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The Effect of Dust Particles On Instability Due To Electron Beam In Ionospheric Plasma

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Abstract

Dusty plasma consists of electrons, ions, and charged dust particles observed in several astro- and space- physical environments, such as nebulas, cometary tails, planetary rings, and planetary ionospheres. This paper takes a look at the effect of these dust particles on the instability growth rate in ionospheric plasma by altering the input parameters of the normal beam plasma instability simulation of the Electrostatic One Dimensional Code (ES1), to include dust species. The results of the simulation which are given in form of plots of diagnostic parameters such as kinetic energy , as well as total energy as a function of time are found. The kinetic energy (E_1) ***of the hot electron beam was found to decrease with time, while the kinetic energies*** (E_2) ***of the warm ion, and*** (E_3) ***for the warm dust where found to increase with time, which is indicating that energy has been transferred from the hot electron beam to the warm plasma ions and dust. The total energy was found to be constant from time*** $t = 10w_p^{-1}tot = 20w_p^{-1}$***, but varies slightly by 6% for*** $t = 30w_p^{-1}$***, which is within the limit of numerical error. The growth rate was found to be approximately*** $0.04w_p$***, which is close to the theoretical value of*** $0.05w_p$***, from literature. The electrostatic electric field energy for mode1 was found to be the strongest for all the times*** $t = 10w_p^{-1}$ ***to*** $t = 30w_p^{-1}$***, followed by mode2 and mode3, which is in agreement with theory.***

Keywords: dusty plasma, ionospheric plasma, particle in cell, weighting, growth rate, electrostatic electric field energy.

1.0 Introduction

Much of known matter in the universe is in plasma state, comprising of about 99% of matter in the universe [1]. This quasineutral gaseous state of matter, consisting of electrons, positive ions and neutral atoms is found in interplanetary spaces, interstellar gas, supernova remnants, interior of stars, asteroid zones, planetary rings or radiation belts, cometary tails, magnetosphere and the earth's ionosphere [2].

Plasma as a collection of charged particles and neutral atoms, in motion, possess certain electrodynamics properties due to their interaction with space electromagnetic field. Theses electrodynamics properties or quantities are measured using data collected from space crafts attached with measuring instruments [3].

The ionosphere is a partially ionized medium, containing electrons with thermal energy, $Et_h \sim 0.03$–1eV. As a Plasma, having a density $n \sim 10^6 cm^{-3}$, it can support electromagnetic waves, and many other waves and instabilities. The ionosphere lies from about 80km above the earth to about 1000km. The ionospheric Plasma is not totally ionized but contains neutral particles such as hydrogen (H), oxygen (O) atoms; N_2, O_2 molecules and also bigger particles as dusty grains in Plasma. Typical magnitude of the earth's magnetic field is 10^{-5} T.

The sun is a source of charged particles. The solar wind is means by which activity in the sun is communicated to the earth through the magnetosphere. This interaction depends on a weak interaction field of few Tesla carried by the Plasma. The precipitation of charged particle has been observed, into the near regions of the magnetosphere and the outer ionosphere characteristic.

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There are a large variety of models for simulating Plasma and basically, they are of two types, particle models and fluid models. In a particle model, the motion of large number of charged particles in their self-consistent electric and magnetic fields are followed. In the fluid model, the Plasma fluid equations are adopted [4].

In this work, a study is made, by the method of particle simulation, of the effects of the incoming electron beams on some properties of dusty ionospheric Plasma system. The particle-in-cell (PIC) type of simulation, developed by Birdsall and Langdon [5] and commonly referred to as ES1, is used in the analysis. The key physical quantities that are gained in this simulation are exchange in energies between the participating particle species and growth rate of the excited waves due to Beam-Plasma instability [6].

1.1 Iono Spheric Plasma

A unique feature of natural plasma as found in the earth's ionosphere is the low density state and hence the infrequency of collisions between the particles (Collision-less plasma) however the long range Electrodynamics interactive forces between particles are effective and their synergism and collectivisim create spectacular and dynamic effects which give rise to phenomena such as aororal lights, radio emissions, currents that produce simultaneous occurrences such as noctilucent clouds (NLC), waves that produce polar mesospheric Echoes (PMSE) [7].

