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Heat Source and Thermal-Diffusion Effects on Unsteady Free Convection Boundary Layer Flow Through Vertical Porous Plates in the Presence of Magnetic Field

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Abstract

This study investigates the free-convective unsteady hydromagnetic flow of viscous incompressible electrically conducting fluid between two infinite vertical porous plates in the presence of constant heat source and thermal-diffusion under slip condition. Solutions for time dependent energy, concentration and momentumequations are obtained using theperturbation method. The effect of various parameters controlling the physical situation is discussed with the aid of line graphs. During the course of computation, an excellent result wasfoundfromvarious physical parameters embedded in the problem.

Keywords: thermal-diffusion; heat source; magnetic field; free convection

1.0 Introduction

Analysis of unsteady free convection boundary layer fluid flow problem has been subject of considerable interest not only of fundamental theoretical interest but also of practical importance as it occurs in many applied problems. Specifically, it is of great application such as geothermal reservoirs, thermal insulation, and enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, metallurgical and polymer extrusion processes. In a pioneering work, Sakiadis [1] investigated the boundary layer flow induced by a moving plate in a quiescent ambient fluid. Thereafter, many authors have investigated various aspects of the problem. Gbadeyan *et al.* [2] analyzed thermal-diffusion and diffusion-thermo effect on mixed convection boundary layer flow filled with a viscoelastic fluid in the presence of magnetic field over a stretching vertical surface.Makinde and Aziz [3] reported that, the fluid flow over a stretching surface is important in applications such as extrusion, wire drawing, metal spinning and hot rolling. Olanrewaju *et al.* [4] identified the influence of buoyancy of steady laminar boundary layer flow over a permeable flat plate in a uniform free stream, with the bottom surface of the plate is heated by convection from a hot fluid.Jashim Uddin *et al.*[5] studied numerically a 2-D steady laminar incompressible free convective boundary layer flow from a heated solid (impermeable) vertical flat plate embedded in media which is filled with nanofluid taking into account the thermal convective and momentum slip boundary conditions. Raju *et al*. [6] studied heat and mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with Newtonian heating.

Heat-source plays significant role in various physical phenomena for instant convection in earth's mantle Mckenzie*et al.* [7] and application in the field of nuclear energy and fire combustion modeling as reported by Faraboschi and Federico [8]. Crepeau and Clearksean [9] used similarity solution approach on natural convection flow with internal heat generation. Chamkha[10]investigated hydromagnetic three-dimensional free convection on a vertical stretching surface with heat generation or absorption. Singh [11] examined the effect of heat sink on Stokes problem for a porous vertical plate using finite difference method.The excellent work of Vajravelu [12]concluded that the heat source/ sink play an important role in delaying the velocity and the temperature to reach the steady state condition. Several interests have been built in the study of flow of heat generating/absorbing fluid because as the temperature differences are increased appreciably, the volumetric heat generation/absorbing term may exert strong influence on the heat transfer and transitively on the flow as reported by Jhah [13]. He further concluded that an increase in heat sink parameter (S) decreases both skin friction (τ) and Nusselt number (Nu).The effects of magnetic field, viscous dissipation and heat generation on natural convection flow of an incompressible, viscous and electrically conducting fluid along a vertical flat plate in the presence of conduction was reported by Mamun [14]. Jha and Ajibade [15] investigated the free convective flow of heat generating/absorbing fluid between vertical parallel porous plates due to periodic heating of the porous plates.

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The thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects may be significant in the areas of geosciences and chemical engineering. Various aspects dealing with the Soret effect on the combined heat and mass transfer problems have been also studied.Makinde and Olanrewaju [16] investigated the unsteady mixed convection flow past a vertical porous flat plate moving through a binary mixture in the presence of Radiative heat transfer and nth-order Arrhenius type of irreversible chemical reaction by taking into account the diffusion-thermo (Dufour) and thermo-diffusion (Soret) effects.Stanford *et al.* [17] investigated the influence of a magnetic field on heat and mass transfer by mixed convection from vertical surface in the presence of Hall, radiation, Soret (thermal- diffusion) and Dufour (diffusion-thermo) effect.Tsai and Huang[18]performed theoretical study of the steady stagnation point flow over a flat stretching surface in the presence ofspecies concentration and mass diffusion under Soret and Dufour effects. He conclude that for some kinds of mixtures (for example, H2-air) with the light and medium molecular weight, the Soret and Dufour effects play a significant role and should be taken into consideration as well. Hayat [19] studied the heat and mass transfer characteristicsin mixed convection boundary layer flow about a linearly stretching vertical surface in a porous medium filled with a viscoelastic fluid, by taking into account the diffusion-thermo (Dufour) and thermal-diffusion (Soret) effects. R. Tsai and Huang [20] carried out numerically the solutions for heat and mass transfer from natural convection flow along a vertical surface with variable heat fluxes embedded in a porous medium due to thermal-diffusion (Soret) and diffusion-thermo (Dufour) effects.

In view of the amount of works done on free convection boundary layer problems with heat/sink or with thermal-diffusion, it becomes interesting to investigate the effects of these two important activities on unsteady free convection due to imposed magnetic filed normal the plate under slip momentum boundary condition.

2.0 Mathematical Analysis

Consider a free convective flow of a viscous incompressible heat source fluid in a vertical channeldue to uniform transverse magnetic filed B_0 and periodic heating of the porous channel plates. The channel walls are taken vertically, parallel to the x -axis separated by a distance h. It isassumed that on one plate $(y=0)$, fluid is injected into the channel with certain constant velocity v_0 and that it is sucked off from theother plate $(y=h)$ at the same rate (see Fig. 1). The heat source term is assumed to be that of Foraboschi and Federico [8].

