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Solution of the Dirac Equation for the q-parameter Poschl-Teller Potential Under Spin and Pseudospin Symmetries

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Abstract

The spin and pseudospin symmetric solutions of the Dirac equation with q-parameter Pöschl-Teller scalar and vector potentials including a coulomb-like tensor interaction term for arbitrary spin-orbit quantum number κ are presented. The energy eigenvalues and the unnormalized wave functions in terms of Jacob polynomials are obtained using the Nikiforov-Uvarov(NU) method by employing Pekeris approximation to the centrifugal term.

Keywords: q-parameter Poschl-Teller potential Nikiforov-Uvarov method, Spin and pseudospin symmetries.

1.0 Introduction

The problem of finding the exact or approximate solutions of the Dirac equation for a number of special potentials has been of great interest in recent time and many researchers have studied the bound states solution of the Dirac equation under the condition of spin and pseudospin symmetries [1-2]. The pseudospin symmetry in the Dirac formulation refers to the case where the magnitude of the attractive Lorentz scalar potential S(r) and the repulsive vector potential V(r) are equal but opposite in sign i.e S(r) = - V(r). However, approximate pseudospin symmetry is when the sum of the potential is $\sum(r) = S(r) + V(r) = c_{ps} = const. \neq 0$ [3-6]. The pseudospin symmetry is used to establish effective shell model [7]. The exact spin symmetry occurs when the scalar potential S(r) and vector potential V(r) are equal i.e S(r)=V(r). In nuclei, however the difference in these potential $\Delta(r) = V(r) - S(r) = c_s = const \neq 0$ [8]. The spin symmetry is relevant in meson [9]. On the other hand, the pseudospin symmetry has been successfully used for many different phenomena in nuclear structure such as the super deformation and identical bands. The pseudospin concept was first introduced in 1969 based on the experimental observation of quasi degeneracy in nuclei between single-nucleon states with non-relativistic quantum numbers $\left(n; l; j = l + \frac{1}{2}\right)$ and $\left(n-1; l+2; j = l + \frac{3}{2}\right)$ where n, l and j are the radial , the orbital and the total angular momentum quantum numbers respectively [10-11] These Symmeteies under various phenomenological potentials have been investigated by using various different mathematical techniques such as the super symmetry quantum mechanics (SUSYQM) [12], exact quantuization rule [13], the Nikiforove-Uvarov (NU) technique [14], asymptotic iteration method (AIM) [15], shape invariance (SI) [16] among others. In recent time, the study of Dirac equation with exponential-type potential models has attracted the attention of many authors in the field. These potentials include harmonic oscillator [17], Wood-Saxon [18], Manning-Rosen [19], Eckart potential [20], modified deformed Hylleraas [21], Pöschl-Teller [22, 23] and others. Maghsoodi et al [24] has investigated the Dirac equation with Pöschl-Teller double-ring-shaped coulomb potential under paeudospin symmetry and obtained the wave functions and the corresponding energy eigenvalues have been calculated.

The main aim of the present paper is to obtain approximate solutions of the Dirac equation with q-parameter Pöschl-Teller potential including the coulomb-like potential for the spin and pseudospin symmetries. We shall attempt to calculate the energy eigenvalues and the wave functions for arbitrary κ-state using Nikiforov-Uvarov method by employing the Pekeris approximation for the spin-orbit coupling term $\frac{\kappa(\kappa \pm 1)}{r^2}$. The organization of the paper is as follows:

The Nikiforov-Uvarov method is presented in section 2. Section 3 present the Dirac theory while the Result and discussion are given in sec.4. Finally, a brief conclusion is given in Sec.5.

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2.0 The Nikiforov-Uvarov Method

