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Comparative Dynamical Analysis of the Ratchet Potential with a Lagrange Polynomially Approximated Value

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Abstract

The ratchet potential gradient (PG) was approximated using Lagrange interpolation, to obtain a (a) Third degree polynomial (b) A fourth degree polynomial (c) A fifth degree Polynomial and (d) A tenth degree polynomial. These polynomials were used to obtain the attractors and time history, and the results were compared with attractor and the time series plot (TS) of the exact potential gradient.

1.0 Introduction

There is an increasing interest recently in the area of transport properties of nonlinear systems that extract usable work from unbiased nonequilibrium fluctuations [1].A ratchet can be modeled by considering a Brownian particle in a periodic asymmetric potential which is acted upon by an external time-dependent force of zero average. Noise-induced directed transport in a spatially periodic system in thermal equilibrium is ruled out by the second law of thermodynamics [2]. Therefore to generate transport requires Brownian motors or ratchet devices can be used for the modeling such systems.

Consider the equation of motion [1] a one-dimensional problem of a particle driven by a periodic time-dependent external force, under the influence of an asymmetric periodic potential of the ratchet type. The time average of the external deterministic force is zero. The dimensionless equation of motion is given by [1].

$$\ddot{x} + b_1\dot{x} + \tfrac{dv}{dx} = F_0\cos(\omega t) \tag{1}$$

where the ratchet potential is given by \cite{Vincent:06}where b_1 is the dimensionless friction coefficient, V(x) is the external asymmetric periodic potential F_0 is the amplitude of the external force and ω is the external driving frequency. The ratchet potential is given by

$$V(x) = c - \tfrac{1}{4\pi^2\delta}[\sin(2\pi(x - x_0)) + 0.25\sin(4\pi(x - x_0))] \tag{2}$$

where the potential is shifted by the amount x_0 in order that the maxima of the potential are located at integers and δ_1 is defined as $\sin(2\pi|x_0|) + 0.25\sin(4\pi|x_0|)$ the constant c is chosen such that V (0) = 0.giving 2C the value – ($\sin(2\pi x_0) + 0.25\sin(4\pi x_0)$ and x_0 is chosen to be 0.82 giving the values of δ_1, c and F_0 as 1.61432324 , 0.0173 and 0.08092845 respectively [4]

$$\ddot{x} + \omega_1^2 x = -b_1\dot{x} + \frac{1}{4\pi\delta}\left[2\cos\left(2\pi\left(x - x_0\right)\right) + \cos\left(4\pi\left(x - x_0\right)\right)\right] + \omega_1^2 x + F_0\cos\omega t \tag{3}$$

A lot of work has been done in recent times in the area of dynamics of oscillators under the influence of the ratchet potential a lot of results were obtained using computer simulations little was done on the analytic solutions due to the complex nature of the ratchet potential. This work is to look toward the analytic solution by approximating the PG with a Lagrange polynomial. We look for the approximate asymptotic solutions of equation (3) by using the method of multiple scales [5, 6] and compare with already existing simulated results.

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2.0 Results and Discussion

The PG were approximated by the Lagrange's polynomials

$f(x) = a_0 + a_1 x + a_2 x^2$ Where a_0 = -0.0105557,a_1= 0.335818 and a_2 = -0.335818,

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4$, where a_0 = -0.0105557,a_1= -0.501061, a_2 = 5.75165, a_3 = -12.4042 and a_4 = 7.15365 , $f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5$ where a_0 = -0.0105557,a_1= 1.69548, a_2 =-11.7413, a_3 = 34.1655, a_5 = -43.5321 and a_6 = 19.4124, and

$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + a_5 x^5 + a_6 x^6 + a_7 x^7 + a_8 x^8 + a_9 x^9 + a_{10} x^{10}$ where a_0 = -0.0105557,a_1= 1.31333, a_2 =-9.63468, a_3 = 77.7829, a_4 = -571.273, a_6 = 2424.3, a_6 = -5748.15, a_7 = 7839.35,a_8 = -6123.42, a_9 =2547.15 and a_{10} = -437.428 [7]

Figure 1 shows the result of the approximated potential gradient using the Lagrange's interpolation of different degrees compared with the exact PG. Equation (1) was evaluated using these approximated PG and the results were compared with the already existing simulated results using the exact PG [4]. The PG approximation from 4th degree polynomial happen to be the worst case of approximation to Equation (1), for the computed attractors and the time series plots figures 2 and 3, even though the approximate PG obtained from the 2nd degree polynomial is worse than the 4th degree approximation as can be seen in Figs1 (a) and (b). Fig 4 and 5 respectively show how the results improve as from the fifth degree polynomial and tenth degree polynomial as expected from Fig 1.