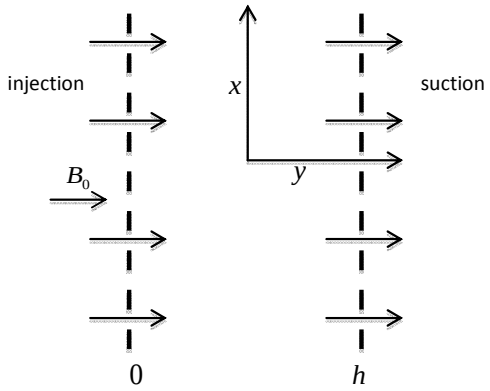

Figure 1: Geometry of the problem

Under these assumptions and usual Boussinesq's approximation, the momentum, energy and mass transfer equations in dimensional form are:

$$\frac{\partial u'}{\partial t'}+v'\frac{\partial u'}{\partial y'}=-\frac{\partial p'}{\partial x'}+\vartheta\frac{\partial^2 u'}{\partial y'^2}-\frac{\sigma_e B_0^2 u'}{\rho v}+g\beta\left(T'-T_0\right) \tag{1}$$

$$\frac{\partial T'}{\partial t'}+v'\frac{\partial T'}{\partial y'}=\frac{k}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}-Q_0\frac{\left(T'-T_0\right)}{\rho C_p}+\frac{D_c}{\rho C_p}\frac{\partial^2 C'}{\partial y'^2} \tag{2}$$

$$\frac{\partial C'}{\partial t'}+v'\frac{\partial C'}{\partial y'}=D^*\frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

Nomenclature			
Symbol	**Description**	**Symbol**	**Description**
A, B	Constants	t'	Dimensional time
B_0	Applied magnetic field	t	Dimensionless time
C_0	Initial concentration at $t'=0$	T_0	Initial temperature of fluid $(t'=0)$
C_p	Specific heat capacity	T_w	Temperature of the plate
C_w	Fluid concentration on the plate $y'=0\,(t'>0)$	u', v'	Dimensional velocities
C	Dimensionless fluid concentration	u, v	Dimensionless velocities
D^*	Dimensional co-efficient of mass-diffusion	v_0	Scale of suction/injection velocity
D_c	Dimensional co-efficient of thermal-diffusion	x', y'	Cartesian co-ordinates along the plate and normal to it
D_f	Thermal-diffusion parameter	**Greek Symbols**	
h	Distance between the plates	β	Co-efficient of thermal expansion
g	Acceleration due to gravity	σ_e	Fluid electrical conductivity
Gr	Grashof number	θ	Dimensionless temperature
M	Magnetic number	ϕ	Dimensionless concentration
Nu	Nusselt number	τ	Skin friction
p	Dimensionless pressure	η	Frequency oscillation
p'	Dimensional pressure	γ	Slip condition
Pr	Prandtl number	ρ	Density of the fluid
s	Heat source parameter	ϑ	Kinematic viscosity of the fluid
Sc	Schmidt number		

The relevant initial and boundary condition for the present physical situation are:

$$t'=0: u'=0,\ T'=T_0,\ C'=C_0 \quad for\ \ 0\le y'\le h$$

$$t'>0:\begin{cases} u'-\lambda\dfrac{\partial u'}{\partial y'}=0,\ T'=T_0,\ C'=C_0 & at\ y'=0 \\ u'=0,\ T'=T_w+(T_w-T_0)\varepsilon e^{\eta t},\ C'=C_w+(C_w-C_0)\varepsilon e^{\eta t} & at\ \ y=h \end{cases} \tag{4}$$

the dimensionless quantities introduced in the above equations are defined as:

$$x=\frac{x'}{h},\ y=\frac{y'}{h},\ \theta=\frac{T'-T_0}{T_w-T_0},\ \phi=\frac{C'-C_0}{C_w-C_0},\ M^2=\frac{\sigma_e B_0^2}{\rho v},\ p=\frac{hp'}{\rho v},$$

$$Gr=\frac{g\beta(T_w-T_0)h^2}{vu},\ Pr=\frac{v\rho C_p}{k},\ D_f=\frac{D_C(C_w-C_0)}{(T_w-T_0)},\ Sc=\frac{v}{D^*},\ \lambda=v_0\frac{h}{v} \tag{5}$$

Substituting equation (5) into equation (1) – (4), the dimensionless boundary layer equations are

$$\frac{\partial u}{\partial t}+\lambda\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+\frac{\partial^2 u}{\partial y^2}-M^2u+Gr\theta \tag{6}$$

$$Pr\left[\frac{\partial\theta}{\partial t}+\lambda\frac{\partial\theta}{\partial y}\right]=\frac{\partial^2\theta}{\partial y^2}-s\theta+D_f\frac{\partial^2\phi}{\partial y^2} \tag{7}$$

$$Sc\left[\frac{\partial\phi}{\partial t}+\lambda\frac{\partial\phi}{\partial y}\right]=\frac{\partial^2\phi}{\partial y^2} \tag{8}$$

with the initial and boundary conditions

$$u=\theta=\phi=0 \quad for \quad 0\le y\le 1$$

$$\left.\begin{aligned} u-\gamma\frac{\partial u}{\partial y}=0,\ \theta=0,\ \phi=0,\ \ at\ y=0 \\ u=0,\ \theta=1+\varepsilon e^{\eta t},\ \phi=1+\varepsilon e^{\eta t},\ at\ y=1 \end{aligned}\right\} \tag{9}$$

For periodic flow, let the pressure gradient be of the form:

$$-\frac{\partial p}{\partial x}=A+Be^{\eta t} \tag{9a}$$

The solution to the dimensionless boundary layer equations set in equations (6) to (9) can be obtained by representing velocity; temperature and mass transfer as follows:

$$\left.\begin{aligned} u(y,t)=u_0(y)+\varepsilon u_1(y)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j u_j e^{(\eta t)^j} \\ \theta(y,t)=\theta_0(y)+\varepsilon\theta_1(y)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j\theta_j e^{(\eta t)^j} \\ \phi(y,t)=\phi_0(y)+\varepsilon\phi_1(y)+\cdots=\sum_{j=0}^{\infty}\varepsilon^j\phi_j e^{(\eta t)^j} \end{aligned}\right\} \tag{9}$$

Substituting equation (9) into equation (6) to (8) and equating like powers of ε, one obtains the harmonic and non-harmonic boundary value problem for $j=0$ and $j=1$ as:

$$\frac{d^2\phi_0}{dy^2}-\lambda Sc\frac{d\phi_0}{dy}=0 \tag{10}$$

$$\frac{d^2\phi_1}{dy^2}-\lambda Sc\frac{d\phi_1}{dy}-\eta Sc\phi_1=0 \tag{11}$$

$$\frac{d^2\theta_0}{dy^2}-\lambda Pr\frac{d\phi_0}{dy}-s\theta_0=-D_f\frac{d^2\phi_0}{dy^2} \tag{12}$$

$$\frac{d^2\theta_1}{dy^2}-\lambda Pr\frac{d\theta_1}{dy}-\left[s+\eta Pr\right]\theta_1=-D_f\frac{d^2\phi_1}{dy^2} \tag{13}$$

$$\frac{d^2u_0}{dy^2}-\lambda\frac{du_0}{dy}-M^2u_0=A-Gr\theta_0 \tag{14}$$

$$\frac{d^2u_1}{dy^2}-\lambda\frac{du_1}{dy}-\left[\eta+M^2\right]u_1=B-Gr\theta_1 \tag{15}$$