The NU method can solve a second-order differential equation of the form [14]

$$\sigma^2(s)\psi_n''(s)+\sigma(s)\tilde{\tau}(s)\psi_n'(s)+\tilde{\sigma}(s)\psi_n(s)=0, \tag{1}$$

where $\sigma(s)$ and $\tilde{\sigma}(s)$ are polynomials, at most of second degree, and $\tilde{\tau}(s)$ is a first-degree polynomial. To make the application of the NU method simpler and more direct, we introduce a more compact presentation of the idea. In order to do this, we rewrite Eq. (1) as follows [26]

$$\psi_n''(s)+\left(\frac{c_1-c_2s}{s(1-c_3s)}\right)\psi_n'(s)+\left(\frac{-\xi_1s^2+\xi_2s-\xi_3}{s^2(1-c_3s)^2}\right)\psi_n(s)=0, \tag{2}$$

in which

$$\psi_n(s)=\phi(s)y_n(s). \tag{3}$$

Comparing Eq. (1) with Eq. (2), we obtain the following identifications:

$$\tilde{\tau}(s)=c_1-c_2s,\quad \sigma(s)=s(1-c_3s),\quad \tilde{\sigma}(s)=-\xi_1s^2+\xi_2s-\xi_3, \tag{4}$$

Following the NU method [14, 26], we obtain the following required parameters:

(i) the relevant constant:

$$\begin{aligned}
&c_4=\frac{1}{2}(1-c_1), && c_5=\frac{1}{2}(c_2-2c_3),\\
&c_6=c_5^2+\xi_1, && c_7=2c_4c_5-\xi_2,\\
&c_8=c_4^2+\xi_3, && c_9=c_3c_7+c_3^2c_8+c_6.\\
&c_{10}=c_1+2c_4+2\sqrt{c_8} && c_{11}=c_2-2c_5+2\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)\\
&c_{12}=c_4+\sqrt{c_8} && c_{13}=c_5-\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)
\end{aligned} \tag{5}$$

(ii) the essential polynomial functions:

$$\pi(s)=c_4+c_5s-\left[\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)s-\sqrt{c_8}\right], \tag{6}$$

$$k=-(c_7+2c_3c_8)-2\sqrt{c_8c_9}, \tag{7}$$

$$\tau(s)=c_1+2c_4-(c_2-2c_5)s-2\left[\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)s-\sqrt{c_8}\right], \tag{8}$$

$$\tau'(s)=-2c_3-2\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)<0. \tag{9}$$

(iii) The energy equation:

$$c_2n-(2n+1)c_5+(2n+1)\left(\sqrt{c_9}+c_3\sqrt{c_8}\right)+n(n-1)c_3+c_7+2c_3c_8+2\sqrt{c_8c_9}=0. \tag{10}$$

(iv) The wave functions

$$\rho(s)=s^{c_{10}}(1-c_3s)^{c_{11}}, \tag{11}$$

$$\phi(s)=s^{c_{12}}(1-c_3s)^{c_{13}},\ c_{12}>0,\ c_{13}>0, \tag{12}$$

$$y_n(s)=P_n^{(c_{10},c_{11})}(1-2c_3s),\ c_{10}>-1,\ c_{11}>-1, \tag{13}$$

$$\psi_{n\kappa}(s)=N_{n\kappa}s^{c_{12}}(1-c_3s)^{-c_{12}-\frac{c_{13}}{c_3}}P_n^{\left(c_{10}-1,\frac{c_{11}}{c_3}-c_{10}-1\right)}(1-2c_3s) \tag{14}$$

where $P_n^{(\mu,\nu)}(x),\ \mu>-1,\ \nu>-1$, and $x\in[-1,1]$ are Jacobi polynomials with

$$P_n^{(\alpha,\beta)}(1-2s)=\frac{(\alpha+1)_n}{n!}\,{}_2F_1(-n,1+\alpha+\beta+n;\alpha+1;s), \tag{15}$$

and $N_{n\kappa}$ is a normalization constant. Also, the above wave functions can be expressed in terms of the hypergeometric function via

$$\psi_{n\kappa}(s) = N_{n\kappa} s^{c_{12}} (1 - c_3 s)^{c_{13}} \; {}_2F_1(-n, 1 + c_{10} + c_{11} + n; c_{10} + 1; c_3 s) \tag{16}$$

where $c_{12} > 0,\ c_{13} > 0$ and $s \in [0, 1/c_3],\ c_3 \neq 0.$ This method has been extensively used to solve various second-order differential equations in quantum mechanics such as Schrödinger, Klein-Gordon, Duffin-Kemmer-Petiau (DKP), spinless-Salpeter and Dirac equations [17-21].

