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A Simple Stochastic Algorithm for Solution to PDE with Financial Application

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Abstract

This paper presents a stochastic algorithm in a drifted financial derivative system for pricing an American options under the Black-Scholes model. With finer discretization, space nodes and time nodes, we demonstrate that the drifted financial derivative system can be efficiently and easily solved with high accuracy, by using a stochastic approximation method which proves to be faster in pricing an American options. An illustrative example is given in concrete setting.

Keywords: Financial PDE; Stochastic algorithm; Drifted system; Option pricing; Spatial discretization.
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1.0 Introduction

As it is well known, the iteration methods play a fundamental role in numerical analysis due to their simple structure and flexibility in practical computation. Theoretically, all kinds of equations including functional equation(s) can be solved by using iteration method. For instance, the solution of first-order ordinary differential equation(s) (ODE) can be defined as the limit of the Picarditeration sequence. However the Picard sequence actually does not work in solving differential equations except for the very simple ODE, because it requires to compute integration repeatedly. An iteration method works in solving differential equations only if special features or special classes of equations are addressed, for which the solution can be obtained by a small number of iterations.

For numerical approximations, the most popular numerical methods for pricing American options can be classified to lattice method, Monte Carlo simulation and finite difference method. Sure, besides finite difference methods, there are other popular numerical method based on discretization for solving PDEs like finite element method, boundary element method, spectral and pseudo-spectral methods and etc. Here we just use finite difference to stand for methods of this kind. In fact, finite difference method ranks as the most popular one among its kind in financial engineering. The lattice method is simple and still widely used for evaluating American options. It was first introduced by Cox et al [1], and the convergence of the lattice method for American options is proved by Amin and Khanna [2]. The Monte Carlo method is also popular among financial practitioners. It is appealing, simple to implement for pricing European options, and especially has advantage of pricing multi-asset options. For pricing American options, Monte Carlo method requests some further modification due to the early – exercise feature. Fu [3,4]Priced American- style options by using Monte Carlo method in conjunction with gradient-based optimization techniques. Duck et al[5] proposed a technique which generates monotonically varying data to enhance the accuracy and reliability of Monte Carlo-based method in handling early exercise features.

The finite difference method for pricing American options was first presented in[6,7,8]. Jaillet et al. [9] showed the convergence of the finite difference method. A comparism of different numerical methods for American options pricing was discussed in[10, 11]. Generally, there still exist some difficulties in using these numerical methods. For finite difference method, the difficulty arises from the early exercise property, which changes the original Black-schools equation to an inequality that cannot be solved via fractional finite difference process. Therefore, finding the early exercise boundary prior to spatial discretization (discretization on underlying asset) is a must in each time step. Horng et al.[12] proposed a simple numerical method base on finite difference and method of lines to overcome this difficulty in American option valuation.

Although the early exercise boundary prior to spatial discretization in each time step has been established, another approach used by Osu and Solomon [13] is proposed in this paper based on the fact that financial derivative experience a drift which hardly can be brought to equilibrium state. The analyses are made based on the discretization of Black-Scholes equation using central finite-difference approximation into first-order ordinary differential equation and later transformed to a drifted

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financial derivative system. We solve the resulting drifted financial derivation system by employing a stochastic algorithm described and analyzed in [13] where each iteration requires the adjustment of the drift parameter based on the dividend yield.

The outline of the paper is the following:In section 2 we review modeling of Black-Scholes, the partial differential equation which financial derivative have to satisfy and formulate Linear Complementary Problem (LCP) for an American option. In section, 3, we discretize the generic PDE into LCP and drift financial derivative system. A stochastic algorithm is formulated in section 4. Numerical experiments are presented in section 5 and conclusions are given in section6.

