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Block Multistep Methods with Chebyshev Basis Functions For Second Order ODEs

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Abstract

This article presents two-step and three-step implicit block methods for direct solution of second order stiff differential equations. Techniques of collocation and interpolation were used to generate the methods. One of the orthogonal polynomials precisely Chebyshev polynomial was used as a basis function in the development of the methods and were implemented in block mode. Analysis of the methods recovered reveals that they are zero stable, consistent and convergent. The stability domain of the block methods were derived and sketch out. The performances of the methods were tested on three numerical examples. The results show that the methods compared creditably well with those of existing methods.

Keywords: Interpolation, Collocation Approximation, Chebyshev Polynomial, Orthogonal Polynomial, Linear Multistep Methods (LMMs), Stiff Equations

1.0 Introduction

Consider second-order ordinary differential equations (ODEs) of the form:

$$y'' = f(x, y, y'),\ y(a) = \eta_0,\ \ y'(a) = \eta_1 \tag{1}$$

where f satisfied Lipstichz condition that guaranteed the existence and the uniqueness of the solution of (1). Equations of these types occur mostly in the modeling of real life situations in areas like physical, chemical, biological and even social sciences. However, the analytical solution of some of the equations is not easily obtainable in real sense hence, the need for a numerical approximation. Earlier attempts in literature employed the technique of reduction of second order ODEs to the system of first order differential equations which has been found to be unsuitable due to some of its drawbacks [1, 2].

Moreover, the Chebyshev polynomials and Rational Chebyshev (RC) tau functions respectively were used with collocation approximation to develop algorithms for higher order differential equations in [3]. Also, an improved self-starting implicit hybrid method, was presented in [4]. In [5] continuous formulation of a class of accurate implicit LMMs with Chebyshev basis function in a collocation technique for the solution of first order ordinary differential equations were proposed. Furthermore, Kayode and Adeyeye [6] worked on two-step two point hybrid methods for general second order ordinary differential equations using Chebyshev polynomial as basis function. The continuous multistep methods developed in [7] were with Chebyshev polynomial as basis function. All authors mentioned above implemented their work in predictor-corrector mode. A class of collocation methods for general second order ordinary differential equations was proposed in [8] and Chebyshev interpolation, an interactive tour, has been considered in [9].

Therefore, this article proposes two-step and three-step LMMs by collocation approximation which will be implemented in block mode adopting Chebyshev polynomial as the basis function for solution of general second order ordinary differential equations. The implementation strategy makes its computation be competitive and especially its implicit type formulas can be used in solving stiff ODEs effectively. The stability domain of the block methods is also sketched.

2.0 Preliminaries

In this section, we give an introduction to the Chebyshev polynomials and their basic properties (see [10] and [11] for more details). This is needed in order to understand the relationship between the applied polynomials. The Chebyshev polynomials of the first and second kind are denoted by $T_n(x)$ and $U_n(x)$ respectively. A Chebyshev polynomial of first kind can be represented as a linear combination of two Chebyshev polynomials of second kind,

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$$T_{n+1}(x)=\frac{1}{2}(U_n(x)-U_{n-2}(x))$$

and the derivative of the first kind can be written as

$$T_n'(x)=nU_{n-1}(x), n=1,\ 2,...$$

Chebyshev polynomials of first kind are denoted by T_n and the first several polynomials are listed below:

$$T_0(x)=1, T_1(x)=x, T_2(x)=2x^2-1, T_3(x)=4x^3-3x,$$

$$T_4(x)=4x^3-3x,$$

A Chebyshev polynomial can be found using the previous two polynomials by the recursive formula

$$T_{n+1}(x)=2xT_n(x)-T_{n-1}(x), \text{ for } n\geq 1$$

The leading coefficient of the T_n is 2^{n-1} for $n\geq 1$. These polynomials are orthogonal with respect to the weight function $\frac{1}{\sqrt{1-x^2}}$ on the interval [-1,1].

$$\int_{-1}^{1}\frac{T_i(x)T_j(x)}{\sqrt{1-x^2}}=\begin{cases}0, i\neq j,\\ \pi, i=j=0\\ \pi/2, i=j\neq 0\end{cases}$$

