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Sensitivity Analysis in a Manpower Planning Model

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Abstract

Sensitivity analysis deals with the investigation into various changes in the optimum solution of a model due to changes in the original data. This paper examines the sensitivity analysis of a manpower planning model using linear programming and dynamic programming techniques which includes changes in the objective function coefficients and changes in the right hand side values of the DP model. It is observed that the number of staff recruited and retrenched or retired are equal to the maximum number of staff anticipated(H) in the manpower system. It is also observed that the objective function value is highest when H is increased by two units and the initial number of staff (h) is increased by one unit.

Keywords: Manpower Planning, Recruitment, Wastage, Linear Programming, Dynamic Programming.

1.0 Introduction

In mathematical modeling, we do take cognizance of significant controllable and uncontrollable variables as well as parameters of the model [1].The parameters as remarked in [2], are input variables which help to specify the relationship between other types of variables and for a given simulation the parameters have a constant value. In modeling we use the model input data to find not only an optimal solution, but we want also to determine what happens to the optimal solution when certain changes are made in the system. We would like to determine the effects of these changes without having to solve a new problem or a series of new problems. Investigations that deal with the changes in the optimum solution due to changes in the original data are called sensitivity analysis. The changes to which the system is subjected in analyzing the incremental behaviour of the optimal solution can represent either real changes that can be made in the operation of the physical system which the model represents or fictitious changes which are made to investigate the effects of uncertainty in the basic data. The practical application of a model can be hampered at times by imperfect knowledge of the necessary data or by a complete lack of it [3].

There are three major factors why many organizations are discouraged from allowing changes in the values of model parameters and uncontrollable variables of their systems. The critical factors are time, cost and risk. Despite the advantages of good planning that accrue in having the knowledge of the effects of changes in values of the model input on the output variables, these three factors often discourage firms from carrying out sensitivity analysis of system models as contained in [4]. This is because many firms always feel that changes in values of some input of new and complex problems is a time-consuming exercise even despite the availability of computer packages.

For practical problems that are often complex, the cost of obtaining optimal solution to series of new problems is enormous and often considered risky [5]. Based on this risk factor, the differences in observed risk propensity and their impact on firm performance is explored in [6]. A decision theoretical model which (a) measures a firm's risk propensity in the form of an "implied" utility function (b) investigates changes in corporate risk propensity with respect to changes in firm size and (c) examines the relationships between firm's risk propensities and alternative dimensions of economic performance is developed in [7]. The risk of a company's income stream for a given year by the variance in security analysts' forecasts of that income is discussed in [8].

Sensitivity analysis can be carried out in LP model and the changes in the LP problem that are usually investigated include:

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(1) changes in the Right Hand Side(RHS) values (2) changes in the coefficients of objective function and (3) changes in coefficients of the matrix (a_{ij}), $i = 1\ (1)\ m$ and $j = 1\ (1)\ n$. A lot of studies have been done in (3). For example, tolerance approach to perturbations in only one row or column of matrix coefficients is used in [9] while [10] went further to obtained the maximum approximate tolerance for multiple rows or columns of a perturbed matrix coefficients is carried out in [10]. Changes in matrix coefficients cannot be investigated in the DP model for manpower planning. This is because the coefficients of the matrix (a_{ij}) are either 0,1 or -1 (in structured arrangement) from the deviation of the constraints. Therefore the changes relevant to the LP model of the manpower problem are changes in the objective function coefficients as carried out in [11, 12] and changes in the RHS values of the DP model which is the focus of this paper.

2.0 Two-factor Dynamic Programming Model

The two factor DP model for manpower planning proposed in [13] has the following notations:

$x_j =$ number of staff that are on wastage in period j.

$y_j =$ number of staff that are recruited in period j.

$c_j =$ average accrued revenue to the organization from each wastage staff in period j by virtue of their exit from the system.

$c'_j =$ average salary per recruited staff in period j.

$h =$ initial number of staff on ground in the organization at the beginning of the time horizon.

$H =$ total number of staff at the end of the time horizon under consideration.

As we are dealing here with a dynamic situation, we divide the time span of interest into time intervals, which we shall assume to be sufficiently short so that we can consider $x_j(t),\ y_j(t),\ c_j(t)\ and\ c'_j(t)$ to be continuous during the time intervals but discontinuous from one time interval to the next.

The problem of manpower planning is to maximize the periodic additional revenue accruable to the organization from the wastage staff wage bill less the periodic salary of recruited staff i.e. $\sum_{j=1}^{n}(c_j x_j - c'_j y_j)$.