2.0 Option Pricing Model

Here, we consider the Black and Scholes Model [14] and Merton [15] and the partial differential equation which financial derivative (stock) have to satisfy. The Black-Scholes Model assumes a market consisting of a single risky asset (S) and a risky-free bank account (r). This market is given by the equations;

$$dS = \mu S dt + \sigma S dz \quad (1)$$

$$dB = rBdt\,. \quad (2)$$

Here(1) is a geometric Borwnian-Motion and (2) a non-stochastic. S is a Brownian-Motion, Z is a Wiener process μ is a constant parameter called the drift. It is a measure of the average rate of growth of the asset price. Meanwhile,σ , is a deterministic function of time. When σ is constant, (1) is the originalBlack-Scholes Model of the movement of a security, S. In this form $\mu$is the mean return of S, and σ is a variance. The quantity dZ is arandom variable having a normal distribution with mean 0 and variance dt.

$$dZ \propto N(0, (\sqrt{dt})^2).$$

For each interval $dt,\ dZ$ isa sample drawn from the distribution $N(0, (\sqrt{dt})^2)$, this is multiplied by σ to produce the term σdZ. The value of the parameters μ and σ may be estimated from historical data.

Under the usual assumptions, Black and Scholes [14] and Merton [15] have shown that the worth$V$ of any contingent claim written on a stock, whether it is American or European, satisfies the famous Black-Scholes equation:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\,\frac{\partial V}{\partial S} - rV = 0\,. \quad (3)$$

Where volatility σ, the risk-free rate r, and dividend yield q are all assumed to be constants. The value of any particular contingent claim is determined by the terminal and boundary conditions. For an American option, notice that the PDE only holds in the not-yet-exercised region. At the place where the option should be exercised immediately, the equality sign in (3) would turn into an inequality one. That means the option value $V(S,t)$ at each time follows either $V(S,t) = \wedge\ (S,t)$ for the early exercised region or (3) for the not-yet-exercised region, where $\wedge\ (S,t)$ is the payoff of an American option at time t.

The generic form of (3) is derived by the change of variable $\tau = T - t$ to

$$\frac{\partial V}{\partial \tau} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (r-q)\,S\,\frac{\partial V}{\partial S} + rV = LV \quad (4)$$

where $V(.,\tau) \equiv V(.,T-\tau),\ \ \sigma\,(.,\tau) \equiv\ \sigma\,(.,T-\tau),\ \ \tau = 0\ to\ \tau = T$

$S_{min} < S < S_{max}$, subject to the initial condition $V(S,0) =\ \wedge(S)$.

For the computations, the unbounded domain is truncated to

$$(S,t)\ \epsilon\ \ (0,S)\ x\ \ (0,T] \quad (5)$$

with sufficiently large $S\ \equiv\ S_{max}$.

The worth V of an American option under Black-Scholes model satisfies an LCP

$$\begin{cases} LV\ \geq 0 \\ V\ \geq \wedge \\ (LV)(V - \wedge) = 0\,, \end{cases} \quad (6)$$

we impose the boundary conditions

$$\begin{cases} V(0,t) = 0 \\ V(S,t) = \wedge\,(S),\ \ S\epsilon\ (0, S_{max})\ . \end{cases} \quad (7)$$

Beyond the boundary $S =\ S_{max}$, the worth$V$ is approximated to be the same as the payoff $\wedge$, that is $V\ (S,t) = \wedge\,(S)$ for $S \geq\ S_{max}$.

3.0 Discretizing the financial PDE for American option

American options can be exercised at any time before expiry. Formally, the value of an American put option with a strike price k is

$$V(0,k) = \sup\,(0 \leq \tau^* \leq T{:}\,E(e^{-\tau\tau^*}(k - S_{\tau*})^+).$$

The optimal exercise time τ^* is the value that maximizes the expected payoff - any scheme to price an American must calculate this.

For American options with payoff $\wedge(s)$, the equivalent of equation (4) is

$$\begin{bmatrix} \frac{\partial V}{\partial \tau} - \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (r-q)S\,\frac{\partial V}{\partial S} + rV \geq 0 \\ V(S,T) \geq \wedge(S) \end{bmatrix} \quad (8)$$

$$\left[\frac{\partial V}{\partial \tau} - \frac{1}{2}\,\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - (r-q)s\frac{\partial V}{\partial s} + rV\right][V - \wedge(S)] = 0.$$

Consider a uniform spatial mesh on the interval$[s_{min},\ s_{max}]$:

$S_j = S_{min} + j\delta S,\ j = O, 1, \ldots, n+1$, where

$$\delta s = \frac{S_{max} - S_{min}}{n+1},\ S_{max} = S_0 \exp\left[\left(r - q - \frac{\sigma^2}{2}\right)T + 6\sigma\sqrt{T}\right]. \quad (9)$$

The truncated domain $\widehat{D}$ has the lower bound $S_{min} = 0$ and upper boundS_{max} as in (9).