T_n denotes the Chebyshev polynomial of degree n. It has the root n. The node can be calculated by

$$x_i=\cos\left(\frac{i-1/2}{n}\right)\pi, \text{ for } i=1,2,...,n.$$

A function can be approximated by an nth degree polynomial $P_n(x)$ expressed in terms of $T_0,\ T_1,\ ...,T_n$

$$P_n(x)=C_0T_0(x)+C_1T_1(x)+...+C_nT_n(x),$$

$$C_j=\frac{2}{n}\sum_{k=1}^{n+1}f(x_k)T_j(x_k), j=0,1$$

and $x_k, k=1,...,n+1$ are zeros of T_{n+1} since

$$T_j(x)=\cos(j\arccos x),$$

$$T_j(x_k)=\cos(j\arccos x_k)=\cos\left(\frac{j(k-1/2)}{n+1}\pi\right),$$

Let f(x) be continuous function defined on the interval [-1,1], that is to be approximated by a polynomial P_n. One can measure how good the approximation is of f(x) by the uniform norm,

$$\left\|f-P_n\right\|=\max_{-1\leq x\leq 1}\left|f(x)-P_n\right|.$$

The paper is organized as follows: section 1 is of an introductory nature. The mathematical formulation was described in section 2. Stability analysis of the new methods was discussed in section 3. In section 4, some numerical experiments and results showing the relevance of the new methods are discussed. Finally, in section 5, some conclusions are drawn.

3.0 Mathematical Formulation

Assuming an approximate solution to (1) by taking the partial sum of Chebyshev polynomia

$$y(x)=\sum_{r=0}^{m+n-1}a_rT_r(x), x_n\leq x\leq x_{n+r} \qquad (2)$$

where x can be used only after certain transformation . The second derivative of (2) gives

$$y''(x)=\sum_{r=0}^{m+n-1}a_r\mathrm{T}_r''(x) \qquad (3)$$

Substituting (3) into (1) gives

$$\sum_{r=0}^{m+n-1}a_rT_r''(x)=f(x, y(x), y'(x)), x_n\leq x\leq x_{n+r} \qquad (4)$$

where $T_r(x)$ is the Chebyshev polynomial of degree r, valid in $x_n \le x \le x_{n+r}$ and a_r's are real unknown parameters to be determined and where m, n are the collocation and interpolation points respectively. Equation (2) and (4) will be adopted for the derivation of the proposed schemes and m, n are the collocation and interpolation points respectively. Equation (2) and (4) will be adopted for the derivation of the proposed schemes is the sum number of collocation and interpolation points. The well-known Chebyshev polynomials are defined on the interval [-1,1].

2.1 Derivation of the Continuous Coefficient Implicit Methods

In this section, the continuous formulation of the general linear multistep method $\overline{y(x)}$ of degree $r = m+n-1, m>0, n>0$, is obtained. Equation (2) and (4) were adopted for the derivation of the proposed schemes. In order to be able to use the polynomial in the $[x_r, x_{n+r}] \subseteq [-1,1]$ some necessary transformation were made. Two cases were considered two-step and three-step methods.

CASE 1: Two-Step Implicit Method

Let $m+n-1 = 4$ in equation (2) and (4) give the following partial sums

$$y(x) = a_0 + \left(\left(\frac{x-x_n}{h}\right)-1\right)a_1 + \left(2\left(\frac{x-x_n}{h}\right)^2 - 4\left(\frac{x-x_n}{h}\right)+1\right)a_2$$
$$+\left(4\left(\frac{x-x_n}{h}\right)^3 - 12\left(\frac{x-x_n}{h}\right)^2 + 9\left(\frac{x-x_n}{h}\right)-1\right)a_3$$
$$+\left(8\left(\frac{x-x_n}{h}\right)^4 - 32\left(\frac{x-x_n}{h}\right)^3 + 40\left(\frac{x-x_n}{h}\right)^2 - 16\left(\frac{x-x_n}{h}\right)+1\right)a_4 \qquad (5)$$

$$f(x,y,y') = 4a_2 + \left(24\left(\frac{x-x_n}{h}\right)-24\right)a_3 + (96\left(\frac{x-x_n}{h}\right)-192\left(\frac{x-x_n}{h}\right)+80)a_4 \qquad (6)$$

Collocating (6) at the grid points, $x = x_{n+i}, i = 0, 1, 2$ and interpolating (5) at the grid points $x = x_{n+i}, i = 0,1$ yields the following system of equations expressed as $AX = B$.