The objective function can be written as:

$$Maximize\ \ z = \sum_{j=1}^{n}(c_j x_j - c'_j y_j) \tag{1}$$

There are two sets of staffing constraints and two sets of nonnegativity constraints in this manpower planning problem.

(i) The overstaffing constraints:

$$\sum_{j=1}^{i}(y_j - x_j) = -\sum_{j=1}^{i}x_j + \sum_{j=1}^{i}y_j \le H - h,\ \ i = 1(1)n \tag{2}$$

(ii) The understaffing constraints

$$\sum_{j=1}^{i-1}(x_j - y_j) + x_i = \sum_{j=1}^{i}x_j - \sum_{j=1}^{i-1}y_j \le h,\ \ i = 1(1)n \tag{3}$$

(iii) Nonnegativity constraints:
The nonnegativity constraints are

$$x_j,\ y_j \ge 0,\ \ j = 1(1)n \tag{4}$$

Equation (1) stated above constitutes the total manpower planning cost from all the n periods while equations (1)-(4) constitute a DP problem which is stated thus:

Primal DP Problem

$$\left.\begin{aligned}
&Maximize\ \ z = \sum_{j=1}^{n}(c_j x_j - c'_j y_j)\\
&s.t.\\
&\quad -\sum_{j=1}^{i}x_j + \sum_{j=1}^{i}y_j \le H - h,\ \ i = 1(1)n\\
&and\ \ \sum_{j=1}^{i}x_j - \sum_{j=1}^{i-1}y_j \le h,\ \ i = 1(1)n\\
&\qquad x_j,\ y_j \ge 0\ ,\ \ j = 1(1)n
\end{aligned}\right\} \tag{5}$$

Further simplification of (5) yields the system in (6).

$$\left.\begin{array}{llll}
Max\ \ z = c_1x_1 + c_2x_2 + \ldots + c_nx_n - c_1'y_1 - c_2'y_2 - \ldots - c_n'y_n & & & \\
s.t. & & & \\
-x_1 & +y_1 & & \leq H-h \\
-x_1 - x_2 & +y_1 + y_2 & & \leq H-h \\
-x_1 - x_2 - x_3 & +y_1 + y_2 + y_3 & & \leq H-h \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
-x_1 - x_2 - x_3 - \ldots - x_n & +y_1 + y_2 + y_3 + \ldots + y_n & & \leq H-h \\
x_1 & & & \leq h \\
x_1 + x_2 & -y_1 & & \leq h \\
x_1 + x_2 + x_3 & -y_1 - y_2 & & \leq h \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots & & & \\
x_1 + x_2 + x_3 + \ldots + x_n & - y_1 - y_2 - y_3 - \ldots - y_{n-1} & & \leq h \\
x_j,\ y_j \geq 0,\ \ j = 1(1)n & & &
\end{array}\right\} \tag{6}$$

Let $d_1, d_2, \cdots, d_n$ be the first n dual variables for the first n constraints in system (6) and $e_1, e_2, \cdots, e_n$ be the last n dual variables for dual DP model of the manpower planning problem:

Dual DP Problem

$$Minimize\ w = (H-h)\sum_{i=1}^{n} d_i + h\sum_{i=1}^{n} e_i \tag{7}$$

s.t.

$$-\sum_{i=k}^{n} d_i + \sum_{i=k}^{n} e_i \geq c_k,\ \ k = 1(1)n \tag{8}$$

$$\sum_{i=k}^{n} d_i - \sum_{i=k+1}^{n} e_i \geq -c_k',\ \ k = 1(1)n \tag{9}$$

$$d_i,\ e_i \geq 0,\ \ i = 1(1)n \tag{10}$$

We define new variables D_k *and* E_k as follows:

$$D_k = \sum_{i=k}^{n} d_i,\ \ k = 1(1)n \tag{11}$$

$$E_k = \sum_{i=k}^{n} e_i,\ \ k = 1(1)n \tag{12}$$

In view of the definition of D_k *and* E_k, we see that non negativity of d_i *and* e_i will be ensured if we augment the dual LP problem, expressed in terms of D_k *and* E_k by the constraints:

$$D_k \geq D_{k+1}\ ,\ \ \ k = 1(1)\ n-1 \tag{13}$$

$$E_k \geq E_{k+1}\ ,\ \ k = 1(1)\ n-1 \tag{14}$$

3.0 Sensitivity Analysis

There is need to examine the effects of different changes in the number of employees initially on ground and at the end of the time horizon denoted by h *and* H respectively.