3.0 Theory of Dirac Equation

The Dirac equation for spin-$\frac{1}{2}$ particles moving in an attractive scalar potential $S(r)$, a repulsive vector potential $V(r)$ and a tensor potential $U(r)$ in the relativistic unit $(\hbar = c = 1)$ is [20]

$$\left[\vec{\alpha}.\vec{p} + \beta(M + S(r)) - i\beta\vec{\alpha}.\hat{r}U(r)\right]\psi(r) = \left[E - V(r)\right]\psi(r), \qquad (17),$$

Where E is the relativistic energy of the system, $\vec{p} = -i\vec{\nabla}$ is the three dimensional momentum operator and M is the mass of the fermionic particle. $\vec{\alpha}, \beta$ are the 4×4 Dirac matrices given as

$$\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma}_i \\ \vec{\sigma}_i & 0 \end{pmatrix}, \beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}, \tag{18}$$

where I is 2×2 unitary matrix and $\vec{\sigma}_i$ are the Pauli three-vector matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{19}$$

The eigenvalues of the spin-orbit coupling operator are $\kappa = \left(j + \frac{1}{2}\right) \succ 0, \kappa = -\left(j + \frac{1}{2}\right) \prec 0$ for unaligned $j = l - \frac{1}{2}$ and the aligned spin $j = l + \frac{1}{2}$, respectively. The set $\left(H, K, J^2, J_z\right)$ forms a complete set of conserved quantities. Thus, we can write the spinors as [21],

$$\psi_{n\kappa}(r) = \frac{1}{r}\begin{pmatrix} F_{n\kappa}(r) Y^{l}_{jm}(\theta,\varphi) \\ iG_{n\kappa}(r) Y^{\tilde{l}}_{jm}(\theta,\varphi) \end{pmatrix} \tag{20}$$

where $F_{n\kappa}(r), G_{n\kappa}(r)$ represent the upper and lower components of the Dirac spinors. $Y^{l}_{jm}(\theta,\varphi), Y^{\tilde{l}}_{jm}(\theta,\varphi)$ are the spin and pseudospin spherical harmonics and *m* is the projection on the z-axis. Using well-known identities [24],

$$\begin{gathered} \left(\vec{\sigma}.\vec{A}\right)\left(\vec{\sigma}.\vec{B}\right) = \vec{A}.\vec{B} + i\,\vec{\sigma}.\left(\vec{A}x\vec{B}\right), \\ \vec{\sigma}.\vec{p} = \vec{\sigma}.\hat{r}\left(\hat{r}.\vec{p} + i\,\frac{\vec{\sigma}.\vec{L}}{r}\right) \end{gathered} \tag{21}$$

as well as the relations

$$\begin{gathered} \left(\vec{\sigma}.\vec{L}\right)Y^{\tilde{l}}_{jm}(\theta,\varphi) = (\kappa - 1)Y^{\tilde{l}}_{jm}(\theta,\varphi) \\ \left(\vec{\sigma}.\vec{L}\right)Y^{l}_{jm}(\theta,\varphi) = -(\kappa - 1)Y^{l}_{jm}(\theta,\varphi) \\ \left(\vec{\sigma}.\hat{r}\right)Y^{l}_{jm}(\theta,\varphi) = -Y^{\tilde{l}}_{jm}(\theta,\varphi) \\ \left(\vec{\sigma}.\hat{r}\right)Y^{\tilde{l}}_{jm}(\theta,\varphi) = -Y^{l}_{jm}(\theta,\varphi) \end{gathered} \tag{22}$$

we find the following two coupled first-order Dirac equation [20],

$$\left(\frac{d}{dr} + \frac{\kappa}{r} - U(r)\right)F_{n\kappa}(r) = \left(M + E_{n\kappa} - \Delta(r)\right)G_{n\kappa}(r), \tag{23}$$

$$\left(\frac{d}{dr}-\frac{\kappa}{r}+U(r)\right)G_{n\kappa}(r)=\left(M-E_{n\kappa}+\Sigma(r)\right)F_{n\kappa}(r), \tag{24}$$

where,

$$\Delta(r)=V(r)-S(r) \tag{25}$$

$$\Sigma(r)=V(r)+S(r) \tag{26}$$

Eliminating $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ in Eqs. (23) and (24), we obtain the second-order Schrödinger-like equations