$$\begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 4 & -24 & 80 \\ 0 & 0 & 4 & 0 & -16 \\ 0 & 0 & 4 & 24 & 80 \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+1} \\ f_n \\ f_{n+1} \\ f_{n+2} \end{pmatrix} \qquad (7)$$

Solving (7) using Gaussian elimination method, we obtained the parameters $a_z's$ and substituting into equation (5) yields the continuous scheme of the form:

$$\overline{y(x)} = \left(1-\left(\frac{x-x_n}{h}\right)\right)y_n + \left(\frac{x-x_n}{h}\right)y_{n+1}$$
$$+\frac{h^2}{24}\left(-7\left(\frac{x-x_n}{h}\right)+12\left(\frac{x-x_n}{h}\right)^2 - 6\left(\frac{x-x_n}{h}\right)^3 + \left(\frac{x-x_n}{h}\right)^4\right)f_n$$
$$-\frac{h^2}{12}\left(3\left(\frac{x-x_n}{h}\right)+4\left(\frac{x-x_n}{h}\right)^3 + \left(\frac{x-x_n}{h}\right)^4\right)f_{n+1}$$
$$+\frac{h^2}{24}\left(\left(\frac{x-x_n}{h}\right)-2\left(\frac{x-x_n}{h}\right)^3 + \left(\frac{x-x_n}{h}\right)^4\right)f_{n+2} \qquad (8)$$

Evaluating (8) at the point $x = x_{n+i}, i = 2$, we recovered Numerov's formula (see [12], pp 255) of the form:

$$y_{n+2} - 2y_{n+1} + y_n = \frac{h^2}{12}(f_n + 10f_{n+1} + f_{n+2}) \qquad (9)$$

Discrete scheme (9) is consistent, zero-stable and of order $p=3$, with the absolute error $C_{p+2} = -\frac{1}{240}$. The first derivative of (8) was found and evaluated at points $x = x = x_{n+i}, i = 0, 1, 2,$, we obtained the following derivatives schemes.

$$hy'_n = -y_n + y_{n+1} - \frac{h^2}{24}(7f_n + 6f_{n+1} - f_{n+2}) \quad (10)$$

$$hy'_{n+1} = -y_n + y_{n+1} + \frac{h^2}{24}(3f_n + 10f_{n+1} - f_{n+2}) \quad (11)$$

$$hy'_{n+2} = -y_n + y_{n+1} + \frac{h^2}{24}(f_n + 26f_{n+1} + 9f_{n+2}) \quad (12)$$

Combining (9) and (10) to form block and solving by using matrix inversion method yields:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_{n+1} \\ y_{n+2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} y_{n-1} \\ y_n \end{pmatrix} + h\begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} y'_{n-1} \\ y_n' \end{pmatrix}$$

$$+ h^2\begin{pmatrix} 0 & \frac{7}{24} \\ 0 & \frac{2}{3} \end{pmatrix}\begin{pmatrix} f_{n-1} \\ f_n \end{pmatrix} + h^2\begin{pmatrix} \frac{1}{4} & -\frac{1}{24} \\ \frac{4}{3} & 0 \end{pmatrix}\begin{pmatrix} f_{n+1} \\ f_{n+2} \end{pmatrix} \quad (13)$$

Writing (13) explicitly, we have

$$y_{n+1} = y_n + y'_n + \frac{h^2}{24}(7f_n + 6f_{n+1} - f_{n+2}) \quad (14)$$

$$y_{n+2} = y_n + 2y'_n + \frac{h^2}{3}(2f_n + 4f_{n+1}) \quad (15)$$

The block method is of order $p = (3, \ 3)^T$ with the error constant $C_{p+2} = \left(\frac{1}{45}, \ \frac{2}{45}\right)^T$, the derivative schemes are:

$$y'_{n+1} = y_n + y'_n + \frac{h}{8}(5f_n + 2f_{n+1} + f_{n+2}) \quad (16)$$

$$y'_{n+2} = 2y'_n + \frac{h}{12}(15f_n + 16f_{n+1} + 11f_{n+2}) \quad (17)$$

Equations (14)-(17) are then applied as simultaneous integrator to (1).