Replacing all derivatives with respect to S by their central finite-difference approximations, we obtain the following approximation to the Black-Scholes PDE (8)

$$\frac{\partial V(\tau,S)}{\partial \tau} = \frac{1}{2}\ \sigma^2(S)\,S^2\frac{V(\tau,\ S+\delta S) - 2V(\tau,S)\ +\ V\,(\tau, S-\delta S)}{\delta S^2}$$
$$+ (r-q)\,S\ \frac{V(\tau,\ S+\delta S) -\ V\,(\tau,\ S-\delta S)}{2\delta S} - rV\,(\tau,S) + \mathrm{O}(\delta S^2). \quad (10)$$

Let $V_j(\tau)$ denote the semi-discrete approximation to $V(\tau, S_j)$. Applying (10) at each internal node S_j, we obtain the following system of first-order ordinary differential equations;

$$\frac{dV_j(\tau)}{d\tau} = \frac{1}{2}\left(\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 - \frac{(r-q)S_j}{\delta S}\right)V_{j-1}(\tau) - \left(-\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 -\ r\right)V_j(\tau)$$
$$+\ \frac{1}{2}\left(\left(\frac{\sigma(S_j)S_j}{\delta S}\right)^2 +\ \frac{(r-q)S_j}{\delta S}\right)V_{j+1}(\tau), j = 1, 2, \ldots, n\,; \quad (11a)$$

with discretized form given as

$$\frac{dV_j(\tau)}{d\tau} = L_{j,j-1}V_{j-1}(\tau) - L_{j,j}V_j(\tau) + L_{j,j+1}V_{j+1}(\tau). \quad (11b)$$

System (11) has n equation in $n+2$ unknown functions,

$V_0(\tau), V_1(\tau), \ldots, V_n(\tau), V_{n+1}(\tau)$. Using the boundary conditions we have the functions $V_0(\tau)$ and $V_{n+1}(\tau)$ which respectively approximate the solution at the boundary nodes $S_0 = S_{min}$ and $S_{n+1} =\ S_{max}$ .As a result, the system of differential equations (11) can be written as the following matrix-vector differential equation with an n-by-n tri-diagonal coefficient matrix L whose entries are defined in (11)

$$\frac{dV(\tau)}{d\tau} = LV(\tau) +\ G(\tau), \quad (12)$$

Subject to the initial condition

$$V(0) =\ \Lambda := [\Lambda(S_1)\ \ , \Lambda(S_2), \ldots, \Lambda(S_{n)})]^T. \quad (13)$$

Here we use the notation:

$$L = \begin{pmatrix} L_{11} & L_{12} & 0 & \cdots & 0 & 0 \\ L_{21} & L_{22} & L_{23} & \cdots & 0 & 0 \\ 0 & L_{32} & L_{33} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & L_{n-1,n-1} & L_{n-1,n} \\ 0 & 0 & 0 & \cdots & L_{n,n-1} & L_{n,n} \end{pmatrix}, V(\tau) = \begin{pmatrix} V_1(\tau) \\ V_2(\tau) \\ \vdots \\ V_{n-1}(\tau) \\ V_n(\tau) \end{pmatrix}.$$

The vector $G(\tau) \epsilon\ R^n$ is given by

$$\left[\left(\frac{\sigma^2(S_0)S_0^2}{2\delta S^2} -\ \frac{(r-q)S_0}{2\delta S}\right)V_0(\tau), 0\ , \ldots, 0, \left(\frac{\sigma^2(S_{n+1})S_{n+1}{}^2}{2\delta S^2} +\ \frac{(r-q)S_{n+1}}{2\delta s}\right)\ V_{n+1}(r)\right]^T.$$

$G\ (\tau)$ contains boundary values of the mesh solution.

The spatial discretization leads to:

Semi-discrete LCP, according to [16] from (9), (12) and (13), we have

$$\begin{cases} L^j V^{j+1}\ \geq\ g^j \\ V^{j+1}\ \geq\ \Lambda \qquad , \\ (V^{j+1}\ - \Lambda)^T(L^j V^{j+1} -\ g^j) = 0 \end{cases} \quad (14)$$

where L is n-by-n tri-diagonal coefficient matrix, g is a vector resulting from the second term in equation (11) V and Λ are vectors containing the grid point values of the worth V and the pay off Λ, respectively. This again must be solved at every time step. A crude approximation is to solve the system $L^j\ X =\ g^j$, then set $L^{j+1} = \max(X, \Lambda)$.