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{\kappa(\kappa+1)}{r^2}+\dfrac{2\kappa U(r)}{r}-\dfrac{dU(r)}{dr}-U^2(r)-\left(M+E_{n\kappa}-\Delta(r)\right)\left(M-E_{n\kappa}+\Sigma(r)\right)\\ +\dfrac{\dfrac{d\Delta(r)}{dr}\left(\dfrac{d}{dr}+\dfrac{\kappa}{r}-U(r)\right)}{\left(M+E_{n\kappa}-\Delta(r)\right)}\end{array}\right\}F_{n\kappa}(r)=0, \tag{27}$$

$$\left\{\begin{array}{l}\dfrac{d^2}{dr^2}-\dfrac{\kappa(\kappa-1)}{r^2}+\dfrac{2\kappa U(r)}{r}+\dfrac{dU(r)}{dr}-U^2(r)-\left(M+E_{n\kappa}-\Delta(r)\right)\left(M-E_{n\kappa}+\Sigma(r)\right)\\ +\dfrac{\dfrac{d\Sigma(r)}{dr}\left(\dfrac{d}{dr}-\dfrac{\kappa}{r}+U(r)\right)}{\left(M-E_{n\kappa}+\Sigma(r)\right)}\end{array}\right\}G_{n\kappa}(r)=0, \tag{28}$$

where $\kappa(\kappa-1)=\tilde{l}(\tilde{l}+1), \kappa(\kappa+1)=l(l+1)$.

4.0 Pseudospin and spin Symmetry Limits under Coulomb-like tensor Interaction

In this section, we investigated Dirac equation with q-parameter Poschl-Teller potential in the presence of Coulomb-like tensor interaction.

4.1 Pseudospin Symmetry with Coulomb-like Tensor Interaction

The pseudospin symmetry occurs in the Dirac equation when $\frac{d\Sigma(r)}{dr}=0$ or equivalently $\Sigma(r)=C_{ps}=const$ [2-4]. To investigate the approximate analytical solution of the q-parameter Poschl-Teller potential, we consider the sum of the scalar and vector potential as [27],

$$\Delta(r)=\frac{\lambda(\lambda-1)e^{2\alpha r}}{\left(q+e^{2\alpha r}\right)^2} \tag{29}$$

Where $r\in(0,\infty)$ and the λ is the dissociation constant while q and α represent deformation and screening parameter respectively and in addition to the Coulomb-like tensor interaction term,

$$U(r)=-\frac{H_c}{r}, \qquad H_c=\frac{z_a z_b e^2}{4\pi\varepsilon_0}, \qquad r\geq R_e, \tag{30}$$

where $R_e=7.78$ fm is the Coulomb radius, z_a and z_b represent the charges of the projectile a and target nuclei b, respectively [25]. Substituting Eqs. (29)-(30) into Eq. (28), give

$$\left\{\frac{d^2}{dr^2}-\frac{\kappa(\kappa-1)}{r^2}-\frac{2\kappa H_c}{r^2}+\frac{H_c}{r^2}-\frac{H_c^2}{r^2}-\varepsilon_{ps}^2-\frac{\beta_{ps}\lambda(\lambda-1)e^{2\alpha r}}{\left(q+e^{2\alpha r}\right)^2}\right\}G_{n,k}^{ps}(r)=0, \tag{31}$$

Where

$$\begin{aligned}\varepsilon_{ps}^2&=\left(M+E_{n,k}\right)\left(M-E_{n,k}+c_{ps}\right)\\ \beta_{ps}&=\left(M-E_{n,k}+c_{ps}\right)\end{aligned} \tag{32}$$

It is well known that the above equation cannot be solved exactly due to the centrifugal term r^{-2} . In order to get rid of the centrifugal term, we make use of Pekeris approximation [33]

$$\frac{1}{r^2} \approx \varsigma \left\{ d_0 + \frac{d_1}{\left(q+e^{2\alpha r}\right)} + \frac{d_2}{\left(q+e^{2\alpha r}\right)^2} \right\}, \qquad \varsigma = \frac{1}{r_c^2} \tag{33}$$

Where r_c, is the equilibrium position.