1.2 Dusty Plasma In Earth's Ionosphere

Plasma regime found in the Earth's ionosphere is a complex one which consists of electrons, positive ions (H^+), neutral atoms and molecules such as N_2^+, 0^+, $N\text{-}0^+$ and other foreign particles such as dielectrics (ice, mica, silicates etc) and metalloids (magnites, graphites etc) [8]. These constitute dust grains found in ionospheric plasma and this regime of plasma is called dusty plasma.

1.3 Previous Works

Plasmas streams can appear in a number of different ways in practice, but in our case here, we are considering stream of electrons from solar wind interacting with dusty Plasma in the Earth ionosphere, generating-instability growth (wave amplification), in the linear region initially and then amplitude fluctuation with characteristic damping in the non-linear region.

In recent studies, the expanding exhaust film of space shuttle was observed to condense into ice grains which are subsequently charged to create dusty Plasma in the Earth ionosphere [9]. Instabilities are excited creating dust acoustic waves due to relative drift between the charged dust particles and the background Plasma ions and electrons [10]. Studies have also shown that the dispersion relation of electrostatic mode in dusty Plasma gotten from experimental results agree with dispersion relation of dust acoustic wave, provided it is modified by a damping rate due to drag from neutrals on dust grains [11].

Kinetic model of Plasma has also attributed list damping to be due to dust-dust collision, which account for the pair correlation function of the dust particles in dusty Plasma [12].

Other studies of collective effects of Plasma in space environment and the Earth lower ionosphere have been carried out in[13], where fluid model has been used and drifting Maxwellian distribution of velocities assumed. Havness [14] examined the conditions for the onset of instability in Plasma model consisting of two charged dust distributions streaming relative to one another. De Angelis et al [7]studied propagation of ion acoustic waves in dusty Plasma in which their result showed the production of the low frequency electrostatic enhancement associated with dusty regions of Halley's comet as observed by space probes. Vega and Giotto in [15] investigated the existence of low frequency electrostatic wave in an un-magnetized collision-less dusty Plasma, which they suggested may have relevance to the low frequency noise in the F-ring of Saturn.

The effect of dust particles on the instability growth rate and threshold drift velocity for excitation of the instability was examined in[16].

Other wave modes including the linear properties of the dust acoustic wave in a Plasma where the ion and electron densities are given by Boltzman's distribution were investigated in[11]. In their observations, they pointed out the possibility of a novel dust acoustic wave in an unmagnetized dusty Plasmas. Rosenbergin [17] investigated the dust-ion acoustic instabilities using the Vlasov approach in which he suggested the possible application of these instabilities to various cosmic environment.

Finally for investigation of the damping waves, Melandso [16] used fluid model to study dust-acoustic wave with dust charge variation and found that this could lead to strong damping of the wave.

In this study we looked at the effect of streaming electron beam on warm dust molecules and ions in an ionospheric plasma by altering input parameters of the three stream instability simulation of the ES 1-D code to include the dust as a third species and study the growth rate of the instability and compare it with the theory. Section two consists of the theory, section three gives the numerical procedure, section four comprises of the results as well as discussions, and section five gives the conclusion.

2.0 Theoretical Frame Work

This section gives the theoretical basis of the instability that occurs in plasma as a result of streaming of charged particles to produce waves that grows.

