

On Bijective Soft Multiset with its Operations

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Abstract

Theory of soft set and multiset are important mathematical tool to handle uncertainties about vague concept. In 1999, Molodtsov proposed it as newly emerging tool that has been studied by researchers in theory and practice. Work on bijective soft set has been carried out by some researchers. In this paper, we extend the concept to soft multiset. We initiate the concept of bijective soft multiset and some of its basic operations such as restricted AND and relax AND operations on bijective soft multiset, dependency between two bijective soft multiset, bijective soft multiset decision system, importance of bijective soft multiset with respect to bijective soft multiset decision system.

Keywords and Phrases: Soft multiset, AND Operation, dependency, decision system

1.0 Introduction

Mathematical modelling and manipulation of various types of uncertainties has become an issue of great relevance in seeking solution to complicated problems arising in many important application areas such as economics, engineering, environment, social sciences, medical science and business management. Although a number of mathematical models like probability theory, fuzzy sets [1], rough sets [2], and interval mathematics [3] are well known and often effective tools for modelling uncertainties but each of them has distinguish advantages as well as certain inherent limitations. One major problem shared by these theories is their incompatibility with the parameterization tools. In 1999, Molodtsov [4] proposed a completely new concept called soft set theory to model uncertainty, which associate a set with a set of parameters and thus is free from the difficulties caused by the aforementioned problem. It has been demonstrated that soft set theory brings about a rich potential for applications in many fields like function smoothness, Riemann integration, decision making, measurement theory, game theory [4] etc.

Soft set theory has received much attention since its introduction by Molodtsov. The concept and basic properties of soft set are presented in [4, 5]. To deal with the fuzziness of problem parameters, Roy and Maji [6] proposed the concept of fuzzy soft set and provide its properties and an application in decision making under an imprecise environment. Chen et al. [7] presented a definition for soft set parameterization reduction and showed an improved application in another decision making problem. Liu and Yan [8] discussed the algebraic structure of fuzzy soft set and gave the definition of fuzzy soft group. In their paper, they defined operations on fuzzy soft groups and improve some results on them.

The applications of soft set theory are also extended to data analysis under incomplete information [9], combined forecast [10], decision making problems, normal parameter reduction [11], and d- algebra [12], demand analysis [13]. These applications showed the promising of soft set theory in dealing with uncertain problems.

Many researchers in the area of multiset and their applications [14, 15] have voiced that there is no good reason for admitting repeated elements into power multiset, violating the Cantor's theorem on power set. However, in this research work we admit repeated elements in to the power multiset to enable us describe any object adequately. The motivation is that there are situations in real life problems that some conditions, events need to be described in details and that is why bijective soft multiset is intended to solve. In this paper, we initiate a new type of soft multiset set called bijective soft multiset, in which every element can be mapped only into one parameter and the union by the parameter set gives the universe multiset.

2.0 Preliminaries and Basic Definition

In this section, we present the notion of soft multiset and some basic definitions in soft multiset.

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Definition 2.1. Soft multiset theory

Let $\{U_i: i \in I\}$ be a collection of universes such that there exist U_j, U_k and $U_j \cap U_k \neq \emptyset$. Suppose $U = \bigsqcup_{i \in I} P(U_i)$, where $P(U_i)$ denotes the power set of U_i , and E be a set of parameters. A pair (F, A) , where $A \subseteq E$, is called a soft multiset over U . F is a mapping given by $F: A \rightarrow U$. That is, a soft multiset over U is a parametrized family of submultisets of U such that for $e \in A$, $F(e)$ is considered as the set of e -approximate element of the soft multiset (F, A) .

Definition 2.2: Multivalue-class.

The class of all value set of a soft multiset (F, A) is called the value class of the soft multiset and is denoted by

$$C_{(F,A)}^* = \{V_1, V_2, \dots, V_n\}.$$

Obviously $C_{(F,A)}^* \subseteq U$. Also, if there exists at least one i such that $V_i = V_j$,

$\forall i, j = 1, 2, \dots, n$, then the value-class of the softmultiset (F, A) is called Multi value-class of the soft multiset (F, A) and is denoted by $C_{(F,A)}^m$. Similarly $C_{(F,A)}^m \subseteq U$.