Substituting Eq. (33) into Eq.(31) and applying the transformation, $s = e^{2\alpha r}$, Equation (31) yields

$$\frac{d^2 G_{n,\kappa}^{ps}}{ds^2} + \frac{\left(1+\frac{s}{q}\right)}{s\left(1+\frac{s}{q}\right)} \frac{dG_{n,\kappa}^{ps}}{ds} + \frac{1}{s^2\left(1+\frac{s}{q}\right)^2}\left[-\gamma_1^{ps} s^2 + \gamma_2^{ps} s - \gamma_3^{ps}\right] G_{n,\kappa}^{ps} = 0, \tag{34}$$

Where,

$$\gamma_1^{ps} = \left\{ \frac{\varepsilon_{ps}^2}{4\alpha^2 q^2} + \frac{\varsigma}{4\alpha^2 q^2} \eta_\kappa (\eta_\kappa - 1) d_0 \right\}, \tag{35}$$

$$\gamma_2^{ps} = -\frac{1}{4}\left\{ \frac{2\varepsilon_{ps}^2}{\alpha^2 q^2} - \frac{\beta_{ps}\lambda(\lambda-1)}{\alpha^2 q^2} + \frac{\varsigma}{\alpha^2 q^2}\eta_\kappa(\eta_\kappa - 1)\left(d_1 + 2qd_0\right) \right\}, \tag{36}$$

$$\gamma_3^{ps} = \frac{1}{4\alpha^2 q^2}\left\{ q^2 \varepsilon_{ps}^2 + \varsigma \eta_\kappa (\eta_\kappa - 1)\left(d_2 + qd_1 + q^2 d_0\right) \right\}, \tag{37}$$

$$H_c^2 + (2\kappa - 1) H_c + \kappa(\kappa - 1) = (H_c + \kappa)(H_c + \kappa - 1) = \eta_\kappa(\eta_\kappa - 1) \rightarrow \eta_\kappa = H_c + \kappa. \tag{38}$$

4.2 Spin symmetry with the Coulomb-like Tensor Interaction

In the spin symmetry limit case, we use:

$$\Sigma(r) = \frac{\lambda(\lambda-1)e^{2\alpha r}}{\left(q+e^{2\alpha r}\right)^2} \tag{39}$$

$$U(r) = \frac{H_c}{r}, \qquad \Delta(r) = C_s \tag{40}$$

Substituting Eqs. (39) and (40) into Eq.(27) yields,

$$\left\{ \frac{d^2}{dr^2} - \frac{\kappa(\kappa+1)}{r^2} - \frac{2\kappa H_c}{r^2} - \frac{H_c}{r^2} - \frac{H_c^2}{r^2} - \varepsilon_s^2 - \frac{\beta_s \lambda(\lambda-1)e^{2\alpha r}}{\left(q+e^{2\alpha r}\right)^2} \right\} F_{n,k}^s(r) = 0 \tag{41}$$

Where

$$\begin{aligned} \varepsilon_s^2 &= \left(M - E_{n,k}^s\right)\left(M + E_{n,k}^s - C_s\right), \\ \beta_s &= \left(M + E_{n,k}^s - C_s\right) \end{aligned}, \tag{42}$$

By replacing the centrifugal barrier term with Pekeris approximation of Eq. (33) in Eq.(41),then Eq.(41) leads to the following second order differential equation with the transformation $s = e^{2\alpha r}$,

$$\frac{d^2 F_{n,k}^s(s)}{ds^2} + \frac{\left(1+\frac{s}{q}\right)}{s\left(1+\frac{s}{q}\right)} \frac{dF_{n,k}^s(s)}{ds} + \frac{1}{s^2\left(1+\frac{s}{q}\right)^2}\left[-\gamma_1^s s^2 + \gamma_2^s s - \gamma_3^s\right] F_{n,k}^s(s) = 0, \tag{43}$$

where,

$$\gamma_1^s = \left\{ \frac{\varepsilon_s^2}{4\alpha^2 q^2} + \frac{\varsigma}{4\alpha^2 q^2} \eta_\kappa \left(\eta_\kappa + 1\right) d_0 \right\}, \qquad (44)$$

$$\gamma_2^s = -\frac{1}{4} \left\{ \frac{2\varepsilon_s^2}{\alpha^2 q} - \frac{\beta_s \lambda (\lambda - 1)}{\alpha^2 q^2} + \frac{\varsigma}{\alpha^2 q^2} \eta_\kappa \left(\eta_\kappa + 1\right)\left(d_1 + 2qd_0\right) \right\}, \qquad (45)$$