Drifted financial derivative system:
According to [17],$G(\tau)$ term in (12) can be treated as an enforced input to the financial derivative system, resulted from boundary condition, defined in (7). With zero boundary condition, equation (12) yields.

$\dot{V} = LV,$ (15)

which represents a pfaftian differential constraints (see [18] for pfaftian differential constraints) but not of kinematic nature arises from the conservation on non-zero financial derivatives. The transformed financial derivative system (15) can be re-expressed as

$L\,V = d$ (16)

System (16) represents a drifted financial derivative system with a drift term d. In such a system the derivative value V can be solved by computing the stochastic algorithm used by Osu and Solomon [13].

4.0 Formulation of Stochastic Algorithm:

We consider the finite dimensional variation problem: find $v \in D(\varphi)$
such that

$Lv + \partial\varphi(v) \ni b$ (17)

Subject to equation (14)
where (φ) is convex function,
$D(\varphi) = [v \in R^n : \varphi(v) < \infty] \neq \emptyset$, then for $v \in D(\varphi)$, the sub gradient $\partial\varphi$ of $\varphi: R^n \to R$ at v is defined as;
$\partial(\varphi) = \langle g \in R^n : f(v+t) - f(v) \geq \langle g, t\rangle\rangle \; \forall v + t \in D(\varphi).$ (18)
It is well known that if a function $f \; on \; R^n$ is differentiable, then there exists $d \,\epsilon\, R^n$ such that $f(v) - f(v_0) = (d \;, v - v_0)\; || \; v - v_0 ||$,
where $d = \frac{\partial f(v)}{\partial v}$is the gradient of the function f.
Denote $\partial f^k = \frac{\partial f(v^k)}{\partial v}, \frac{\partial^2 f(v^k)}{\partial v_r \partial v_s} = \partial^2_{r,s} f^k$, as in Okoroafor and Osu [19], we constructed a sequence of random vector $d^k \epsilon R^n$ that strongly approximate $\partial f^k = \partial f(v^k)$ for each k in the sense that

$$E\left\|{d_j}^k - \partial f^k\right\| = 0$$

and their expected Euclidean distance

$$E\left\|{d_j}^k - \partial f^k\right\|^2 = M^{-1}\sigma^2$$

is minimum so that a search in the direction of the randomsequence$\langle {d_j}^k \rangle$approximate a search through the true gradient∂f^k and this is expected to lead to the non-zero global minimizing factor if it exists. To this end, we consider the natural Taylor's expansion of a quadratic function f about point v_0 given by

$f(v) - f(v_0) = \langle \partial f(v_0), v - v_0 \rangle + {}^1/_2 \,(v - v_0) H(v_c)(v - v_0)$ (19)

where v_c is on the line segment between$v$ and v_0 and $H(V_c)$ is the Hessian of f at v_c.
Given that
$E\,\left(e(v_j)\right) = 0$ for each j.
and
$E\,\left(e(v_i)\, e(v_j)\right) = \sigma^2 \delta_{ij} \; 0 < \sigma^2 < \infty.$
Let $Y(v_1)\,, Y\,(v_2), \ldots, Y\,(v_m)$ be real-valued independent observable random variable performed on $v_1, v_2, \ldots, v_n,\; n+2 < m < \frac{1}{2}\, n(n+1)$ chosen in the neighbourhood ofv^k for a fixed K, then

$Y_j = Y\,(v_j) = \; f\,(v + \; t_j) - \; f(v_j)$

$= \langle \partial f(v^k), t_j \rangle + \frac{1}{2} \sum_{k=1} \sum_{r=1} t_{kj} t_{rj} \partial^2_{kr}\, f + e(v_j)$ (20)

is identifiable with (17) so the fixed $t_j \epsilon R^n$ satisfying $\Sigma_{i=1}^m t_{ij} = 0,\; M^{-1} \sum_{i=1}^\infty t_{ij}^2$linearizes f, Okoroafor and Osu [20] and hence the least square approximation.