2.1 Two Stream Instability

Consider a uniform cold Plasma in which the ions are stationary and the electrons have a velocity $\mathbf{V}_o$ relative to the ions. The linearized equations of motion for ions and electrons are then given by

$$Mn_o\frac{\partial\vec{v}_{i1}}{\partial t} = en_o\vec{E}_1 \quad (1)$$

$$mn_o\left[\frac{\partial\vec{v}_{e1}}{\partial t} + (\vec{v}_o\cdot\nabla)\vec{v}_{e1}\right] = -en_o\vec{E}_1 \quad (2)$$

We look for electrostatic waves of the form:

$$\vec{E}_1 = Ee^{i[kx-\omega t]}\hat{x} \quad (3)$$

Where $\hat{x}$ is the direction of $\vec{v}_{i1}$ and k. Substitution in the above equations yields

$$-i\omega Mn_o\vec{v}_{i1} = en_o\vec{E}_1,\ \text{or}\ \vec{v}_{i1} = \frac{Eie}{M\omega}\hat{x} \quad (4)$$

$$mn_o(-i\omega + ikv_o)\vec{v}_{e1} = -en_o\vec{E}_1,\ \text{or}\ \vec{v}_{e1} = -\frac{ie}{m}\frac{E\hat{x}}{\omega - k\vec{v}_o} \quad (5)$$

The ion equation of continuity gives

$$\frac{\partial n_{i1}}{\partial t} + n_o\nabla\cdot\vec{v}_{i1} = 0,$$

or

$$n_{i1} = \frac{k}{\omega}n_o\vec{v}_{i1} = \frac{ien_okE}{M\omega^2} \quad (6)$$

The electron equation of continuity is

$$\frac{\partial n_{e1}}{\partial t} + n_o\nabla\cdot\vec{v}_{e1} + (v_o\cdot\nabla)\vec{v}_{e1} = 0 \quad (7)$$

i.e $$(-i\omega + ikv_o)n_{e1} + ikv_o\vec{v}_{e1} = 0 \quad (8)$$

$$n_{e1} = \frac{kn_o\vec{v}_{e1}}{(\omega - kv_o)} = -\frac{iekn_o}{m(\omega - kv_o)^2}E \quad (9)$$

Since the unstable waves are highfrequency Plasma oscillators, we may not use the Plasma approximation but use Poison's equation given by

$$\varepsilon_o\nabla\cdot\vec{E}_1 = e(n_{i1} - n_{e1}) \quad (10)$$

i.e

$$ik\varepsilon_oE = (ie^2n_okE)\left[\frac{1}{M\omega^2} + \frac{1}{m(\omega - kv_o)^2}\right] \quad (11)$$

The dispersion relation is found upon dividing equation (11) by $ik\varepsilon_oE$to get

$$1 = \omega_p^2\left[\frac{m/M}{\omega^2} + \frac{1}{(\omega - kv_o)^2}\right] \quad (12)$$

where

$$\omega_p^2 = \frac{n_0e^2}{\varepsilon_0m} \quad (13)$$

Let us see if oscillations with real k are stable or unstable. Upon multiplying through by the common denominator, one would obtain a fourth-order equation for ω. If the roots ω_j are real, each root would indicate possible oscillation of the form:

$$\vec{E}_1 = Ee^{i[kx-\omega t]}\hat{x}$$

If some of the roots are complex, they will occur in complex conjugate pairs. Let these complex roots be written as:

$$\omega_i = \alpha_i + i\gamma_i \quad (14)$$

Where α and γ are re(ω) and Im(ω) respectively. The time dependence will now be given by

$$\vec{E}_1 = Ee^{i[kx-\alpha_1t]e^{\gamma_1t}}\hat{x} \quad (15)$$

Positive Im(ω) indicates an exponentially growing wave; negative Im(ω) indicates a damped wave. Since the roots ω_j occur in conjugate pairs, one of these will always be unstable unless all the roots are real. The damped roots are not self- excited and not of interest now.

The dispersion relation (12) can be analysed without actually solving the fourth-order equation. Let us define:

$$x = \frac{\omega}{\omega_p}\ \text{and}\ y = \frac{kv_o}{\omega_p} \quad (16)$$

Then equation (12) becomes:

$$1 = \frac{m/M}{x^2} + \frac{1}{(x-y)^2} = F(x,y) \quad (17)$$

For any given value of y, we can plot function F(x,y) against x. This function will have singularities at x = 0 and x = y .