Definition 2.3. Soft submultiset.

Let (F, A) and (G, B) be two softmultisets over U , we say that (F, A) is a softmultisubset of (G, B) written as $(F, A) \subseteq (G, B)$ if

- i. $A \subseteq B$
- ii. $M_{(F,A)}(x) \leq M_{(G,B)}(x)$ for all $x \in U$.

Definition 2.4. Equality of two soft multisets.

Two Soft multisets (F, A) and (G, B) over U are said to be equal if and only if (F, A) is a softmultisubset of (G, B) and (G, B) is a softmultisubset of (F, A) .

Definition 2.5. NOT Set of a set parameters.

Let E be a set of parameters. The NOT set of E denoted by $\neg E$ is defined by $\neg E = \{\neg e_1, \neg e_2, \dots, \neg e_n\}$ where $\neg e_i = \text{not } e_i, \forall i$.

Proposition 2.6.

- 1. $\neg(\neg A) = A$
- 2. $\neg(A \cup B) = (\neg A) \cap (\neg B)$
- 3. $\neg(A \cap B) = (\neg A) \cup (\neg B)$

Definition 2.7. Similar Soft multisets

Two Soft multisets (F, A) and (G, B) are said to be ‘Cognate’ or similar if

$$\forall x (x \in (F, A) \Leftrightarrow x \in (G, B)) \text{ where } x \text{ is an object. Therefore, similar Soft multisets have equal root sets but need not be equal themselves.}$$

Definition 2.8. Union of two soft multisets.

Let (F, A) and (G, B) be two Softmultisets over U . $(F, A) \cup (G, B)$ is the softmultiset defined by

$$M_{(F,A) \cup (G,B)}(x) = M_{(F,A)}(x) \cup M_{(G,B)}(x) = \text{maximum} (M_{(F,A)}(x), M_{(G,B)}(x)) \text{ being the union of two numbers.}$$

Definition 2.9. Intersection of two soft multiset.

Let (F, A) and (G, B) be two soft multisets over U . Then, the intersection of (F, A) and (G, B) written as $(F, A) \cap (G, B)$ is the Soft multiset defined by

$$M_{(F,A) \cap (G,B)}(x) = M_{(F,A)}(x) \cap M_{(G,B)}(x) = \text{minimum} (M_{(F,A)}(x), M_{(G,B)}(x)) \text{ being the intersection of two numbers.}$$

That is, an object x occurring a times in (F, A) and b times in (G, B) , occurs minimum (a, b) in $(F, A) \cap (G, B)$, which always exists.

Definition 2.10. Absolute soft multiset.

A Soft multiset (F, A) over universe U is said to be absolute soft multiset denoted by \tilde{A} if for all $e \in A, F(e) = U$

Definition 2.11. Null soft multiset.

A Soft multiset (F, A) over universe U is said to be null soft multiset denoted by $\tilde{\emptyset}$ if for all $e \in A, F(e) = \emptyset$

Definition 2.12. Difference.

Let (F, A) and (G, B) be two soft multisets over U , and

$$(G, B) \subseteq (F, A). \text{ Then } M_{(F,A)-M_{(G,B)}}(x) = M_{(F,A)}(x) - M_{(F,A)} \cap M_{(G,B)}(x)$$

for all $x \in U$.

It is sometimes referred to as the arithmetic difference of (G, B) from (F, A) . Note that, even if $(G, B) \subset (F, A)$, this definition still holds good

Definition 2.13. Direct Sum.

Let (F, A) and (G, B) be two Soft multisets defined by

$$M_{(F,A) \cup (G,B)}(x) = M_{(F,A)}(x) + M_{(G,B)}(x), \text{ for any } x \in U, \text{ direct sum of two numbers. That is, an object } x \text{ occurring } a \text{ times in } (F, A) \text{ and } b \text{ times in } (G, B), \text{ occurs } a + b \text{ times in } (F, A) \cup (G, B).$$

Definition 2.14. OR operation.

Let (F, A) and (G, B) be two soft multisets over U . Then (F, A) OR (G, B) denoted by $(F, A) \vee (G, B)$ is defined as $(F, A) \vee (G, B) = (H, A \times B)$ where $H(a, b) = F(a) \cup G(b)$

Definition 2.15. AND operation on two soft multisets.