CASE 2: The Three-steps Implicit Method

Let $m + n - 1 = 5$ in the partial sums of (2) and (3) gives. Collocating the partial sums of (4) at all the grid points, $x = x_{n+i},\ i = 1, 2, 3$ and interpolating the partial sums of (2) at the first two grid points $x = x_{n+i},\ i = 0, 1.$ These yield the following system of equations expressed as $AX = B$

$$\begin{pmatrix} 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{1}{3} & -\frac{7}{9} & \frac{23}{27} & \frac{17}{81} & \frac{241}{243} \\ 0 & 0 & \frac{16}{9} & \frac{32}{3} & \frac{320}{9} & \frac{-800}{9} \\ 0 & 0 & \frac{16}{9} & \frac{-32}{9} & \frac{-64}{27} & \frac{3040}{243} \\ 0 & 0 & \frac{16}{9} & \frac{32}{9} & \frac{-64}{27} & \frac{-3040}{243} \\ 0 & 0 & \frac{16}{9} & \frac{32}{3} & \frac{320}{9} & \frac{800}{9} \end{pmatrix}\begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix} = \begin{pmatrix} y_n \\ y_{n+1} \\ f_n \\ f_{n+1} \\ f_{n+2} \\ f_{n+3} \end{pmatrix} \quad (18)$$

Solving (18) and substituting the parameter a_z's into the partial sums of (2) yields the continuous scheme

$$\overline{y(x)} = \left(1-\left(\frac{x-x_n}{h}\right)\right)y_n + \left(\frac{x-x_n}{h}\right)y_{n+1}$$

$$-\frac{h^2}{360}\left(97\left(\frac{x-x_n}{h}\right)-177\left(\frac{x-x_n}{h}\right)^2+110\left(\frac{x-x_n}{h}\right)^3-30\left(\frac{x-x_n}{h}\right)^4\right)f_n$$

$$-\frac{h^2}{120}\left(38\left(\frac{x-x_n}{h}\right)-60\left(\frac{x-x_n}{h}\right)^3+25\left(\frac{x-x_n}{h}\right)^4-3\left(\frac{x-x_n}{h}\right)^5\right)f_{n+1}$$

$$+\frac{h^2}{120}\left(13\left(\frac{x-x_n}{h}\right)-30\left(\frac{x-x_n}{h}\right)^3+20\left(\frac{x-x_n}{h}\right)^4-3\left(\frac{x-x_n}{h}\right)^5\right)f_{n+2}$$

$$+\frac{h^2}{360}\left(-8\left(\frac{x-x_n}{h}\right)+20\left(\frac{x-x_n}{h}\right)^3-15\left(\frac{x-x_n}{h}\right)^4+3\left(\frac{x-x_n}{h}\right)^5\right)f_{n+3} \qquad (19)$$

The discrete scheme of (19) is obtained as:

$$y_{n+3} = -2y_n + 3y_{n+1} + \frac{h^2}{24}\left(4f_n + 42f_{n+1} + 24f_{n+2} + 2f_{n+3}\right) \qquad (20)$$

The order of the (20) is $p = 4$, with the error constant $C_{p+2} = \frac{1}{15360}$. Evaluating (19) at the points $x = x_{n+i}, i = 1, 2,$ we obtained the following discrete schemes.

$$y_{n+3} = -2y_n + 3y_{n+1} + \frac{h^2}{24}\left(4f_n + 42f_{n+1} + 24f_{n+2} + 2f_{n+3}\right) \qquad (21)$$

$$y_{n+2} = -y_n + 2y_{n+1} + \frac{h^2}{12}\left(f_n + 10f_{n+1} + f_{n+2}\right) \qquad (22)$$

Differentiating and evaluating (19) at the grid points $x = x = x_{n+i}, i = 0, 1, 2, 3$, we obtained the following derivative schemes.

$$hy'_n = -y_n + y_{n+1} - \frac{h^2}{360}\left(97f_n + 114f_{n+1} - 39f_{n+2} + 8f_{n+3}\right) \qquad (23)$$

$$hy'_{n+1} = -y_n + y_{n+1} + \frac{h^2}{360}\left(38f_n + 171f_{n+1} - 36f_{n+2} + 7f_{n+3}\right) \qquad (24)$$

$$hy'_{n+2} = -y_n + y_{n+1} + \frac{h^2}{360}\left(23f_n + 366f_{n+1} + 159f_{n+2} - 8f_{n+3}\right) \qquad (25)$$

$$hy'_{n+3} = -y_n + y_{n+1} + \frac{h^2}{360}\left(38f_n + 291f_{n+1} + 444f_{n+2} + 127f_{n+3}\right) \qquad (26)$$