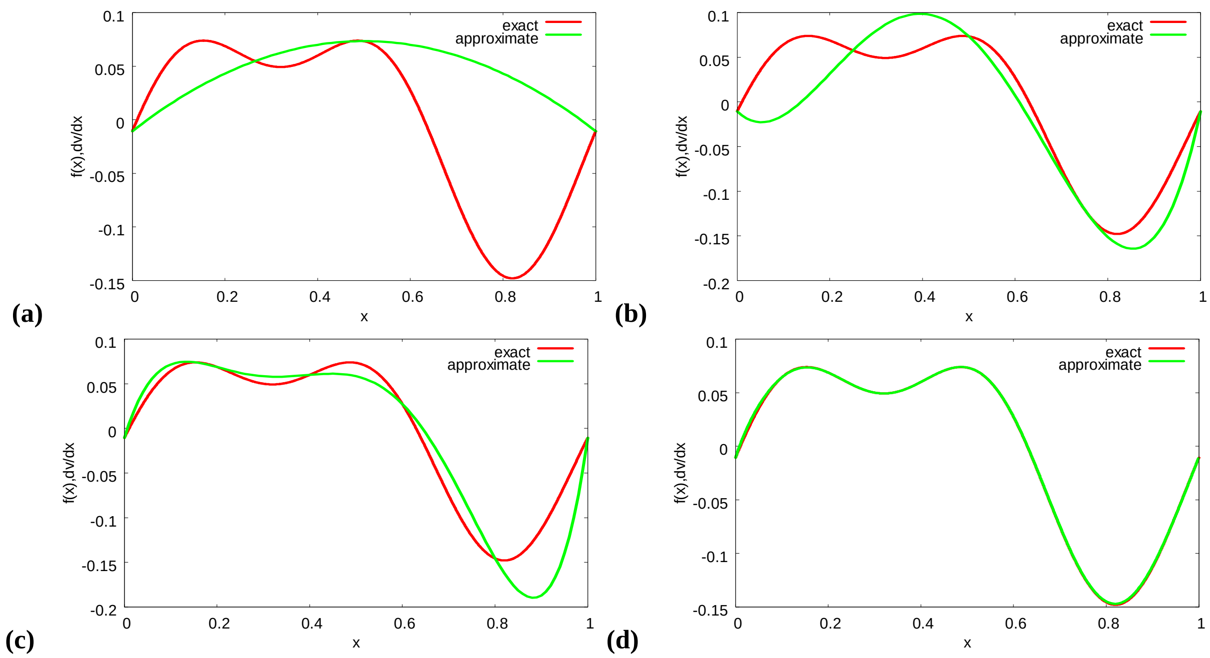

Figure 1:

(a) Is the potential gradient (PG) approximated by a 3 points polynomial compared with the exact PG**.** **(b)** Is the PG approximated by a 4 points polynomial compared with the exact PG

(c) Is the PG approximated by a 5 points polynomial compared with the exact PG **(d)** Is the PG approximated by a 10 points polynomial compared with the exact PG.