(a) Effect of keeping H unchanged and increasing only h:

Theorem 1

If H is kept unchanged and h is increased to H, then the objective function value (z) of the primal LP problem is increased to

$$z' = z + (H-h)(E_1 - D_1) \tag{15}$$

Proof

When h is increased by 1 unit, each of the upper half constraint limit of the primal LP model in system (6) is reduced by 1 unit while each constraint limit in the lower half is increased by 1 unit. The resultant effect is the increase by

$(\sum_{i-1}^{n} e_i - \sum_{i=1}^{n} d_i)$ of the primal objective function value (z). When h is increased to H, i.e. h increased by $(H-h)$, the new primal objective function value is $z' = z + (H-h)(E_1 - D_1)$, where $E_1 = \sum_{i=1}^{n} e_i \ and \ D_1 = \sum_{i=1}^{n} d_i$ as earlier defined. This completes the required proof. This theorem is numerically illustrated in section four.

Note: The primal objective function is $z = \sum_{j=1}^{n}(c_j x_j - c'_j y_j)$. When $(E_1 - D_1) > 0$, the net accruable revenue to the organization from human resources is increased per unit increase in h which is an advantage. The financial increment can be enhanced up to $(H-h)(E_1 - D_1)$ when h is increased to H. It will be economically disadvantageous to the organization to increase h (i.e. to start with higher periods with $h' = h + p,\ 0 < p \le (H-h) \ if \ (E_1 - D_1) < 0$.

(b) Effect of increasing H by two units and increasing h by one unit.

In this case, both the upper half and lower half constraints in system (6) will be increased by one unit. This leads to a higher new primal objective function value

$$z' = z + (H-h)(E_1 + D_1) \qquad (16)$$

This is so because there will be more output through increased upper boundary limits of all manpower constraints making it possible for the organization to attain full capacity production earlier than anticipated.

(c) Similarly, the following effects are considered.

(i) Effect of increasing H and keeping h unchanged

$$z' = z + \sum_{i=1}^{n} d_i = z + D_1 \qquad (17)$$

where D_1 is the financial increase per unit increase in H when h is unchanged.

(ii) Effect of increasing both h and H

$$z' = z + \sum_{i=1}^{n} e_i = z + E_1 \qquad (18)$$

where E_1 is the financial increase per unit increase in both h and H.

(iii) Effect of reduction in both h and H

$$z' = z - \sum_{i=1}^{n} e_i = z - E_1 \qquad (19)$$

where E_1 is the financial decrease per unit decrease in both h and H. This should be completely avoided.

4.0 Numerical illustration

Links between personnel and vacancy flows in a graded personnel system, focusing on outside hiring within a university community is discussed [14].The need for models that should estimate projected manpower for between ten and twenty years planning horizon is emphasized in [15]. Based on these critical remarks, we obtained the following data from XYZ College of Education in Nigeria (name of institution withheld for confidentiality) for both junior and senior staff of the institution.

Table 1: Average monthly salary of **junior staff** on wastage and recruitment.

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
c_j	33286	32045	35770	35918	36637	37552	38437	39126	33065	32281	38084	40124
c'_j	30148	32281	33665	34305	37545	34305	37894	36157	32981	30467	37688	36645

For Table 1,h=162 and H=393.

Table 2: Average monthly salary for **senior staff** on wastage and recruitment

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
c_j	127104	131223	135211	140421	142995	159213	162372	179084	180512	182750	184152	187289
c'_j	74372	76911	80625	83179	88370	91372	94246	96960	99124	102629	113893	118413

For Table 2, h=230 and H=600

We considered average monthly salary for a period of up to 12 years so that our results can give good estimates of staff wastage (x_j) and recruitment (y_j). Based on the present salary trend, we want to determine the optimal annual number of staff on wastage and recruitment that will maximize the total accruable revenue to the institution in the next 12years (i.e by the year 2024).