The intersections of this curve with line F(x,y) = 1 give the values of x satisfying the dispersion relation. There are four intersections, so there are four real roots ω_i. However, if we choose a smaller value of y, now there are only two intersections and, therefore, only two real roots. The other two roots must be complex, and one of them must correspond to unstable wave. Thus, for sufficiently small $kv_0$the Plasma is unstable. For any given $V_o$the Plasma is always unstable to long wavelength oscillations. The maximum growth rate predicted by equation (12) is, for m/M$\ll$ 1:

$$Im\left(\frac{\omega}{\omega_p}\right) = \left(\frac{m}{M}\right)^{1/3} \tag{18}$$

Since a small value of$kv_0$is required for instability, one can say that for a given k, v_0 has to be sufficiently small for instability.

2.2 Particle In Cell Codeand ES1 Simulation

The model used here is the particle in cell(PIC) type [5], which originated from the work of Dawson[18]. In this model the plasma is represented by macroparticles, where each macro particle represents thousands of real particles. PIC code is able to integrate in time the trajectory of a huge number of charged particles in their self-consistent electrostatic fields.

The computational cycle starts with the knowledge of the initial particle position x_i and velocity v_i, from which the charge and current density at grid point is obtained by weighting or interpolation

$$\rho_j = \sum_i q_i S(x_j - x_i) \tag{19}$$

in which S is the particle shape function which replaces point charges with a finite sized charge cloud. Next the electric field E_j at the grid points is obtained by solving the Poisson equation

$$\nabla E(x_j) = -\nabla^2 \varphi = \rho(x_j)/\varepsilon, \tag{20}$$

using the fast Fourier Transform. The electric field at the grid point is then weighted or interpolated to the particles positions as $E(x_i)$ and then used to find the force $F(x_i)$ that is used to move the particles to new positions and velocities by integration of the Lorentz force using the leap-frog method

$$dv_i/dt = q_i\, E_i/m_i \tag{21a}$$

$$dx_i/dt = \ v_i \tag{21b}$$

The process is repeated for the total number of time steps. The computer runs producing snap shot of the diagnostics such as kinetic energy versus time, potential, electric field, e.t.c

3.0 Numerical Procedure

Our model of electron beam - dusty plasma system consists of three interacting particle species; a super-thermal electron beam, thermal ions, and thermal dust with Maxwellian velocity distributions. An input file in the ES1 code was created by modifying one of the available input files to suite our model. The input file in the required format is as shown below:

```
THREE STREAM INSTABILITY
nsp-----------l---------dt----------nt-----mmax----l/a
3      6.283185307    0.20       600    3          0
ng----iw----epsi-------a1------a2-------E0-----w0
1024   3    1.00       0.00    0.00      0        0

SPECIES 1: Super Thermal Electron Beam
n----nv2---nlg---mode
1024    0     1     1
wp----wc-----qm-----vt1----vt2---v0
1.0  0.00   -1.00  0.50   0.00  1.00
x1----v1---thetax--thetav
0.01  0.00   0.00    0.00

SPECIES 2: Warm Ion
n----nv2---nlg---mode
1024   0     1     1
wp----wc-----qm-----vt1----vt2---v0
1.00  0.00   1.00  0.10   0.00  0.00
x1----v1---thetax--thetav
0.01  0.00   0.00    0.00

SPECIES 3: Warm Dust Ion
n----nv2---nlg---mode
1024   0     1     1
wp----wc-----qm-----vt1----vt2---v0
0.001 0.00   0.1   0.10   0.00  0.00
```