Subject to the boundary conditions

$$\left.\begin{aligned} \phi_0=0,\ \theta_0=0,\ u_0-\gamma\frac{du_0}{dy}=0,\ \ at\ y=0 \\ \phi_0=1,\ \theta_0=1,\ u_0=0,\qquad at\ y=1 \end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} \phi_1=0,\ \theta_1=0,\ u_1-\gamma\frac{du_1}{dy}=0,\ \ at\ y=0 \\ \phi_1=1,\ \theta_1=1,\ u_1=0,\qquad at\ y=1 \end{aligned}\right\} \tag{17}$$

the values of $\phi_0(y)$ and $\phi_1(y)$ from (10) and (11) using the boundary condition (16) and (17) are:

$$\phi_0(y)=\frac{1}{\left[e^{\lambda Sc}-1\right]}\left(e^{\lambda Scy}-1\right) \tag{18}$$

$$\phi_1(y) = \frac{1}{\left[e^{\lambda Sc} - 1\right]}\left(e^{-x_2 y} - e^{x_1 y}\right) \tag{19}$$

Substituting (18) and (19) into equation (12) and (13) the require solution of $\theta_0(y)$ and $\theta_1(y)$ subject to boundary condition (16) and (17) can be presented as:

$$\theta_0(y) = A_1 e^{x_3 y} + A_2 e^{-x_4 y} + f_1 e^{\lambda Scy} \tag{20}$$

$$\theta_1(y) = A_3 e^{x_5 y} + A_4 e^{-x_6 y} + f_2 e^{x_1 y} + f_3 e^{-x_2 y} \tag{21}$$

Since equation (14) and (15) are coupled with $\theta_0(y)$ and $\theta_1(y)$, using result obtained in equation (20) and (21), the solutions of $u_0(y)$ and $u_1(y)$ are:

$$u_0(y) = A_5 e^{x_7 y} + A_6 e^{-x_8 y} + h_1 e^{x_3 y} + h_2 e^{-x_4 y} + h_3 e^{\lambda Scy} - \frac{A}{M^2} \tag{22}$$

$$u_1(y) = A_7 e^{x_9 y} + A_8 e^{-x_{10} y} + h_4 e^{x_1 y} + h_5 e^{-x_2 y} + h_6 e^{x_5 y} + h_7 e^{-x_6 y} - \frac{B}{\eta + M^2} \tag{23}$$

The complete solution of velocity, temperature and mass transfer equations are:

Velocity equation

$$u(y) = A_5 e^{x_7 y} + A_6 e^{-x_8 y} + h_1 e^{x_3 y} + h_2 e^{-x_4 y} + h_3 e^{\lambda Scy} - \frac{A}{M^2} + \left\{A_7 e^{x_9 y} + A_8 e^{-x_{10} y} + h_4 e^{x_1 y} + h_5 e^{-x_2 y} + h_6 e^{x_5 y} + h_7 e^{-x_6 y} - \frac{B}{\eta + M^2}\right\} e^{\eta t} \tag{24}$$

Temperature equation

$$\theta(y) = A_1 e^{x_3 y} + A_2 e^{-x_4 y} + f_1 e^{\lambda Scy} + \left\{A_3 e^{x_5 y} + A_4 e^{-x_6 y} + f_2 e^{x_1 y} + f_3 e^{-x_2 y}\right\} e^{\eta t} \tag{25}$$

Concentration equation

$$\phi(y) = \frac{1}{\left[e^{\lambda Sc} - 1\right]}\left(e^{\lambda Scy} - 1\right) + \left\{\frac{1}{\left[e^{\lambda Sc} - 1\right]}\left(e^{-x_2 y} - e^{x_1 y}\right)\right\} e^{\eta t} \tag{26}$$

Skin friction

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=1} = x_7 A_5 e^{x_7} - x_8 A_6 e^{-x_8} + x_3 h_1 e^{x_3} - x_4 h_2 e^{-x_4} + \lambda Sc h_3 e^{\lambda Sc} + \left\{x_9 A_7 e^{x_9} - x_{10} A_8 e^{-x_{10}} + x_1 h_4 e^{x_1} - x_2 h_5 e^{-x_2} + x_5 h_6 e^{x_5} - x_6 h_7 e^{-x_6}\right\} e^{\eta t} \tag{27}$$

Nusselt number

$$Nu_1 = \left.\frac{d\theta}{dy}\right|_{y=1} = x_3 A_1 e^{x_3} - x_4 A_2 e^{-x_4} + \lambda Scf_1 e^{\lambda Sc} + \left\{x_5 A_3 e^{x_5} - x_6 A_4 e^{-x_6} + x_1 f_1 e^{x_1} - x_2 f_3 e^{-x_2}\right\} e^{\eta t} \tag{28}$$

3.0 Results and Discussion

In this paper, we have examined the free-convective unsteady hydromagnetic flow confined through a vertical porous channel plates in the presence of constant heat source and thermal-diffusion under slip condition. The system of governing equationsof the physical situation presented in (6) to (8) is solved employing the Perturbation method subject to the boundary conditions (9). The concentration is coupled to the temperature by composition gradients parameter D_f and temperature is coupled to the velocity by the free convection parameter Gr as illustrated in equations (7) and (8) respectively.

The investigation is performed using the basic dimensionless parameters governed the flow in the channel. To be realisticthe following choices were made: Prandtl number Pr is choosing as 0.71 which correspond to air. In air the diffusing chemical species of common interest have Schmidt number in the range 0.1 – 1.0 Jha and Ajibade (2011): the work consider three species of interest H_2, NH_3 and CO_2 respectively. Grashof number is $(Gr > 0)$ is considered, which is employed for

practical applications in nuclear technology and also in geophysical and naval energy system applications Singh (2010). However in Figure 2b Grashof numberis taken as $Gr<0$ correspond to an external heating of the channel plate. The suction/injection parameter (λ), the heat source parameter (s), the diffusion-thermo parameter (D_f), the magnetic parameter (M), and the slip number (γ) are choosing arbitrary.