Let (F, A) and (G, B) be two softmultisets over U . Then (F, A) AND (G, B) denoted by $(F, A) \wedge (G, B)$ is defined as $(F, A) \wedge (G, B) = (H, A \times B)$ where $H(a, b) = F(a) \cap G(b)$

3.0 Bijective Soft Multiset

The concept of bijective soft multiset can be created by giving a relevant example. It will be used to illustrate some notion of this section.

Example 3.1. Let $U_1 = \{S_1, S_2, S_3\}$ be a set of state with availability of land and $U_2 = \{S_1, S_4, S_5, S_6\}$ be a set of states with availability of raw materials. Let $U = \cup U_i = \{S_1, S_1, S_2, S_3, S_4, S_5, S_6\}$ be a common universe multiset of six different states. Suppose that the six states are characterized by soft multiset (F, A) over the common universe U . Let A be a set of decision parameters related to the universe U . $A = A_1 \cup A_2 \cup A_3 \cup A_4 = \{a_1 = peaceful, a_2 = accessible, a_3 = market, a_4 = armed robbery, a_5 = kidnapping, a_6 = labour, a_7 = densely populated, a_8 = sparsely populated, a_9 = good weather\}$. (F_i, A_i) is a soft submultiset of (F, A) , where $i = 1, 2, 3, 4$.

The mapping of each soft multiset over U is defined as follows:

$$F_1(peaceful) = \{S_1, S_6\}, F_1(accessible) = \{S_2, S_3, S_5\}, F_1(market) = \{S_1, S_4\}.$$

$$F_2(armed robbery) = \{S_1, S_2, S_3\}, F_2(kidnapping) = \{S_1, S_4, S_5, S_6\}.$$

$$F_3(labour) = \{S_1, S_2, S_3, S_4\}, F_3(densely populated) = \{S_1, S_5, S_6\},$$

$$F_4(sparsely populated) = \{S_1, S_3, S_6\}, F_4(good weather) = \{S_1, S_2, S_4, S_5\}.$$

Concept of bijective soft multiset

Definition 3.2. Let (F, A) be a soft multiset over a common universe U , where F is a mapping $F: A \rightarrow P(U)$ and A is a non-empty set of decision parameters. We say that (F, A) is a bijective soft multiset if the following condition holds:

- (i) $\cup_{a \in A} F(a) = U$, where U is the universe multiset.
- (ii) For any two parameters $a_i, a_j \in A, a_i \neq a_j$, then either $F(a_i) \cap F(a_j) = \emptyset$ or $F(a_i) \cap F(a_j) \neq \emptyset$

In other words, let $Y \subseteq P(U)$ and $Y = \{F(a_1), F(a_2), \dots, F(a_n)\}, a_1, a_2, \dots, a_n \in A$. From the definition 3.2, the mapping $F: A \rightarrow P(U)$ can be transformed to the mapping $F: A \rightarrow Y$, which is a bijective soft multiset function. That is for every $y \in Y$, there exists exactly one parameter $a \in A$ such that $F(a) = y$ and no unmapped element is left in A and Y .

Example 3.3. Suppose that $U = \{S_1, S_1, S_2, S_3, S_4, S_5, S_6, S_7\}$ be a common universe, (F, A) is a soft multiset over U and $A = \{a_1, a_2, a_3, a_4\}$. The mapping of (F, A) is given below:

$$F(a_1) = \{S_1, S_2, S_3\}, F(a_2) = \{S_4, S_5, S_6\}, F(a_3) = \{S_7\}, F(a_4) = \{S_4, S_5, S_6, S_7\}.$$

From the definition 3.2. $(F, \{a_1, a_2, a_3\})$ and $(F, \{a_1, a_4\})$ are bijective soft multisets while $(F, \{a_1, a_2\})$ and $(F, \{a_1, a_3\})$ are not bijective soft multisets.

Theorem 3.4. Let (F, A) and (G, B) be two bijective soft multiset over the common universe U . $(F, A) \wedge (G, B) = (H, C)$ is also a bijective soft multiset.