In block form, we have

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}y_{n+1}\\y_{n+2}\\y_{n+3}\end{pmatrix} = \begin{pmatrix}0&0&1\\0&0&1\\0&0&1\end{pmatrix}\begin{pmatrix}y_{n-1}\\y_{n-2}\\y_n\end{pmatrix} + h\begin{pmatrix}0&0&1\\0&0&2\\0&0&3\end{pmatrix}\begin{pmatrix}y'_{n-1}\\y'_{n-2}\\y'_n\end{pmatrix}$$

$$+h^2\begin{pmatrix}0&0&\frac{97}{360}\\0&0&\frac{28}{45}\\0&0&\frac{39}{40}\end{pmatrix}\begin{pmatrix}f_{n-1}\\f_{n-2}\\f_n\end{pmatrix} + h^2\begin{pmatrix}\frac{19}{60}&-\frac{13}{120}&\frac{1}{45}\\\frac{22}{15}&-\frac{2}{15}&\frac{2}{45}\\\frac{27}{10}&\frac{27}{40}&\frac{3}{20}\end{pmatrix}\begin{pmatrix}f_{n+1}\\f_{n+2}\\f_{n+3}\end{pmatrix} \qquad (27)$$

Writing (27) explicitly

$$y_{n+1} = y_n + y_n' + \frac{h^2}{360}\left(97 f_n + 114 f_{n+1} - 39 f_{n+2} + 8 f_{n+3}\right) \quad (28)$$

$$y_{n+3} = y_n + 3y_n' + \frac{h^2}{40}\left(39 f_n + 108 f_{n+1} + 27 f_{n+2} + 6 f_{n+3}\right) \quad (29)$$

$$y_{n+2} = y_n + 2y_n' + \frac{h^2}{45}\left(28 f_n + 66 f_{n+1} - 6 f_{n+2} + 2 f_{n+3}\right) \quad (30)$$

$$y'_{n+1} = y_n' + \frac{h}{24}\left(9 f_n + 19 f_{n+1} - 5 f_{n+2} + f_{n+3}\right) \quad (31)$$

$$y'_{n+2} = y_n' + \frac{h}{3}\left(f_n + 4 f_{n+1} + f_{n+2}\right) \quad (32)$$

$$y'_{n+3} = y_n' + \frac{h}{8}\left(3 f_n + 9 f_{n+1} + 9 f_{n+2} + 3 f_{n+3}\right) \quad (33)$$

The order of the block method (28)-(30) is $p = (4, \; 4, \; 4)^T$, with the error constant $C_{p+2} = \left(-\frac{7}{480}, -\frac{1}{30}, -\frac{9}{160}\right)^T$.Equations (28)-(33) are then applied as simultaneous integrator to the initial value problems (1).

4.0 Stability Analysis

To investigate the stability properties of methods for solving the initial value problem (1), the three-step block method (27) was normalized for easy analysis [13]. The first characteristic polynomial is of the form:

$$\rho(\lambda) = \det[\lambda A^0 - A^1]$$

$$= \det\left[\lambda\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} - \begin{bmatrix}0 & 0 & 1\\ 0 & 0 & 1\\ 0 & 0 & 1\end{bmatrix}\right]$$

$$= \det\begin{bmatrix}\lambda & 0 & 0\\ 0 & \lambda & 0\\ 0 & 0 & \lambda\end{bmatrix}$$

$$\rho(\lambda) = \lambda^2(\lambda - 1) \quad = 0$$

$$\Rightarrow \quad \lambda_1 = \lambda_2 = 0, \lambda_3 = 1,$$

This satisfies the condition for method (27) to be zero –stable. Hence the block method is zero stable. The block method is also consistent, as it has the order p greater than 1. Hence the convergence of the method is asserted as in the theorem 3.1 below.

Theorem 3.1:([14])

The necessary and sufficient conditions for a linear multistep to be convergent are that it be consistent and zero-stable

4.1 Region of Absolute Stability of the method

The Locus method is used to determine the region of absolute stability (13) and (27). The boundary locus method

$$h(\theta) = \frac{\rho(r)}{\sigma(r)} = \frac{\rho\left(e^{i\theta}\right)}{\sigma\left(e^{i\theta}\right)}$$

where $\rho(r)$ and $\sigma(r)$ are the first and second characteristics polynomial respectively as:

$$r = e^{i\theta} = \cos\theta + i\sin\theta$$

Using a Matlab program, the absolute stability region of the new methods is plotted below. Below are the graphical representations of stability domain of the block methods.