$$\gamma_3^s = \frac{1}{4\alpha^2 q^2} \left\{ q^2 \varepsilon_s^2 + \varsigma \eta_\kappa \left(\eta_\kappa + 1\right)\left(d_2 + qd_1 + q^2 d_0\right) \right\}, \qquad (46)$$

$$H_c^2 + \left(2\kappa + 1\right) H_c + \kappa\left(\kappa + 1\right) = \left(H_c + \kappa\right)\left(H_c + \kappa + 1\right) = \eta_\kappa \left(\eta_\kappa + 1\right) \to \eta_\kappa = H_c + \kappa. \qquad (47)$$

4.3 Pseudospin and Spin Symmetry Solutions with Coulomb Interaction

In this section, using the parametric generalization of the NU method [26] the solutions of Eqs. (34) and (43) are as follows:

4.3.1 Pseudospin Symmetry Solution with Tensor Interaction

By comparing Eq.(2) with Eq.(34),we obtain

$$c_1 = 1, c_2 = c_3 = -\frac{1}{q}, \qquad (48)$$

Other parameters can be obtain from Eq.(5) as,

$$\begin{aligned} &c_4 = 0, c_5 = \frac{1}{2q}, c_6 = \tfrac{1}{4q^2} + \gamma_1^{ps}, c_7 = -\gamma_2^{ps}, c_8 = \gamma_3^{ps}, \\ &c_9 = \tfrac{1}{q^2}\left(\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} + q\gamma_2^{ps}\right), c_{10} = 1 + 2\sqrt{\gamma_3^{ps}}, \\ &c_{11} = \frac{2}{q}\left(-1 + \sqrt{\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} + q\gamma_2^{ps}} - \sqrt{\gamma_3^{ps}}\right), \\ &c_{12} = \sqrt{\gamma_3^{ps}}, c_{13} = \frac{2}{q}\left(-1 + \sqrt{\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} + q\gamma_2^{ps}} - \sqrt{\chi_3^{ps}}\right) \end{aligned} \qquad (49)$$

Substituting Eqs.(48) and (49) into Eq.(10) the energy eigenvalues equation and its solution are obtained respectively as

$$n^2 + (n + \frac{1}{2}) - 2\left(n + \frac{1}{2}\right)\left(\sqrt{\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} + q\gamma_2^{ps}} - \sqrt{\gamma_3^{ps}}\right) + q\gamma_2^{ps} + 2\gamma_3^{ps} - 2\sqrt{\gamma_3^{ps}\left(\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} + q\gamma_2^{ps}\right)} = 0. \qquad (50)$$

Solving equation (50) completely, we obtain the energy eigenvalues for the deformed Poschl Teller potential with Pekeris approximation in the Dirac theory as follows:

$$\varepsilon_{ps}^2 = \alpha^2 \left(\left(n + \delta_{ps}\right) + \frac{\frac{\varsigma}{\left(4\alpha^2 q^2\right)} \eta_\kappa \left(\eta_\kappa - 1\right)\left(d_2 + qd_1\right)}{\left(n + \delta_{ps}\right)} \right)^2 - \frac{\varsigma}{q^2}\left[\eta_\kappa \left(\eta_\kappa - 1\right)\left(d_2 + qd_1 + q^2 d_0\right)\right], \qquad (51)$$

where,

$$\delta_{ps} = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{\varsigma}{4\alpha^2 q^2}\left\{\left(d_2 + (q-1)d_1 + 2q(q-1)d_0\right)\eta_\kappa\left(\eta_\kappa - 1\right) - \beta_{ps}\lambda(\lambda - 1)\right\}} \qquad (52)$$

The corresponding lower spinor wave function can be obtained from Eq.(14) using Eqs.(48-49) as

$$\psi_{n,k}^{ps}(r) = N_{n,k}^{ps} e^{2\alpha\sqrt{\gamma_3^{ps}}\, r} \left(1 + \frac{1}{q} e^{2\alpha r}\right)^{\left(\frac{1}{2} - \sqrt{\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} - q\gamma_2^{ps}}\right)} P_n^{2\sqrt{\gamma_3^{ps}},\, 2\sqrt{\frac{1}{4} + q^2\gamma_1^{ps} + \gamma_3^{ps} - q\gamma_2^{ps}}} \left(1 + \frac{2}{q} e^{2\alpha r}\right), \qquad (53)$$

and the other component can be obtained simply from

$$F_{n,k}^{ps}(r) = \frac{1}{M - E_{n,k}^{ps} + c_{ps}} \left(\frac{d}{dr} - \frac{k}{r} - \frac{H}{r} \right) \psi_{n,k}^{ps}(r), \tag{54}$$