In Figure 2 it is observed that the fluid velocity reduce as M increases for fixed values of the other parameters. This physically indicates that the hydromagnetic drag embodies in the term $-M^2u$ of the momentum equation(6) decreases the velocity in the cooling channel plates of the free convection current $Gr>0$. It is also observed from Figure 2a, b that the values of the fluid velocity elevated with the rise in the values of heat source (s) and diffusion-thermo (D_f). This is an important controlling mechanism in flow and heat transfer processes so that the finished product meets the desire quality specification.

Figure 3 present the influence of the Grashof number on the flow when other parameters are fixed. It is seen form Figure 3athat $Gr>0$ significantly increases the velocity of air. This corresponds to the external cooling of the channel plates, which results in thickening the boundary layer and consequently assist the velocity. In Figure 3b it is observe that, for $Gr<0$ the velocity of air takes reverse flow to the negative direction. The Figure also reveals that when injection $(\lambda>0)$ takes place at, $y=0$ the fluid velocities of air is higher compared with suction $(\lambda<0)$. In addition increase in s and D_f with increasing Grashof number enhances the velocity incase of injection where the contrast is observed incase of injection see Figure 3a and 3b.

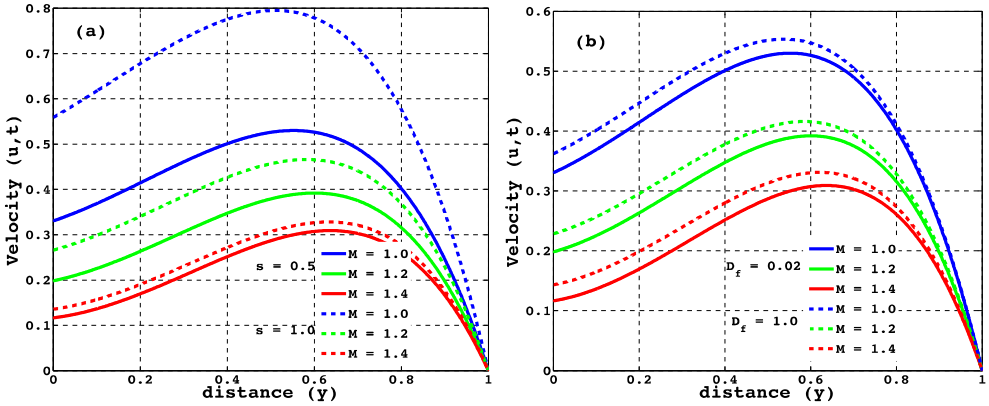

Figure 2: Effect of magnetic parameter on velocity field when $\lambda=0.5$, $Gr=5$, $t=0.5$, $\gamma=1$, $Sc=0.22$, $\eta=1$ $\varepsilon=1$, $A=1$ and $B=1$

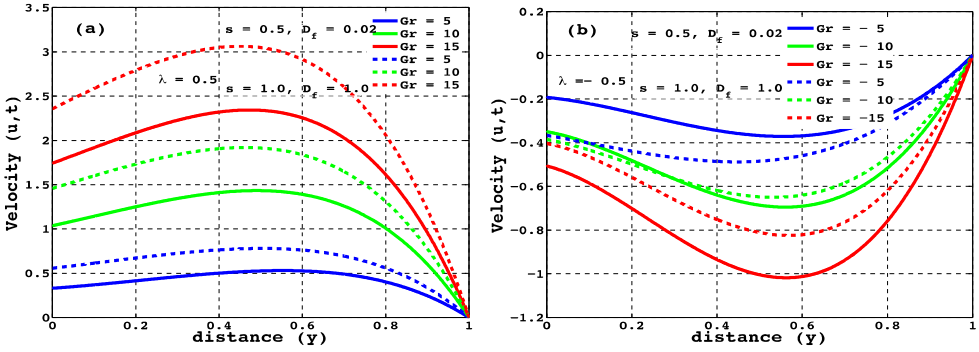

Figure 3: Effect of Grashof number on velocity field when $\lambda=\pm0.5$, $M=1$, $t=0.5$, $\gamma=1$, $Sc=0.22$, $\eta=1$ $\varepsilon=1$, $A=1$ and $B=1$

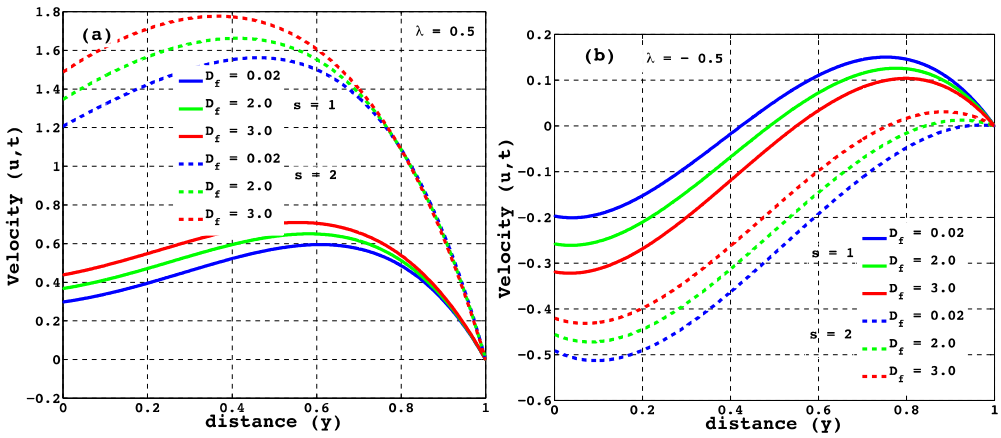

Figure 4:Effect of thermal-diffusion parameter on velocity field when $\lambda = \pm 0.5$, $Gr = 5$, $t = 0.5$, $\gamma = 1$, $Sc = 0.22$, $M = 1, \eta = 1$ $\varepsilon = 1$, $A = 1$ and $B = 1$

Figure 4 illustrates the disparity in velocity due to a change in thermal-diffusion parameter (D_f) when other parameters are treated constant. From Figure 4a it is noted the effect of D_f is to increase the velocity of the air when injection $\lambda > 0$ takes place at $y = 0$. A reverse flow is recognised due to suction $(\lambda < 0)$ as demonstrated in Figure 4b when heat source parameter $s = 1$. However, anomalous phenomenon is observed due to suction $(\lambda < 0)$ to when the heat source parameter $s = 2$ see Figure 4b.

In Figure 5a, b the values of velocity is seen to increase as γ increases for $S > 0$ when values of other parameter are fixed. Furthermore, increasing s or D_f lessen the velocity of air throughout the channel.