Proof : It is a well known fact in soft multiset that $(F, A) \wedge (G, B) = (H, A \times B)$, where $H(a, b) = F(a) \cap G(b)$, $(a, b) \in A \times B$. Suppose $e \in A \times B$ is a parameter of (H, C) .

$$\text{Since } H(e) = F(a) \cap G(b).$$

$$\text{Therefore, } \cup_{e \in A} H(e) = \min(\cup_{a \in A} \cup_{b \in B} F(a) \cap G(b)) = \min(\cup_{a \in A} F(a) \cap (\cup_{b \in B} G(b))) = \min(U \cap U) = U$$

Suppose that $a_i, a_j \in C, a_i \neq a_j$. a_i is the Cartesian product of $a \in A$ and $b \in B, a_j$ is the Cartesian product of $c \in A, d \in B$. Then $H(a_i) \cap H(a_j) = (F(a) \cap G(b)) \cap (F(c) \cap G(d)) = \emptyset$.

Therefore, $(F, A) \wedge (G, B) = (H, C)$ is a bijective soft multiset.

Theorem 3.5. Suppose that (F, A) is a bijective soft multiset over U and (G, B) is a null soft multiset over U . Then $(F, A) \tilde{\cup} (G, B) = (H, C)$ is a bijective soft multiset.

Proof: recalling the definition of union of two soft set, we can write

$$(H, C) = (F, A) \tilde{\cup} (G, B) = \begin{cases} F(a), \text{ if } a \in A - B \\ \emptyset, \text{ if } a \in B - A \\ F(a) \cup \emptyset, \text{ if } a \in A \cap B \end{cases}$$

Where $a \in C$ and $(F, A) \tilde{\cup} (G, B) = (F, A \cup B)$ is a null soft multiset. Obviously $(H, C) = (F, A \cup B)$ is a bijective soft multiset over U .

Operations on Bijective soft multiset

Definition 3.6. Restricted AND Operation on Bijective soft multiset and a subset of Universe.

Let $U = \{S_1, S_2, S_2, S_3, S_4, \dots, S_n\}$ be a common universe, X be a subset of U and (F, A) be a bijective soft multiset over U .

The operation of “(F, A) restricted AND X” is denoted by $(F, A) \underline{\wedge} X$ is defined by $\cup_{a \in A} \{F(a) : F(a) \subseteq X\}$.

Example 3.7. Let (F, A) be a bijective soft multiset over a common universe U, $U = \{S_1, S_1, S_2, S_3, S_4\}$. Suppose $(F, A) = \{a_1 = \{S_1\}, a_2 = \{S_2, S_3\}, a_3 = \{S_1, S_4\}\}$. $X = \{S_2, S_3\}$.

From the definition 3.6, we can write $(F, A) \underline{\wedge} X = \{S_2, S_3\}$.

Definition 3.8. Relax AND Operation on bijective soft multiset and subset of Universe.

Let $U = \{S_1, S_2, S_3, S_3, S_4, S_5, \dots, S_n\}$ be a common universe, X be a subset of U and (F, A) be a bijective soft multiset over U.

The operation of "(F, A) relaxed AND X" denoted by $(F, A) \bar{\wedge} X$, is defined by $\cup_{a \in A} \{F(a) : F(a) \cap X \neq \emptyset\}$.

Example 3.9. Let (F, A) be a bijective soft multiset over a common universe U, $U = \{S_1, S_1, S_2, S_3, S_4\}$. Suppose that $(F, A) = \{a_1 = \{S_1\}, a_2 = \{S_1\}, a_3 = \{S_2, S_3, S_4\}\}$ and $X = \{S_2, S_3\}$. From the definition 3.8, we can write $(F, A) \bar{\wedge} X = \{S_2, S_3, S_4\}$.

The boundary region of the bijective soft multiset (F, A) with respect to X is $(F, A) \bar{\wedge} X - (F, A) \underline{\wedge} X = \{S_4\}$

Dependency between two bijective soft multisets

Definition 3.10. Suppose that (F, A), (D, C) are two bijective soft multiset over a common universe U, where $A \cap C = \emptyset$.

(F, A) is said to depend on (D, C) to a degree k ($0 \leq k \leq 1$), denoted by $(F, A) \rightarrow_k (D, C)$, if $k = \frac{\tilde{J}((F, A), (D, C))}{|\cup_{a \in C} (F, A) \underline{\wedge} D(a)|}$

Where $|\cdot|$ is the cardinal number of a set.