Solution by Linear Programming Approach

The primal LP model based on wastage and recruitment factors (for senior staff) is given below with the optimal tableau after 24 iterations in Fig. 1

$$
\begin{aligned}
Max\ z = {} & 127104x_1 + 131223x_2 + 135211x_3 + 140421x_4 + 142995x_5 + 159213x_6 + 162372x_7 + 179084x_8 + 180512x_9 + 182750x_{10} + 184152x_{11} + 187289x_{12} \\
& -74372x_{13} - 76911x_{14} - 80625x_{15} - 83179x_{16} - 88370x_{17} - 91372x_{18} - 94246x_{19} - 96960x_{20} - 99124x_{21} - 102629x_{22} - 113893x_{23} - 118413x_{24}
\end{aligned}
$$

$s.t.$

$$
\begin{array}{lll}
-x_1 & +x_{13} & \le 370 \\
-x_1 - x_2 & +x_{13} + x_{14} & \le 370 \\
-x_1 - x_2 - x_3 & +x_{13} + x_{14} + x_{15} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 & +x_{13} + x_{14} + x_{15} + x_{16} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} & \le 370 \\
-x_1 - x_2 - x_3 - x_4 - x_5 - x_6 - x_7 - x_8 - x_9 - x_{10} - x_{11} - x_{12} & +x_{13} + x_{14} + x_{15} + x_{16} + x_{17} + x_{18} + x_{19} + x_{20} + x_{21} + x_{22} + x_{23} + x_{24} & \le 370 \\
x_1 & & \le 230 \\
x_1 + x_2 & -x_{13} & \le 230 \\
x_1 + x_2 + x_3 & -x_{13} - x_{14} & \le 230 \\
x_1 + x_2 + x_3 + x_4 & -x_{13} - x_{14} - x_{15} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 & -x_{13} - x_{14} - x_{15} - x_{16} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} & \le 230 \\
x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10} + x_{11} + x_{12} & -x_{13} - x_{14} - x_{15} - x_{16} - x_{17} - x_{18} - x_{19} - x_{20} - x_{21} - x_{22} - x_{23} & \le 230
\end{array}
$$

$$x_1, x_2, \cdots, x_{24} \ge 0$$

Output
SIMPLEX
METHOD

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ON 0

BASE VAR	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24	X25	X26	X27	X28	X29	X30	X31	X32	X33	X34	X35	X36	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48
X25	370.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X26	370.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X27	370.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X28	370.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X29	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X30	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X31	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X32	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X33	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X34	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X35	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X36	370.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	-1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X37	230.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X38	230.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X39	230.00	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

X40 230.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
X41 230.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
X42 230.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00
X43 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00
X44 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
X45 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
X46 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00
X47 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 0.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00
X48 230.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00 -1.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00
z 0.0074372.0076911.0080625.0083179.0088370.0091372.0094246.0096960.0099124.00102629.00113893.00118413.00-127104.00-131223.00-135211.00-140421.00-142995.00-
179084.00-180512.00-182750.00-184152.00-187289.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

ITERATION 24

BASE VAR	VALUE	X1	X2	X3	X4	X5	X6	X7	X8	X9	X10	X11	X12
X13	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X14	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X15	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X16	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X17	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X18	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X19	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X20	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X21	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X22	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X23	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X24	600.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X1	230.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	600.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	600.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X4	600.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

BASE VAR	X13	X14	X15	X16	X17	X18	X19	X20	X21	X22	X23	X24
X13	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X14	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X15	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X16	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X17	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X18	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
X19	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
X20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
X21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
X22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
X23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
X24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
X1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

BASE VAR	X25	X26	X27	X28	X29	X30	X31	X32	X33	X34	X35	X36
X13	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X14	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X15	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X16	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X17	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X18	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
X19	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
X20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
X21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
X22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
X23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
X24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
X1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X4	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

BASE VAR	X37	X38	X39	X40	X41	X42	X43	X44	X45	X46	X47	X48
X13	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X14	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X15	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X16	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X17	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X18	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
X19	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00
X20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00
X21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
X22	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00
X23	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00
X24	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
X1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X2	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X3	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
X4	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

X5 600.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
X6 600.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00
X7 600.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00
X8 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00
X9 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00
X10 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00
X11 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00
X12 600.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 1.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00
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z 502856840.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00 0.00

0.0050193.0050598.0052032.0052051.0051623.0064967.0065412.0079960.0077883.0068857.0065739.00187289.0052732.0054312.0054586.0057242.0054625.0067841.0068126.008212
070259.0068876.00
MINIMUM AT z=
-502856840.00

Fig. 1: LP solution

From the optimal tableau (24th iteration), the optimal solution based on only the decision variables is given as: $x_1 = 230,\ x_2 = x_3 = x_4 = x_5 = x_6 = x_7 = x_8 = x_9 = x_{10} = x_{11} = x_{12} = 600,\ y_1(i.e.\ x_{13}) =$ $y_2(i.e. x_{14}) = y_3(i.e. x_{15}) = y_4(i.e. x_{16}) = y_5(i.e. x_{17}) = y_6(i.e. x_{18}) = y_7(i.e. x_{19}) = y_8(i.e. x_{20}) =$ $y_9(i.e. x_{21}) = y_{10}(i.e. x_{22}) = y_{11}(i.e. x_{23}) = y_{12}(i.e. x_{24}) = 600$ and z=₦502,856,840.