Figure 6a, b explain the effect of increasing heat source parameter (s) for different values of thermal-diffusion parameter. With all other parameters constant, the velocity of air increases with increase in s. However, there is freak behavior on the values of velocity when values of s and D_f tends to 1. Indeed the anomalous phenomenon is well pronounce in Figure 6b due suction in comparison with 6a.

Figure 7a, b show the influence of Schmidt number (Sc) on velocity. The velocity is observed to descend with Schmidt number and increase with time when other numerical values are treated constant.

Figure 8 depict the effect of frequency oscillation (η) on velocity due suction/injection (λ) for fixed values of other controlling parameters. From Figure 8a it is reported that velocity reduces with increase in η for $\lambda > 0$. The reverse is the case for $\lambda < 0$ as demonstrated in Figure 8b.

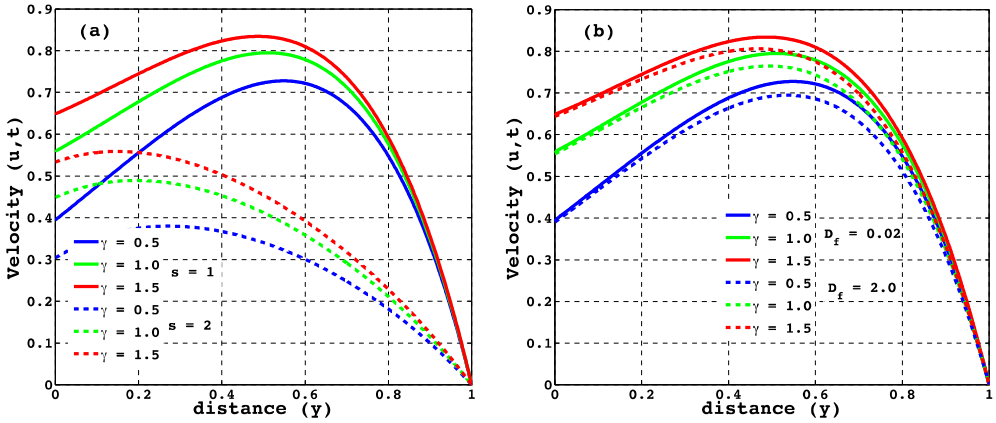

Figure 5: Effect of slip parameter on velocity field when $\lambda = 0.5$, $M = 1$, $Gr = 5$, $t = 0.5$, $Sc = 0.22$, $\eta = 1$ $\varepsilon = 1$, $A = 1$ and $B = 1$

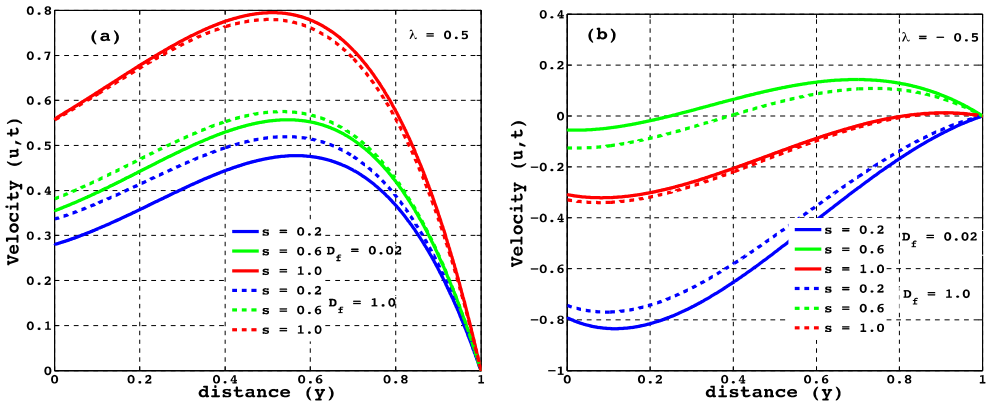

Figure 6: Effect of heat source parameter on velocity field when $Gr = 5$, $t = 0.5$, $\gamma = 1$, $Sc = 0.22$, $\eta = 1$ $\varepsilon = 1$, $A = 1$ and $B = 1$

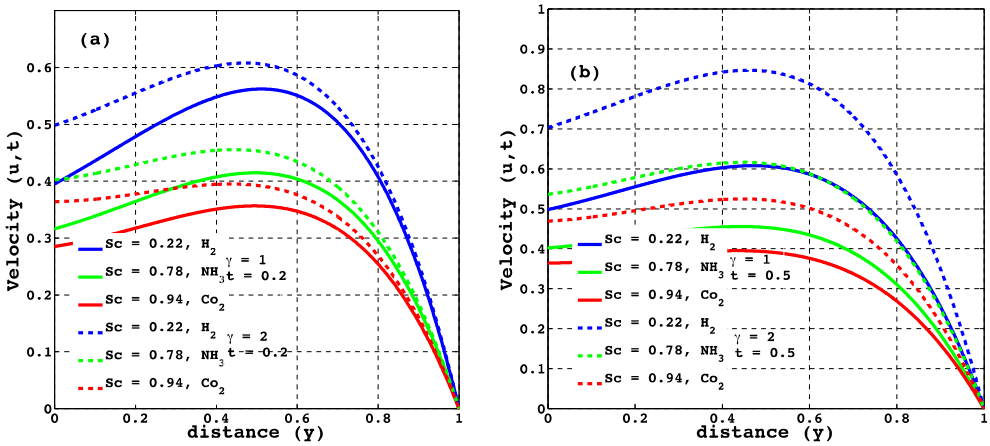

Figure 7: Effect of Schmidt number on velocity field when $\lambda = 0.5, D_f = 0.02$, $s = 1$, $Gr = 5$, $t = 0.5$, $M = 1$, $\eta = 1$, $\varepsilon = 1$, $A = 1$ and $B = 1$