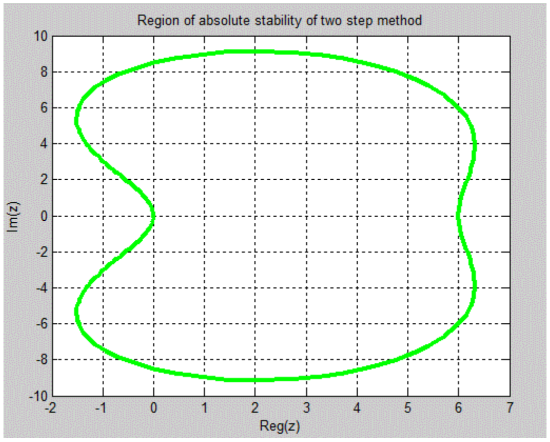

Figure 1: Stability domain of block method (13).

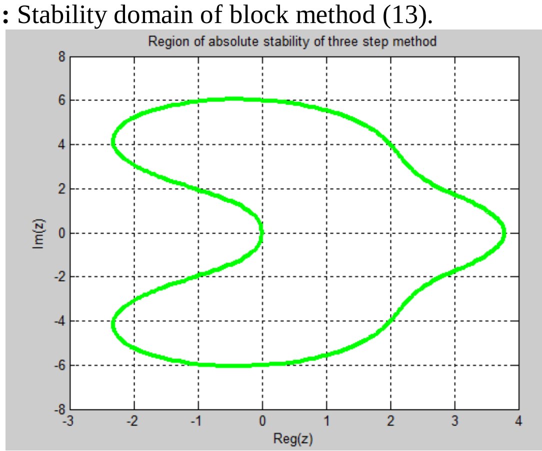

Figure 2: Stability domain of block method (27).

5.0 Numerical Experiments and Results.

In this section, the new methods were implemented as block method and applied on three problems. The absolute errors computed are compared with those obtained in [4] and [8] which are implemented in predictor-corrector mode. The results for the problems are shown in tables 1, 2 , 3 and 4. We also considered nonlinear and real life problem 4.

Problem 1:

Consider a non linear second order problem.

$$y'' = x(y')^2, y(0) = 1,\ y'(0) = \frac{1}{2},\ h = .0025,$$

Exact solution: $y(x) = 1 + \frac{1}{2}\log\left(\frac{2+x}{2-x}\right)$

Problem 2:

Consider second order problem of [4]

$$y'' = 2y - y',\ y(0) = 0,\ y'(0) = 1,\ h = 0.1$$

Exact solution: $y(x) = \dfrac{e^x - e^{-2x}}{3}$

Problem 3:

Consider a nonlinear second order problem of [8].

$$y'' - 3y' = 8e^{2x},\ y(0) = 1,\ y'(0) = 1,\ h = 0.005$$

Exact solution: $y(x) = -4e^{2x} + 3e^{3x} + 2$

Problem 4: A solid cylinder partially submerged in water having weight density 62.5Ib/ft^3, with its axis vertical, oscillates up and down within a period of 0.6 sec. With ρ = 62.5Ib/ft^3 and m=2/32, we have

$$y'' + 1000\pi r^2 y = 0, y(0) = 0, y'(0) = 1$$

$Exact \ = \frac{\sqrt{10}}{100}\frac{\sin(10\sqrt{10\pi}y)}{\sqrt{y}r}$Let r = 0.187ft.

Table 1: The y-exact , y-approximate and error in problem 1 using two-step method

X	y-exact solution	y-computed for k=2	Error in k=2
0.1	1.050041729	1.050041729	6.64e-11
0.2	1.100335348	1.100335347	5.50e-10
0.3	1.151140436	1.151140434	1.93e-09
0.4	1.202732554	1.202732549	4.81e-09
0.5	1.255412812	1.255412802	1.00e-08
0.6	1.309519604	1.309519585	1.87e-08
0.7	1.365443754	1.365443721	3.28e-08
0.8	1.42364893	1.423648875	5.49e-08
0.9	1.484700279	1.484700189	8.94e-08
1	1.549306144	1.549306001	1.44e-07