Where $N_{n,\kappa}^{ps}$ is the normalization constant and $E_{n,\kappa}^{ps} \neq M + C_{ps}$

4.3.2 Spin Symmetry Solution with the Tensor Interaction

By comparing Eq. (2) with Eq.(43),the following parameters are obtained

$$c_1 = 1, c_2 = c_3 = -\frac{1}{q},$$

and

$$\begin{aligned} &c_4 = 0, c_5 = \tfrac{1}{2q}, c_6 = \tfrac{1}{4q^2} + \gamma_1^s, c_7 = -\gamma_2^s, c_8 = \gamma_3^s, c_9 = \frac{1}{q^2}\left(\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s \right), \\ &c_{10} = 1 + 2\sqrt{\gamma_3^s}, c_{11} = -\frac{2}{q} - \left(-1 + \sqrt{\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s} - \sqrt{\gamma_3^s} \right), \\ &c_{12} = \sqrt{\gamma_3^s}, c_{13} = \frac{1}{q}\left[\tfrac{1}{2} - \sqrt{\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s} + \sqrt{\gamma_3^s} \right]. \end{aligned} \tag{55}$$

The energy eigenvalues equation, its solution and the corresponding upper wave function of the Dirac theory for the q-parameter Poschl-Teller potential in the presence of Coulomb tensor interaction are obtained respctively as,

$$n^2 + (n + \frac{1}{2}) - (2n+1)\left(\sqrt{\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s} - \sqrt{\gamma_3^s} \right) + q\gamma_2^s + 2\gamma_3^s - 2\sqrt{\gamma_3^s\left(\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s \right)} = 0, \tag{56}$$

$$\varepsilon_s^2 = \alpha^2 \left[(n + \delta_s) + \frac{\frac{\varsigma}{4\alpha^2 q^2}\eta_\kappa(\eta_\kappa + 1)(d_2 + qd_1)}{(n + \delta_s)} \right]^2 - \frac{\varsigma}{q^2}\left[\eta_\kappa(\eta_\kappa + 1)\left(d_2 + qd_1 + q^2 d_0 \right) \right], \tag{57}$$

where,

$$\delta_s = \frac{1}{2} - \sqrt{\frac{1}{4} + \frac{\varsigma}{4\alpha^2 q^2}\left\{ \left(d_2 + (q-1)d_1 + 2q(q-1)d_0 \right)\eta_\kappa(\eta_\kappa - 1) - \beta_s\lambda(\lambda - 1) \right\}} \tag{58}$$

The corresponding wave function for the spin symmetry limit can be obtained as follows:

$$\psi_{n,\kappa}^s(r) = N_{n,\kappa}^s e^{2\alpha\sqrt{\gamma_3^s}r}\left(1 + \frac{1}{q}e^{2\alpha r} \right)^{\left(\frac{1}{2} - \sqrt{\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s} \right)} \times P_n^{\left(2\sqrt{\gamma_3^s}, -2\sqrt{\frac{1}{4} + q^2\gamma_1^s + \gamma_3^s + q\gamma_2^s} \right)}\left(1 + \frac{2}{q}e^{2\alpha r} \right) \tag{59}$$

where $N_{n,\kappa}^s$ is the normalization constant and the other component of the Dirac spinor can be found as,

$$G_{n,\kappa}^s(r) = \frac{1}{M + E_{n,\kappa}^s - C_s}\left(\frac{d}{dr} + \frac{\kappa}{r} + \frac{H}{r} \right)\psi_{n,\kappa}^s(r) \tag{60}$$

4.4 Conclusion

We have used the NU method to obtain approximate solutions of the Dirac equation for q-parameter-Poschl-Teller potential within the framework of spin and pseudospin symmetry limits. Based on the existing literature we should note that the Dirac equation with this potential under the Coulomb tensor interaction has not been considered before by using the NU method with Pekeris approximation. We have obtained explicitly the energy eigenvalues equations in a closed form and the corresponding wave functions expressed in terms of the Jacobi polynomials for the q-parameter-Poschl-Teller potential with Coulomb-like tensor interactions within the spin and pseudospin symmetry limits.