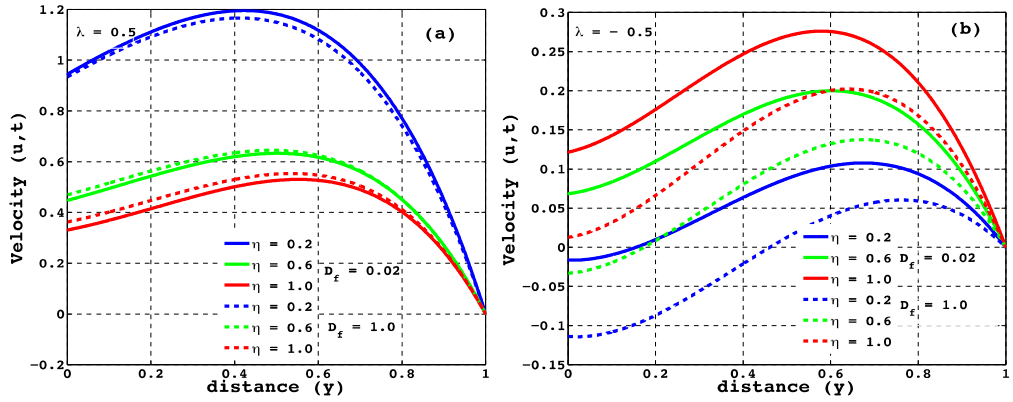

Figure 8: Effect of frequency oscillation parameter on velocity field $\lambda = 0.5$, $Gr = 5$, $t = 0.5$, $\gamma = 1$, $Sc = 0.22$, $M = 1$ $\varepsilon = 1$, $A = 1$ and $B = 1$

Figure 9 represent the effect of thermal-diffusion on temperature for fixed values of other parameters. It is observing that quantitatively, when $S = 0.5$ and D_f increases from 0.02 to 3.0 there is increase in the temperature value as shown in Figure 9a. For the reverse phenomenon, $S = -0.5$ the temperature decreases with increase in D_f from 0.02 to 3.0.

Figure 10 illustrates temperature fielddue to a change in heat source parameter (s) for fixed values of the other parameters. It is observed that the temperature field increases throughout the flow field with an increase in the heat source parameter. In

fact, the presence of a heat sink parameter assists the velocity field due to thinning of the thermal boundary layer see Figure 10a, b respectively.

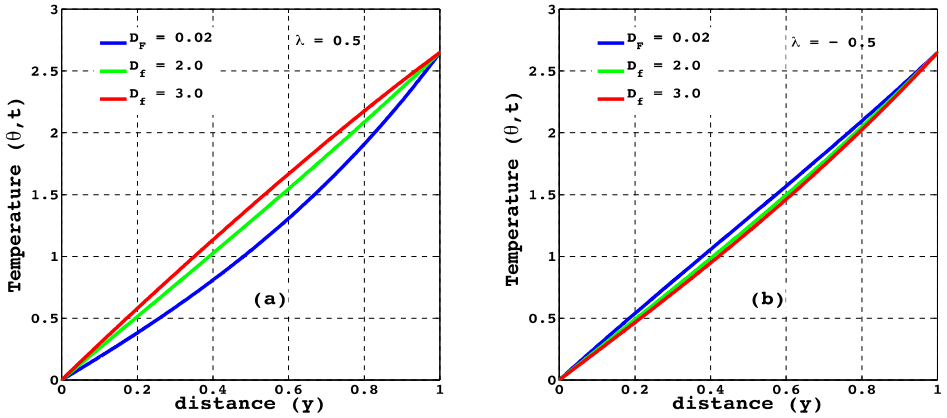

Figure 9: Effect of thermal-diffusion parameter on temperature field when $s=1$, $t=0.5$, $Sc=0.22$, $\eta=1$and $\varepsilon=1$

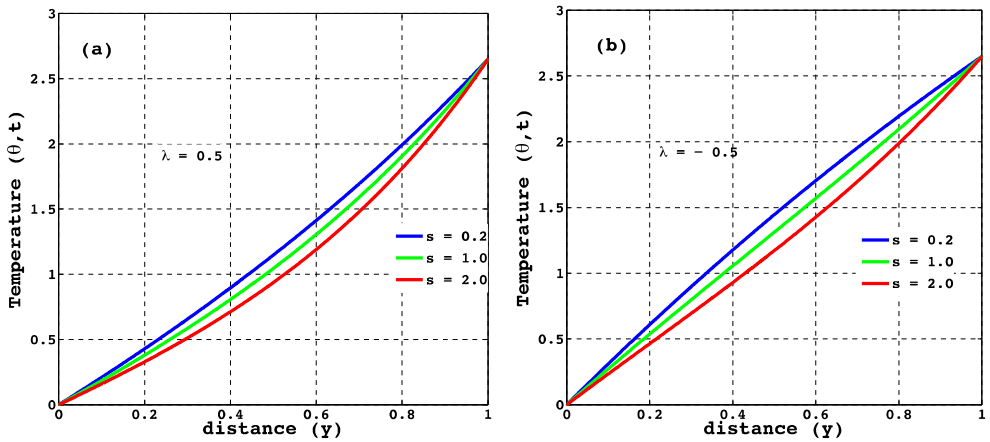

Figure 10: Effect of heat source parameter on temperature field when $D_f=0.02$, $t=0.5$, $Sc=0.22$, $\eta=1$and $\varepsilon=1$

Figure 11and 12 shows the description profiles for skin friction for different values of s and D_f for fixed values of other constants. Figure 11a, b reflect that, for small values ofthermal-diffusion parameter $0\le D_f\le 0.2$ the skin friction increases and later decreases for higher values of $D_f>0.2$. It is interesting to emphasize that the numerical value of a skin friction is lower for injection $(\lambda>0)$ when s increases and ascend at point where values of $D_f\ge 0.5$ see Figure 11a.Incase of suction $(\lambda<0)$ acting at $y=1$ the skin friction increases with increase in heat source (s) for small values of $0\le D_f\le 0.1$ while the contrast is the case for higher values of $D_f>0.1$. Furthermore, in Figure 12a, b the skin friction is plot against heat source for varying values of thermal-diffusion parameter (D_f) when other parameters are retain constant. It is observed from Figure 12a that skin friction tend to increase as the values of s increases for 0 to 0.6 and subsequently takes the reverse case for $s>0.6$. In addition increasing D_f suffered the values of skin friction for $0\le s\le 0.6$ and enhances skin friction for $0.6\le s\le 1$ as illustrated. From Figure 12b it is narrated skin friction decrease at interval $0\le s\le 0.6$ and rises with higher values of D_f. It is interesting to mention that skin friction magnifies for higher values of D_f and $0.6\le s\le 1$ respectively.