The concept of dependency is to describe a degree of bijective soft multiset in classifying the other one.

If $k = 1$, we say that (F, A) depends fully on (D, C),

if $k = 0$, we say that (F, A) does not depend on (D, C).

To show this concept, we give a relevant example below:

Example 3.11: Let us consider the bijective soft multiset given in example 3.1.

$F_4(\text{sparsely populated}) = \{S_1, S_3, S_6\}$,

$F_4(\text{good weather}) = \{S_1, S_2, S_4, S_5\}$,

$(F_1, A_1) = \{\{S_1, S_6\}, \{S_2, S_3, S_5\}, \{S_1, S_4\}\}$,

$(F_1, A_1) \underline{\wedge} F_4(\text{sparsely populated}) = \{S_1, S_6\}$,

$(F_1, A_1) \underline{\wedge} F_4(\text{good weather}) = \{S_1, S_4\}$.

From the definition 3.10

$$k = \tilde{J}((F_1, A_1), (F_4, D_4)) =$$

$$\frac{|(F_1, A_1) \underline{\wedge} F_4(\text{sparsely populated}) \cup (F_1, A_1) \underline{\wedge} F_4(\text{good weather})|}{|U|} = \frac{4}{7} = 0.57$$

The result 0.57 shows that the soft multiset (F_1, A_1) depends reasonably on the soft multiset (F_4, D_4)

Bijective Soft Multiset Decision System

Definition 3.12. Bijective Soft Multiset Decision System

Let (F_i, A_i) , ($i = 1, 2, 3, \dots, n$) be n bijective soft multiset over a common universe U, where $A_i \cap A_j = \emptyset$, ($i = 1, 2, 3, \dots, n; j = 1, 2, 3, \dots, n; i \neq j$), (G, B) is a bijective soft multiset over a common universe U. $B \cap A_i = \emptyset$, ($i = 1, 2, 3, \dots, n$), and we call it the decision soft multiset. Suppose $(F, A) = \cup_{i=1}^n (F_i, A_i)$, the triple $((F, A), (G, B), U)$ is called bijective soft multiset decision system over a common universe U.

In example 3.1, we consider a bijective soft multiset decision system $(\cup_{i=1}^n (F_i, A_i), (F_4, A_4), U)$. This bijective soft multiset decision system describes the sparsely populated nature of the states, weather condition of the states and other relevant information that may be required by a prospective investor.

Definition 3.13. Bijective soft multiset Decision system Dependency

Let $((F, A), (G, B), U)$ be a bijective soft multiset decision system, where $(F, A) = \cup_{i=1}^n (F_i, A_i)$ and (F_i, A_i) is a bijective soft multiset. (F, A) is described or called condition soft multiset. The bijective soft dependency between $(F_1, A_1) \wedge (F_2, A_2) \wedge \dots \wedge (F_n, A_n)$ and (G, B) is called bijective soft multiset decision system dependency of $((F, A), (G, B), U)$, denoted and defined by $k = \tilde{J}(\bigwedge_{i=1}^n (F_i, A_i), (G, B))$.

Theorem 3.14. Let $((F, A), (G, B), U)$ be a bijective soft multiset decision system, where $(F, A) = \cup_{i=1}^n (F_i, A_i)$ and (F_i, A_i) is a bijective soft multiset. K is a bijective soft multiset decision system dependency of $((F, A), (G, B), U)$. The dependency between $(\bigwedge_{i=1}^m (F_i, A_i))$ where $m \leq n$ and (G, B) is $\tilde{J}(\bigwedge_{i=1}^m (F_i, A_i), (G, B))$ and $\tilde{J}(\bigwedge_{i=1}^n (F_i, A_i), (G, B)) \leq k$.

In other words, the condition soft multiset of bijective soft decision of system can be explain the most detailed classification of decision soft multisets. Removing some bijective soft multisets of it can lose some vital information of the decision soft multiset. For instance, a peaceful state may be state not characterized by violence and armed robbery. However, if we only know that the state is free from attack by armed robbery, we cannot judge its peaceful condition completely for the absent information of other factors that affect the peaceful condition of the state. Therefore, more information (bijective soft

multiset) cannot result in to a bigger dependency on decision soft multiset.