5.0 References

[1] J.N.Ginocchio, Phys.Rev.C 69,034318 (2004)
[2] A. N. Ikot, S. Zarrinkamar, B. H. Yazarloo, and H. hassanabadi, Chin. Phys.**B V**ol.23, 10 (2014)
[3] J.N.Ginocchio,Phys.Rev.Lett.78,436 (1997)
[4] P.R.Page, T.Goldman and J.N.Ginocchio,Phys.Rev.Lett 66,204 (2001)
[5] J.N.Ginocchio, Phys.Rep. 414, 165 (2005)
[6] J. N. Ginocchio and A. Leviatan, Phys. Lett.,Ser. B**425**, 1 (1998)
[7] M.K.Bahar and F.Yasuk, Chin.Phys.B 22,010301 (2012)
[8] E.Maghsoodi, H.Hassanabadi, S.Zarrinkamar and H.Rahimov, Phys.Scr.85,055007 (2012)
[9] F.Taskin, Int.J.Theo.Phys.48,1142(2000)
[10] C.S.Jia, P.Guo and X.L.Peng, J.Phys.A:Math.Gen.39,7737(2006)
[11] J.N.Ginocchio,Phys.Rev.Lett.95,252501(2005)
[12] E.Maghsoodi, H.Hassanabadi and S.Zarrinkamar, Few-Body Syst.53,535(2012)
[13] A.N.Ikot, E.Maghsoodi, S.Zarrinkamar and H.Hassanabadi, Few-Body Syst. Doi.10.1007/s00601-013-0701-6
[14] H.Ciftci,R.L.Hall and N.Saad, J.Phys.A36,11807(2003)
[15] A.F.Nikiforov and V.B.Uvarov, Special Functions of Mathematical Physics, (Birkhauser, Basel,1988)
[16] M.R.Setare and Z.Nazari, Acta Phys.Pol.B 40,2809(2009)
[17] H.Hassanabadi, S.Zarrinkamar and B.H.Yazarloo, Chin.J.Phys.50,788(2012)
[18] H.Hassanabadi, S.Zarrinkamar, S.Zarrinkamar and H.Rahimov, J.Math.Phys.53 022104(2012)
[19] O.Aydogdu and R.Sever, Few-Body Syst.47,193(2010)
[20] H.Hassanabadi, S.Zarrinkamar and H.Rahimov, Few-Body Syst.37,209(2012)
[21] A. N. Ikot, E. Maghsoodi, S. Zarrinkamar, L. Naderi and H. Hassanabadi, Few- body Syst. (2014)55:241-253
[22] O.Aydogdu, E.maghsoodi and H.Hassanabadi, Chin.Phys.B 22,010302(2013)
[23] H. Hassanabadi, B. H. Yazarloo, S. Rahimov, Commun. Theor. Phys. **57**,339 (2012)
[24] H. Hassanabadi, E. Maghsoodi, S. Zarrinkamar and H. Rahimov, Chin. Phys. **B 12,**120302 (2012)
[25] M. Eshghi, H. Mehraban.J. Sci. Res. **3**(2), 239 (2011)
[26] H. Hassanabadi, E. Maghsoodi and S.Zarrinkamar,Commun.Theor.Phys.58,807 (2012)
[27] S. M. Ikhdair, J.Math.Phys.52,052303(2012)
[28] A. N. Ikot, Few-Body Syst.53,549(2012)
[29] C. Tezcan and R. Sver, Int. J. Theor. Phys. 48,337(2009)
[30] O. A. Awoga, A. N. Ikot and J. B. Emah, Rev.Mex.de.Fisica,59,229(2013)
[31] F. Taskin and G. Kocak, Chin. Phys.B 19,090314(2010)
[32] H. Yukawa, Math. Soc.Jpn 17,48(1935)
[33] C. I. Pekeris, Phys.Rev. 45, 98(1934)
[34] E. Maghsoodi, H. Hassanabadi, H. Rahimov and S. Zarrinkamar, Chin. Phys. C 37, 043105(2013)