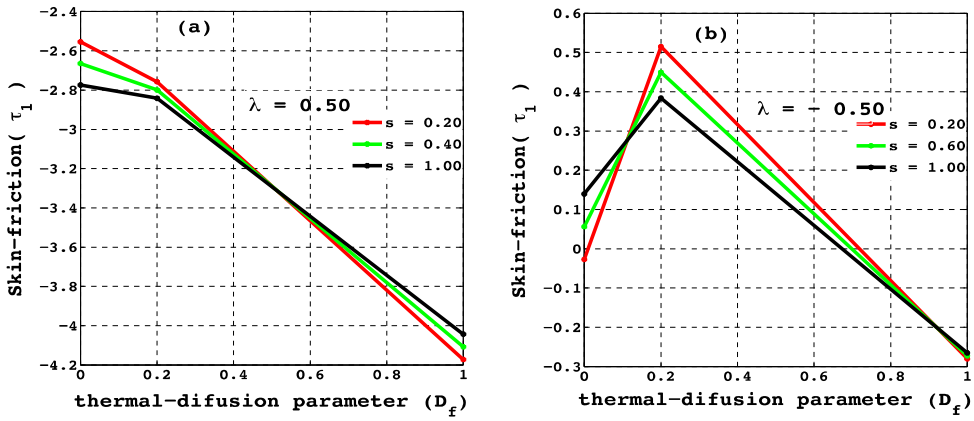

Figure 11: Skin-friction against D_f at $y=1$

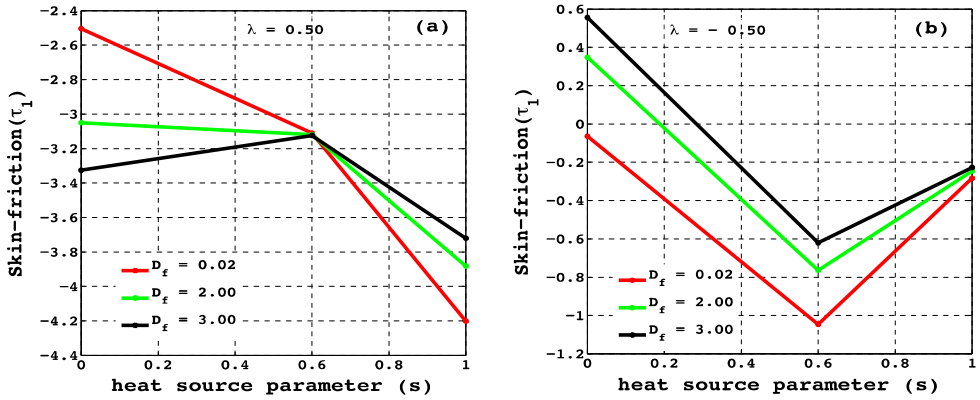

Figure 12: Skin-friction against s at $y=1$

Figures 13 and 14also delineate influence of thermal-diffusion and heat source on Nusseltnumber (rate of heat transfer) at plates $y=1$. In Figures 13a and bNusselt number qualitatively increase with big values of D_f. However Nusselt number holds back as s increases due to injection $(\lambda=0.5)$ and build up in case of suction $(\lambda-0.5)$.

Nusselt number is plotted against heat source in Figure 14a, b. From Figure 14a and b as heat source parameter increases the values of Nusselt number increases. The Figures also narrated that the effect of D_f is to decrease the Nusselt number when injection $(\lambda=0.5)$ takes place at $y=1$ and qualitatively takes that contrast due to suction $(\lambda-0.5)$.

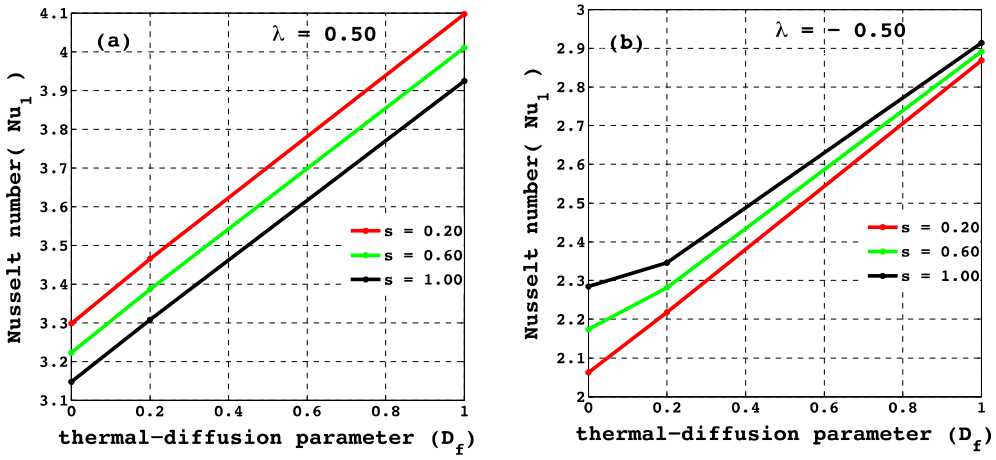

Figure 13: Nusselt number against D_f at $y=1$

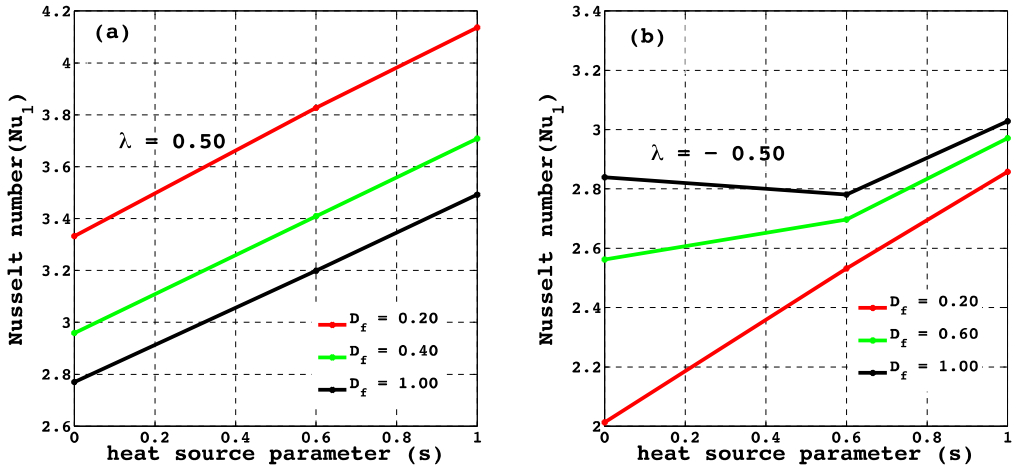

Figure 14: Nusselt number against s at $y=1$

4.0 Conclusion

We studied analytically unsteady laminar incompressible free-convective hydromagnetic boundary layer flow confined through a vertical porous channel plates in the presence of constant heat source and thermal-diffusion under slip boundary condition. The governing boundary layer equations of the physical situation are solved employing the Perturbation method. The results of velocity, temperature, skin friction and Nusselt number under the influence of the material parameters of the flow problem are narrated on line graphs. We conclude from these result that