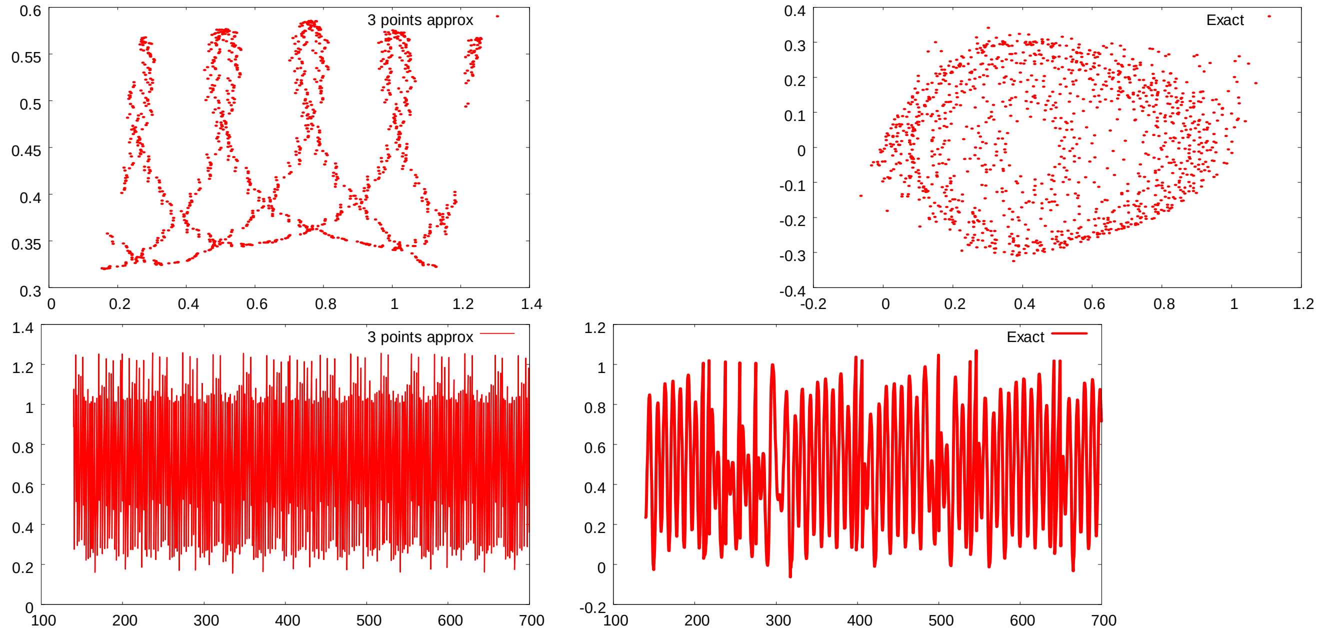

Figure 2:
(a) Is the attractor for the potential gradient (PG) approximated by a 3 points polynomial **(b)**is the attractor for the exact PG **(c)** Is the time series plot for the (PG) approximated by a 3 points polynomial **(d)** is the time series plot for the exact PG

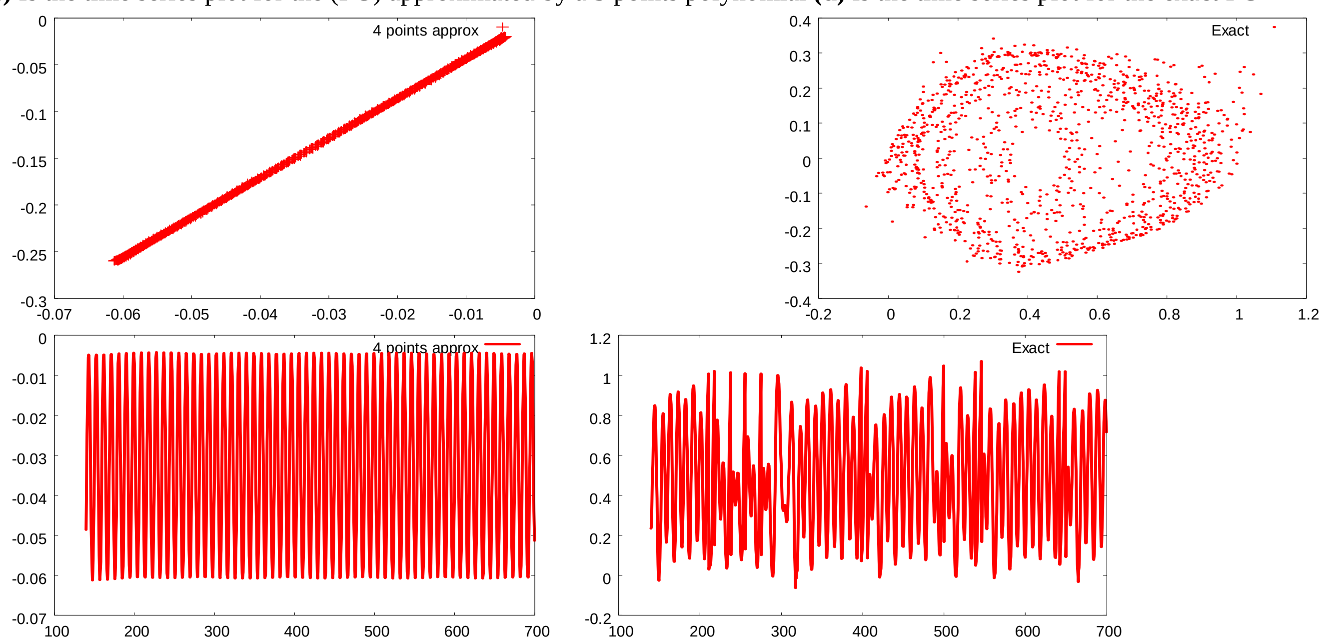

Figure 3:
(a) Is the attractor for the potential gradient (PG) approximated by a 4 points polynomial **(b)** is the attractor for the exact PG **(c)** Is the time series plot for the (PG) approximated by a 4 points polynomial **(d)** is the time series plot for the exact PG