From the optimal tableau (24th iteration), the values of the dual variables in two groups are given in Table 3.

Table 3: Periodic optimal dual variables **(senior staff)**

Year	2001 (1)	2002 (2)	2003 (3)	2004 (4)	2005 (5)	2006 (6)	2007 (7)	2008 (8)	2009 (9)	2010 (10)	2011 (11)	2012 (12)
d_i	50193	50598	52032	52051	51623	64967	65412	79960	77883	68857	65739	187289
e_i	52732	54312	54586	57242	54625	67841	68126	82124	81388	80121	70259	68876

$$D_1 = \sum_{i=1}^{12} d_i = 866{,}604 \text{ and } E_1 = \sum_{i=1}^{12} e_i = 792{,}232,\ E_1 - D_1 = -74{,}372 \text{ and } E_1 + D_1 = 1658836$$

For **postoptimality** (i.e. sensitivity) analysis we examine:

(a) Effect of keeping H constant and increasing only h:

The objective function value becomes:

$$z' = z + (H - h)(E_1 - D_1)$$
$$= 502856840 + 370(-74372)$$
= ₦475,339,200

(b) Effect of increasing H by two units and increasing h by one unit:

The objective function value becomes:

$$z' = z + (H - h)(E_1 + D_1)$$
$$= 502856840 + 370(1658836)$$
= ₦1,116,626,160

c(i) Effect of increasing H and keeping h constant:

The objective function value becomes:

$$z' = z + D_1$$
$$= 502856840 + 866604$$
= ₦503,723,444

c(ii) Effect of increasing both h and H:

The objective function value becomes:

$$z' = z + E_1$$
$$= 502856840 + 792232$$
= ₦503,649,072

c(iii) Effect of reduction in both h and H:

The objective function value becomes:

$$z' = z - E_1$$
$$= 502856840 - 792232$$
= ₦502,064,608

Similar results have been worked out for junior staff and presented in Table 4.which shows the effect of the different types of changes on the objective function value in decreasing order for both junior and senior staff.

Table 4: New objective function values for different types ofchanges

Types of changes	Objective function value (z’)	
	Junior staff	Senior staff
(2) Reducing H by 2 units & increasing h by 1 unit	₦37,218,576	₦1,116,626,160
3(a) Increasing H and h is constant	₦17,726,896	₦503,723,444
3(b) Increasing both h and H	₦17,696,748	₦502,649,072
3(c) Reducing both h and H	₦17,642,268	₦502,064,608
(1) Increasing h and H constant	₦10,705,320	₦475,339,200

4.0 Discussion and Conclusion

In both the junior and senior staff problems of the xyz College of Education, based on the computer solutions, an increase in the RHS values of the first set of primal constraints always result in greater increase in the objective function value than that of the second set constraints (i.e. $D_1 > E_1$). In the two manpower planning problems (for junior and senior staff) examined, it is observed that many wastages (x_j) and recruitments (y_j) are equal to the expected capacity (H) of the organization. This means that the dynamic programming and linear programming models are also applicable to manpower planning problems relating to the number of people who are attending workshops, conferences, military courses and skill acquisition training programs that are often organized in batches and sponsored by Governments, NGOs and professional associations. This is because all the participants are often recruited and disengaged from such training programs in the same number.

For the post optimality analysis, Table 4 clearly shows that in both junior and senior staff categories, the objective function value is highest when H is increased by two units and h is increased by one unit. By this type of change every RHS value of the primal LP problem is increased by only one unit. We must bear in mind that h cannot be increased beyond H which is the target at the end of the time horizon. Each of the remaining four types of changes affect only one set of the RHS values of the primal DP problem. That is why their new objective function values are even lower than half of the highest new objective function value in both junior and senior categories. The lowest of all of them is the case of increasing h while H is unchanged. For this case the new objective function value (z') is lower than the optimal objective function value (z). This sensitivity analysis could not investigate whether the optimality conditions remain satisfied or not for the present optimal solutions in Fig.1 after the different types of changes. This is because some of the cases considered involve increasing h to H.

5.0 Refernces

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