i. Velocity decreases with increase in M and Sc while increases as Gr increases

ii. The values of velocity increases with D_f, γ, s when $\lambda > 0$ and takes reverse action for $\lambda < 0$

iii. Temperature increases with D_f for $\lambda > 0$ and decreases for $\lambda < 0$ while increase in s assist temperature whether suction/injection take place

iv. The effect of small or large values of heat source and thermal-diffusion,influences the values skin friction

v. The values of Nusselt number undergo a change at a apoint due to heat source $(s=0.2)$ and thermal-diffusion $\left(D_f = 0.6\right)$ for both suction and injection.

5.0 Appendices

$$c_1 = \frac{(\lambda Sc)^2}{e^{\lambda Sc}-1};\ c_2 = \frac{(x_1)^2}{e^{-x_2}-e^{x_1}};\ c_3 = \frac{-D_f(x_2)^2}{e^{-x_2}-e^{x_1}}\ f_1 = \frac{-c_1 D_f}{(\lambda Sc)^2 - \lambda^2 PrSc - s};$$

$$f_2 = \frac{c_2 D_f}{(x_1)^2 - x_1\lambda Pr - [\lambda Pr + s]};\ f_3 = \frac{c_2 D_f}{(x_2)^2 + x_2\lambda Pr - [\lambda Pr + s]}$$

$$A_1 = \frac{1 - f_1\left(e^{\lambda Sc} - e^{-x_4}\right)}{e^{x_3} - e^{-x_4}};\ A_2 = \frac{1 - f_1\left(e^{\lambda Sc} - e^{x_3}\right)}{e^{-x_4} - e^{x_3}};\ A_3 = \frac{1 - f_3\left(e^{-x_2} - e^{-x_6}\right) - f_2\left(e^{x_1} - e^{-x_6}\right)}{e^{x_5} - e^{-x_6}}$$

$$A_4 = \frac{1 - f_3\left(e^{-x_2} - e^{x_5}\right) - f_2\left(e^{x_1} - e^{x_5}\right)}{e^{-x_6} - e^{x_5}};\ A_5 = \frac{S_4 e^{-x_8} - S_5\left(1+\gamma x_8\right)}{\left(1-\gamma x_7\right)e^{-x_8} - \left(1+\gamma x_8\right)e^{x_7}};$$

$$A_6 = \frac{S_4 e^{x_7} - S_5\left(1-\gamma x_7\right)}{\left(1+\gamma x_8\right)e^{x_7} - \left(1-\gamma x_7\right)e^{-x_8}};\ A_7 = \frac{S_6 e^{-x_{10}} - S_7\left(1+\gamma x_4\right)}{\left(1-\gamma x_9\right)e^{-x_{10}} - \left(1+\gamma x_{10}\right)e^{x_9}};$$

$$A_8=\frac{S_6e^{x_9}-S_7\left(1-\gamma x_9\right)}{\left(1+\gamma x_{10}\right)e^{x_9}-\left(1-\gamma x_9\right)e^{-x_{10}}};\ S_4=\frac{A}{M^2}-h_3\left(1-\gamma Sc\right)-h_2\left(1+\gamma x_4\right)-h_1\left(1-\gamma x_3\right)$$

$$S_5=\frac{A}{M^2}-h_3e^{\lambda Sc}-h_2e^{-x_4}-h_1e^{x_3}$$

$$S_6=\frac{B}{\eta+M^2}+h_4\left(\lambda x_1-1\right)-h_5\left(\lambda x_2-1\right)+h_6\left(\lambda x_5-1\right)-h_7\left(\lambda x_6-1\right);$$

$$S_6=\frac{B}{\eta+M^2}-h_4e^{x_1}-h_5e^{-x_2}-h_6e^{x_5}-h_7e^{-x_6}\quad;\quad h_1=\frac{-f_1}{\left(x_3\right)^2-x_3\lambda-M^2}\quad;\quad h_2=\frac{-A_2}{\left(x_4\right)^2+x_4\lambda-M^2}\quad;$$

$$h_3=\frac{-A_1}{\left(\lambda Sc\right)^2-\lambda^2Sc-M^2}\quad;\quad h_4=\frac{-f_2Gr}{\left(x_1\right)^2-x_1\lambda-\left[\eta+M^2\right]}\quad;\quad h_5=\frac{-f_3Gr}{\left(x_2\right)^2+x_2\lambda-\left[\eta+M^2\right]}\quad;$$

$$h_6=\frac{-A_3Gr}{\left(x_5\right)^2-x_5\lambda-\left[\eta+M^2\right]};\ h_7=\frac{-A_3Gr}{\left(x_6\right)^2+x_6\lambda-\left[\eta+M^2\right]};\ x_1=\frac{\lambda Sc+\sqrt{\left(\lambda Sc\right)^2+4\eta Sc}}{2};$$

$$x_2=\frac{-\lambda Sc+\sqrt{\left(\lambda Sc\right)^2+4\eta Sc}}{2};\ x_3=\frac{\lambda Pr+\sqrt{\left(\lambda Pr\right)^2+4s}}{2};\ x_4=\frac{-\lambda Pr+\sqrt{\left(\lambda Pr\right)^2+4s}}{2};$$

$$x_5=\frac{\lambda Pr+\sqrt{\left(\lambda Pr\right)^2+4\left(s+\lambda Pr\right)}}{2};\ x_6=\frac{-\lambda Pr+\sqrt{\left(\lambda Pr\right)^2+4\left(s+\lambda Pr\right)}}{2};$$

$$x_7=\frac{\lambda+\sqrt{\left(\lambda\right)^2+4M^2}}{2};\ x_8=\frac{-\lambda+\sqrt{\left(\lambda\right)^2+4M^2}}{2};\ x_9=\frac{\lambda+\sqrt{\left(\lambda\right)^2+4\left(\eta+M^2\right)}}{2};$$

$$x_9=\frac{-\lambda+\sqrt{\left(\lambda\right)^2+4\left(\eta+M^2\right)}}{2}$$

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