x1----v1---thetax--thetav
0.001 0.00 0.00 0.00

Fig. 1a: The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t = 10w_p^{-1}

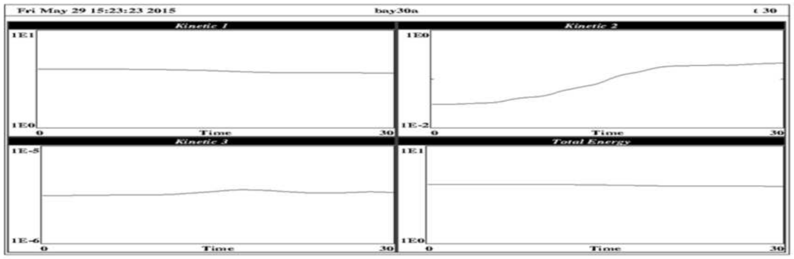

Fig. 1b: The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t = 20w_p^{-1}

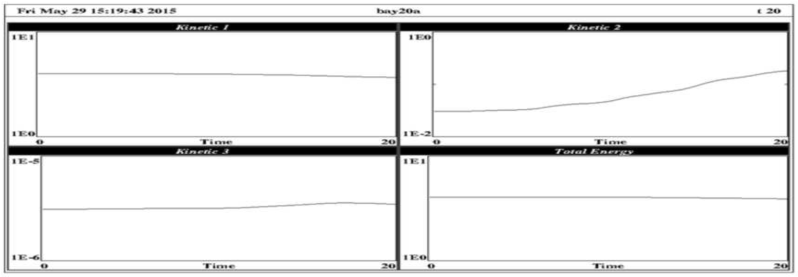

Fig. 1c: The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t = 30w_p^{-1}

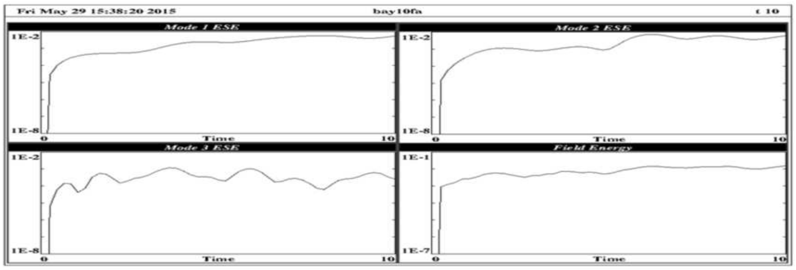

Fig. 2a: Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t = 10w_p^{-1}

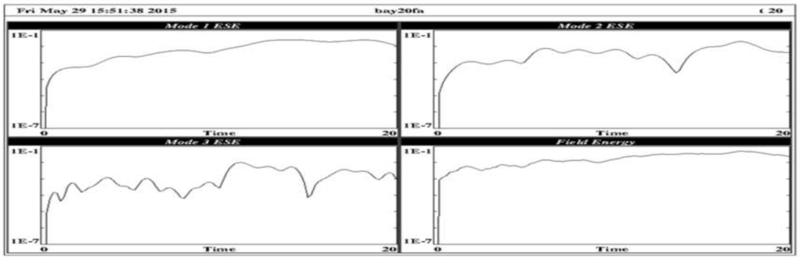

Fig. 2b: Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t = 20w_p^{-1}

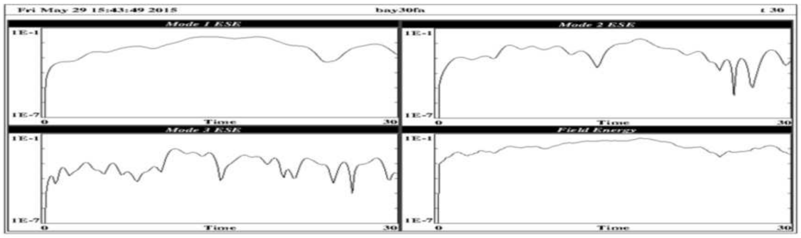

Fig. 2c: Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t = 30w_p^{-1}

4.0 Results and Discussions

The results of the simulation are given in form of the plots of the characteristics parameters used to describe the physical properties of the plasma model as a function of time. Figure 1a-1c show the evolution of the kinetic energies at t=10s.

Figure 1a. The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t=10w_p^{-1}.

Figure 1b. The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t=20w_p^{-1}.