Proof: Let $(H, C) = \bigwedge_{i=1}^n (F_i, A_i)$, $(J, K) = \bigwedge_{i=1}^m (F_i, A_i)$. By definition 3.10 and 3.12

$$\begin{aligned}
 k = (\bigwedge_{i=1}^n (F_i, A_i), (G, B)) &= \frac{|\cup_{e \in B} (H, C) \Delta D(e)|}{|U|} \\
 &= \frac{|\cup_{e \in B} \cup_{e \in C} \{H(a): H(a) \subseteq D(e)\}|}{|U|} \\
 &= \frac{|\cup_{e \in B} (J, K) \Delta D(e)|}{|U|} \\
 &= \frac{|\cup_{e \in B} \cup_{e \in C} \{J(a): J(a) \subseteq D(e)\}|}{|U|}
 \end{aligned}$$

From definition

$$\begin{aligned}
 H(e_1, e_2, e_3, \dots, e_n) &= F_1(e_1) \cap F_2(e_2) \cap F_3(e_3) \cap \dots \cap F_m(e_m) \cap \dots \cap F_n(e_n), \\
 \forall (e_1, e_2, e_3, \dots, e_n) &\in E_1 \times E_2 \times E_3 \times \dots \times E_n \\
 J(e_1, e_2, e_3, \dots, e_m) &= F_1(e_1) \cap F_2(e_2) \cap F_3(e_3) \cap \dots \cap F_m(e_m)
 \end{aligned}$$

$\forall (e_1, e_2, e_3, \dots, e_m) \in E_1 \times E_2 \times E_3 \times \dots \times E_m$, since $n > m$.

Therefore, $H(e_1, e_2, e_3, \dots, e_n) \subseteq J(e_1, e_2, e_3, \dots, e_m)$ and $\cup_{e \in C} H(e) = U$, $\cup_{e \in K} J(e) = U$.

Therefore, $|\cup_{e \in C} \{H(e): H(e) \in D(e)\}| \geq |\cup_{e \in K} \{J(e): J(e) \subseteq D(e)\}|$.

Thus, $\hat{J}(\bigwedge_{i=1}^m (F_i, A_i), (G, B)) \leq k$.

Reduction of Soft Multisets to Decision

Definition 3.15. Let $((F, A), (G, B), U)$ be a bijective soft multiset decision system, where $(F, A) = \cup_{i=1}^n (F_i, A_i)$ and (F_i, A_i) is bijective soft multiset, $\cup_{i=1}^m (F_i, A_i) \subseteq (F, A)$, k is the bijective soft multiset decision system dependency of $((F, A), (G, B), U)$. If $\hat{J}(\bigwedge_{i=1}^m (F_i, A_i), (G, B)) = k$, we say that $\cup_{i=1}^m (F_i, A_i)$ is a reduct of bijective soft multiset decision system $((F, A), (G, B), U)$.

Definition 3.16. Significance of Soft Multisets to decision soft multiset

Suppose that $(\cup_{i=1}^n (F_i, A_i), (G, B), U)$ is a bijective soft multiset decision system. The significance of bijective soft multiset to decision soft multiset, denoted by $\Delta((F_j, A_j) \cup_{i=1}^n (F_i, A_i), (G, B))$, is defined as follows:

$$\Delta \left((F_j, A_j) \cup_{i=1}^n (F_i, A_i), (G, B) \right) = k - \hat{J}((H, C), (G, B)).$$

Where, $(H, C) = \bigwedge_{i=1}^n (F_i, A_i)$, $(i \neq j)$.

The concept of significance of soft multisets to decision soft multiset is the decrease of soft multiset dependency when (F_j, A_j) is removed.

4.0 Conclusion

In this paper, we have introduced the concept of bijective soft multiset and defined some basic operation on it, such as the restricted AND, relax AND operation on bijective soft multiset, boundary region of bijective soft multiset with respect to a subset of universe, dependency between two bijective soft multisets, importance of bijective soft multiset with respect to bijective soft multiset decision system. Reduction of bijective soft multiset, with respect to soft multiset decision system.

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