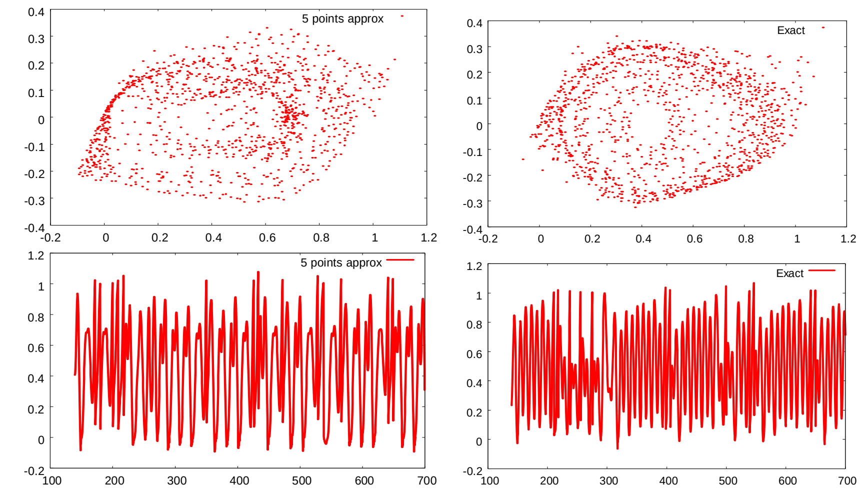

Figure 4:
(a) Is the attractor for the potential gradient (PG) approximated by a 5 points polynomial,**(b)i**s the attractor for the exact PG **(c)**Is the time series plot for the (PG) approximated by a 5 points polynomial**(d)**is the time series plot for the exact PG

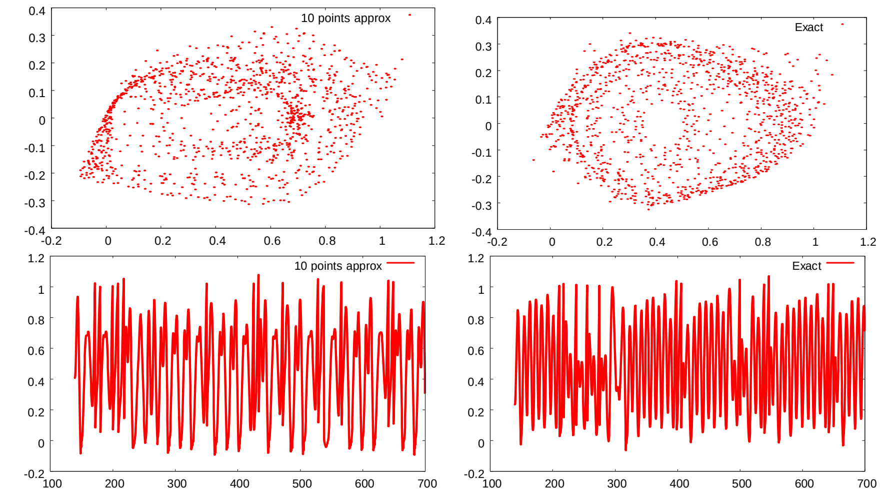

Figure 5:
(a) Is the attractor for the potential gradient (PG) approximated by a 10 points polynomial,**(b)i**s the attractor for the exact PG **(c)**Is the time series plot for the (PG) approximated by a 10 points polynomial**(d)**is the time series plot for the exact PG.

3.0 Conclusion

From Fig. 1 it can be seen the PG improves with the increase in the degree of the Lagrange interpolation polynomial, resulting in the improvement of the attractors and the time series plots for the cases of third, fifth and tenth degree polynomial. This improvement is not found for the case of the four point polynomial approximation, where both the attractor and the TS showed a periodic orbit as opposed to the exact chaotic attractor. In the case of the 3 point polynomial approximation a chaotic attractor was found which is completely different from the exact attractor. Remarkable resemblance was found for the five point and ten points polynomial approximation of the attractors and the exact ratchet potential equation given by equation (1).

4.0 Acknowledgment

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5.0 References

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