Figure 1c. The graph of the Kinetic energy (KE) of each particle species and the total Kinetic energy at t=30w_p^{-1}.

The kinetic energy KE1, of the electron beam is seen to be decreasing with time. However, it is observed that the kinetic energy KE2 of the warm plasma ion increased with time, so also the kinetic energy KE3, of the warm dust particles. Hence onecan conclude that energy has been transferred to the warm dust particles and the warm plasma ion, from the hot electron beam. The total energy curve is seen to be almost constant with time.

Figure 2a. Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t=10w_p^{-1}.

Figure 2b. Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t=20w_p^{-1}.

Figure 2c. Graphs showing the time evolutions of the electrostatic energies of the first three excited modes, and the field energy at t=30w_p^{-1}

The electrostatic energies, named bayf10a,20a, and 30a, are modes of the Fourier decomposed electrostatic field energy. Figure 2 is the diagnostics for the Fourier transforms of the electrostatic energies, $ESE(k)i$ $(i = 1,2,3)$, of the first three excited modes of oscillation and the field energy FE.

In ESE1 linear growth and damping that occurs is due to collective interactions. Instability occurs due to the gain of energy from the electrons by the wave. Some of this energy is subsequently lost to the dust particles through Landau damping process. ESE2, and ESE3 shows some successive nonlinear growths and damping fluctuations which occur after the saturation of the instabilities.

5.0 Growth Rate

The growth rate is measured on the field energy semi-log plot, and is given+ by

$$\gamma = \frac{\Delta FE}{\Delta t}, \qquad (22)$$

the maximum growth rate is given theoretically in[19] as:

$$w_i = \frac{\sqrt{3}}{2}\left(\frac{m_e}{2m_i}\right)^{1/3} w_{pe} \approx 0.05 w_{pe} \qquad (23)$$

at the peak.

Table 1: Numerical values obtained from the simulation, where time is measured in units of plasma frequency, w_p.

Plasma diagnostics	t= 10 w_p^{-1}	t=20 w_p^{-1}	t=30 w_p^{-1}
KE1	4.0370	3.8310	3.7000
KE2	4.891x10^{-2}	1.874x10^{-1}	2.154x10^{-1}
KE3	3.162x10^{-6}	3.636x10^{-6}	3.391x10^{-6}
TOTAL KE	4.1080	4.1800	3.8990
ESE(K)1	7.305x10^{-3}	3.162x10^{-2}	2.848x10^{-2}
ESE(K)2	9.006x10^{-3}	2.565x10^{-2}	2.565x10^{-2}
ESE(K)3	1.520x10^{-3}	1.110x10^{-2}	1.233x10^{-2}
FE	0.01874	0.05926	0.05337
TE	4.1080	4.1800	3.8990
□	0.03$\square_{\square}$	0.035$\square_{\square}$	0.04$\square_{\square}$

The growth rate was found to be approximately 0.04$\square_{\square}$, which is close to the theoretical value of 0.05$\square_{\square}$. The total energy TE was found to be constant from t=10$\square_{\square}^{-1}$ and 20$\square_{\square}^{-1}$, but varies slightly for t=30$\square_{\square}^{-1}$by about 6%, which is within the limit of numerical error. The electrostatic electric field energy for mode 1 is the strongest for all the times t=10$\square_{\square}^{-1}$,20$\square_{\square}^{-1}$, and 30$\square_{\square}^{-1}$, followed by mode 2, and mode 3. This is consistent with theory.

6.0 Conclusion

A simulation of the electron beam-dusty plasma instability was carried out using the ES1-D code which uses the PIC numerical procedure. It was found that energy is transferred from the super-fast electron beam to the warm dusty plasma, but the total energy remains constant for all the time intervals considered. The electrostatic energy was found to be highest for the first mode, but decreases with the number of modes considered as shown in Table 1, and this is consistent with theory [5]. The growth rate of the instability was found from the field energy plots to be approximately 0.04$\square_{\square}$, which is close to the theoretical value of 0.05$\square_{\square}$, obtained from literature [19].

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