Table 2 : Accuracy comparison of two-step method and [4] for problem2

X	y-exact solution	y-computed for k=2	Error in k=2	Error in [4]
0.1	0.095480055	0.95478889	1.17e-06	9.77e-06
0.2	0.183694237	0.183671111	2.31e-05	1.83e-05
0.3	0.267015724	0.26707862	6.29e-05	6.51e-05
0.4	0.347498578	0.347672668	1.74e-04	1.02e-04
0.5	0.426947277	0.427253455	3.06e-04	
0.6	0.506974863	0.507431469	4.57e-04	
0.7	0.589051915	0.589676093	6.24e-04	
0.8	0.674548137	0.675356932	8.09e-04	
0.9	0.764768074	0.765779322	1.01e-03	
1	0.860982182	0.86224398	1.26e-03	

In tables 2, the maximum absolute error of the new two-step compared favourably with those of [4] for problems 2.

Table 3 : The y-exact, y-computed and error in three-step method for problem 1.

X	y-exact solution	y-computed for k=3	Error in k=3
0.1	1.050041729	1.050041729	6.55e-11
0.2	1.100335348	1.100335347	5.50e-10
0.3	1.151140436	1.151140434	1.93e-09
0.4	1.202732554	1.202732549	4.80e-09
0.5	1.255412812	1.255412802	1.00e-08
0.6	1.309519604	1.309519585	1.87e-08
0.7	1.365443754	1.365443722	3.27e-08
0.8	1.42364893	1.423648875	5.48e-08
0.9	1.484700279	1.482742666	8.80e-08
1	1.549306144	1.549306001	1.44e-07

Table 4: Accuracy comparison of three-step method and [8] for problem 3

X	y-exact	y-approximate	**Error in k=3**	**Error in [8]**
0.005	1.005138526	1.005138525	1.14E-11	3.15900E(-07)
0.01	1.010558242	1.010558241	3.70E-10	1.27090E(-06)
0.015	1.016265444	1.016265441	2.82E-09	8.65540E(-06)
0.02	1.022266543	1.022266493	5.01E-08	2.59148E(-05)
0.025	1.028568067	1.028567953	1.14E-07	3.395058E(-05)
0.03	1.035176665	1.035176469	1.96E-07	5.990417E(-05)
0.035	1.042099106	1.042098809	2.97E-07	8.884833E(-05)
0.04	1.049342284	1.049341868	4.16E-07	

In tables 4, the maximum absolute error of the new three-step is higher and more accurate than those of [8] for problems 3.

Table 5: y-exact, y-computed and error in problem 4.

X	y-exact	y-computed	Error in k=2
0.1	0.082670103	0.08267004	6.27E-08
0.2	0.08253607	0.082535942	1.28E-07
0.3	-0.000267849	-0.000267853	3.70E-09
0.4	-0.082803485	-0.082803233	2.51E-07
0.5	-0.082401386	-0.082401067	3.19E-07
0.6	0.000535696	0.000535701	5.42E-09
0.7	0.082936214	0.082935773	4.40E-07
0.8	0.082266053	0.082265542	5.11E-07
0.9	-0.000803539	-0.000803546	7.40E-09
1	-0.083068289	-0.083067659	6.30E-07

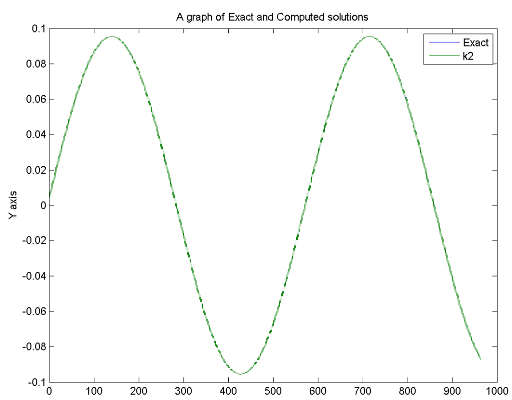

Figure 3: Graph of y-exact and y-computed of problem 4.

6.0 Conclusion

The newly derived collocation approximation with Chebyshev basis functions approach to self-starting implicit LMMs for stiff and non stiff equations were implemented in block mode. The block methods have the advantages of being self -starting, are uniformly of the same order of accuracy and do not need predictors, having good accuracy as shown on numerical results of tables 1, 2, 3, 4 and 5. The block method is recommended for general purposed use. Finally, the stability domain of the block methods of two-step and three-step method were presented in figure 1 and 2. Maple and Matlab software package were employed to generate the schemes and results.

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