)(n+2)(n+1)(n)} \qquad (3.16)$$

$\frac{1}{(n+4)(n+5)(n+6)(n+7)}[[\alpha_7(n+4)[n^3+11n^2-58n-630]+840\alpha_7)-(\beta_6(n+4)(n^2+29n+180)-120\beta_6)-(\gamma_5(n+4)(3n+25)]-20\gamma_5)-(2\lambda_4(n+2)+\mu_3)a_n-\tau_1 C_{n+7}^{m+7}$

$$= \frac{f_n}{(n+7)(n+6)(n+5)(n+4)(n+3)(n+2)(n+1)} \qquad (3.17)$$

4.0 A Numerical Experiment

We consider here the following problems for experimentation with our results of the preceding sections. The exact error is defined by $\varepsilon^* = \max_{0\le x\le 1}[|Y(x_k)-Y_n(x_k)|],\ 0\le x\le 1,\ [x_k]=[0.01k],\ k=0(1)\,100$

Example

$$Ly(x) := y^{iv}(x) + \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right)y^n(x) - y(x) = -1 + 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 \qquad (4.1)$$

$y(0) = -1,\ y'(0)=0,\ y''(0)=2,\ y'''(0)=0,$ (4.2)

with analytical solution

$y(x) = 1 - 2\cos x,\ m=4,\ s=3$ (4.3)

The linear equations obtained were solved by matlab package.

Example 4: Table of Numerical Result (case $n = 7$)

x	Exact	Approximate ($n = 7$)	Error
0.0	–1.00000000000000	– 1.0000000000000	0.0000000000
0.1	– 0.990108330556051	– 0.990008330556079	$2.75335310100388\times10^{-14}$
0.2	– 0.960133055682483	– 0.960133155696527	$1.4043988194468084\times10^{-11}$
0.3	– 0.910672978251212	– 0.910672978887955	$5.367429833924575\times10^{-10}$
0.4	– 0.842121988005770	– 0.842121995095116	$7.089346221878516\times10^{-9}$
0.5	– 0.755165123780746	– 0.755165176032072	$5.225132670982902\times10^{-8}$
0.6	– 0.650671229819356	– 0.650671495834239	$2.660148825661679\times10^{-7}$
0.7	– 0.529684374568977	– 0.529685422767345	$1.048198367659126\times10^{-6}$
0.8	– 0.393413418964000	– 0.393416839947294	$3.421252963264898\times10^{-6}$
0.9	– 0.124321993654100	– 0.243229599820660	$9.663279330829333\times10^{-6}$
1.0	– 0.080804611736280	– 0.080628945638246	$2.433390196665553\times10^{-5}$

Example 4: Table of Numerical Result (case *n* = 8)

x	Exact	Approximate (*n* = 7)	Error
0.0	–1.000000000000000	– 1.000000000000000	0.000000000
0.1	– 0.9900083310556051	– 0.990008330556079	$2.76445533131164 \times 10^{-14}$
0.2	– 0.96013315682483	– 0.960133155696527	$1.404409921690331 \times 10^{-11}$
0.3	– 0.910672978251212	– 0.910672978787955	$5.357430944147500 \times 10^{-10}$
0.4	– 0.842121988005770	– 0.842121995195116	$7.089345999133911 \times 10^{-9}$
0.5	– 0.755165123780746	– 0.755165176032060	$5.225131471942035 \times 10^{-8}$
0.6	– 0.650671229819356	– 0.650671495834051	$2.660146949384767 \times 10^{-7}$
0.7	– 0.529684374568977	– 0.529685422765450	$1.048196472619445 \times 10^{-6}$
0.8	– 0.393413418964	– 0.393416839933247	$3.421238918499547 \times 10^{-6}$
0.9	– 0.243219936541	– 0.243229599738472	$9.663197143017221 \times 10^{-6}$
1.0	– 0.080604611736280	– 0.080628945239064	$2.433350278485680 \times 10^{-5}$

Example 4: Table of Numerical Result (case *n* = 9)

x	Exact	Approximate (*n* = 7)	Error
0.0	–1.000000000000000	– 1.000000000000000	0.000000000
0.1	– 0.990008330556051	– 0.990008330556079	$2.764455331316640 \times 10^{-14}$
0.2	– 0.960133155682483	– 0.960133155696527	$1.404409921690331 \times 10^{-11}$
0.3	– 0.910672978251212	– 0.910672978787955	$5.367430944147600 \times 10^{-10}$
0.4	– 0.842121988005770	– 0.842121995095111	$7.089345110955492 \times 10^{-9}$
0.5	– 0.755165123780746	– 0.755165176032030	$5.225128474339869 \times 10^{-8}$
0.6	– 0.650671229819356	– 0.650671495833497	$2.660141408261652 \times 10^{-7}$
0.7	– 0.529684374568977	– 0.527685422758922	$1.048189945396238 \times 10^{-6}$
0.8	– 0.393413418694331	– 0.393416839877965	$3.42118363449947 \times 10^{-6}$
0.9	– 0.243219936541329	– 0.243229599374524	$9.66833195145382 \times 10^{-6}$
1.0	– 0.080604611736820	– 0.080628943274986	$2.43315387062903 \times 10^{-5}$

5.0 Conclusion

The derivation of an approximation scheme for a fourth order differentiation with third degree overdetermination by the integrated formulation of the tau method has been presented. The approach involves four-time integration of the class of ordinary differential equation under consideration and then perturbing the resulting equation. This is to guarantee an improved accuracy of the desired approximation viz-a-viz those of the recursive and the differential formulations [4, 6]. Numerical evidences from the application of the integration scheme show that it is accurate and effective.

6.0 References

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