$d^k = \; M^{-1} \sum_{j=1}^m t_j Y_j, \quad M = \; \sum_{j=1}^m t_j t_j'$ (21)

exist and is adequate for approximating ∂f such that Euclidean distance
$E||d^k - \; \partial f(v^k)|| \; = 0$ for each k, also yield
$E||\; d^k \; - \partial f(v^k)||^2 \; = \; M^{-1}\sigma^2.$
In the sequel we assume without loss of generality that $\sigma^2 = 1. \langle d^k \rangle$ is thus, a sequence of independently and identically distributed random vector and determines the direction of search. It follows that by letting v^0 be an initial point, the sequence of path produce by$\{v^k\}_{k=0}^\infty$ through its definition
$V^{k+1} = \; V^k - \; P^k d^k$
by successive iteration, is the trajectory of the point v^0 and any limiting point of the sequence is therefore attractor of v^0.

4.1 Getting the domain of attraction

Let $R_t^n - N(O)$ be petitioned into exclusive segment, S_j,
$j = 1,2,\dots,t,\ n < t \le 2^n$. Let$v_j$ be chosen randomly in S_j, such that $f(v_j) > 0\ \forall\ j$. Let $P_j = P(v_j = \alpha)$ be the probability that $v_j = \alpha$ so that$P_j \ge 0, \sum_{j=1}^{t} P_j = 1$ (22)

Put

$$P_j = \frac{f(v_j)}{\sum_{j=1}^{t} f(v_j)}$$

So that$\bar{v} = \sum_{j=1}^{t} v_j P_j = \sum_{j=1}^{t} \frac{v_j f(v_j)}{\sum_{j=1}^{t} f(v_j)}$. (23)

It is shown in Okoroafor and Osu [19] that if

$$\hat{v} = \bar{v} - pd,\ p > 0 \quad (24)$$

Where d is as (21), then
$f(\hat{v}) = min\{f(v_j): v_j\, \epsilon S\}$. It follows that the segment S_T if when $\hat{v}\, \epsilon S_T$ contains$v > 0$ for which $f(v)$ is minimum and hence we have
$\varphi(U_v) \subset S_T$ so that if $\langle 0\rangle$ is the attractor of the point $\bar{v}$ and
$\varphi((0) \cap \varphi(V_x)) = \emptyset$ then $N(0) \cap N(U_{\bar{v}})$.
Where $U_{v^*} = \{V^* \epsilon R^n : V^* > 0: \partial f(v^*) = 0\}$ (25)
is a way of stochastically solving problem (21). Thus we have

Lemma 4.1

Suppose that $U_{\hat{v}} \neq \emptyset$. Thus there exist a neighborhood $N(U_{\hat{v}}) \subseteq D(\partial f)$ of $U_{\hat{v}}$ such that for any initial guess $\hat{v}\, \epsilon\, \varphi\, (U_{\bar{v}})$, the non-negative minimizer$U_{\hat{v}}$ is obtained as a limit of iteratively constructed sequence $\langle v^j\rangle_{j=1}^{\infty}$ generated from $\hat{v}$by $V^{j.+1} = V^j - P^j d^j$. Then with$\hat{v}$as our starting point we search for the minimizer of f as follows:
Starting at $\hat{v}$ as in equation (24)

1. Compute the d^k as in equation (21)
2. Compute the corresponding p as specified below
3. Compute $V^{k+1} = V^k - p^k d^k$.

Has the process converge? i.e. $\left||V^{k+1} - V^k|\right| < \sigma, \sigma > 0,$ if yes then
$V^{k+1} = V^k$. If no return to (1)

Theorem 4.1

Let (p^k) be a real sequence such that

i. $p^0 = 1,\ 0 < p^k < 1, \forall\ k > 1$
ii. $\sum_{k=0}^{\infty} p^k = \infty$
iii. $\sum_{k=0}^{\infty} p^{2k} < \infty$.

Then the sequence $\langle v^k\rangle_{k=0}^{\infty}$ generated by $\hat{v} \in \varphi\,(U_0) \subseteq D\,(\partial f)$ and defined iteratively by$V^{j+1} = V^j - p^j d^j$ remain in $D\,(\partial f)$ and coverage strongly to $U_{\hat{v}}$.

Proof:

Let $b^k = p^k\, ||d^k - \partial f^k||$
Then $\langle b_k\rangle_{k=1}^{\infty}$is a sequence of independent random variable and $E(b_k) = 0$, for each k.
Noticing that the sequence of partial sums $\langle S_k\rangle_{k=1}^{\infty}, S_k = \sum_{j=1}^{k} b_j$ is a martingale. Therefore

$$E(S_k^2) = \sum_{j=1}^{k} E(b_j^2) = \sum_{j=1}^{k} P^{2j}\, E||\, d^j - \partial f^j||^2 = M^{-1}\sigma^2 \sum_{j=1}^{k} p^{2j}$$

and
$\sum E\, b_j^2 < \infty$,since $\sum_{j=1}^{k} p^{2j} < \infty$.
Hence by a version of martingale convergence theorem [21], we have
$\text{Log}_{k\to\infty} S_k = \sum_{j=1}^{\infty} b_j < \infty$,
so that
$\log_{k\to\infty} p^k||\, d^k - \partial f^k|| = 0$.

Noticing that in (17), L is positive definite so that $f(v)$ is convex and hence ∂f is monotone. But an earlier result in theory of monotone operators, due to chidume [22], shows that the sequence $\langle V^k \rangle$ generated by $V^0 \epsilon\, D(\partial f)$ and defined iteratively by
$V^{k+1} = V^k - p^k\, \partial f^k$
remain in $D(\partial f)$ and converges strongly to $\langle V^* : \partial f(v^*) = 0 \rangle$. It follows from this result that our sequence converges strongly to $U_{v^*}\ if\ \ U_{v^*} \neq 0$.

5.0 Numerical Experiment:

In our numerical example, we price American put options. The parameters for the Black-Scholes model are the same as in [16] and they are defined below:

Table 1: Estimated parameters for the Black-Scholes model

Parameter	Notation	Value
Risk free interest rate	r	0.2
Dividend yield	q	0.1
Strike price	k	7
Volatility	σ	0.3
Time to expiry	T	2
Spot price	S_0	10
Ratio of Nodes	ϑ	30

We illustrate the method in a concrete setting, using the parameter in table 1 and substitute in (10 and 11), with time nodes $3x10^3$ and space nodes $9x10^4$ satisfying the ratio of nodes ϑ as stipulated, we have the financial matrix (3 by 3 tri-diagonal coefficient matrix) .

$$L = \begin{pmatrix} 0.2 & 0.05 & 0 \\ -0.1 & 0.2 & 0.1 \\ 0 & -0.15 & 0.2 \end{pmatrix}.$$

By using the equation of total investment return;

$$r = d + q \qquad (26)$$

where r is the risk adjusted discount rate for V (the worth); q is the dividend yield (or convenience yield in case of commodities) and d is the drift (or capital gain rate). Hence $d = 0.1$ for $q = 0.1$ and $d = 0.2$ for $q = 0.0$ (No dividend yield).

From (16), we have

$$\begin{pmatrix} 0.2 & 0.05 & 0 \\ -0.1 & 0.2 & 0.1 \\ 0 & -0.15 & 0.2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 0.2 \\ 0.2 \\ 0.2 \end{pmatrix},$$

the actual solution by [16] is $V(S,t) = 1.171339$, the PDE result is 0.14459568, which Bjerksunet Stensland gives 0.14275. Approximations such as Bjerksunet and Stensland (2002) [23] are not accurate enough to test the accuracy of the finite different scheme. The above procedure starting at $V_0 = (0\ \ 0\ \ 0)$ gives after one iteration$V^*(S,t) = 1.2$, for both values of the drift. This solution is the same as in [16].

This shows that a stochastic approximation method can be used on a discretized financial PDE to price an Americanoption and European option with a considerable success.

6.0 Conclusion:

In this paper we considered a stochastic algorithm on a drifted financial derivative system for pricing American options under the Black-Scholes model. For the Black-Scholes partial derivative, we employed central finite-difference approximation into first-order ordering differential equation and later transformed to a drifted financial derivative system. In numerical experiment, we formed a financial matrix and the value of the drift parameter using Table 1.With finer discretization, space nodes, and time nodes. We demonstrate that the drifted financial derivative system can be efficiently and easily solved with stochastic approximation method. This approach in turn, yields a fast method of pricing American option.

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