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Comparison of the Reliability of Dry Cell Batteries.

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Abstract

The earlier use of the term, reliability, was purely qualitative; for example, aerospace engineers recognized the desirability of having more than one engine on an airplane and drivers keep spare tires in their vehicles without any precise measurement of the failure rate. As used today, however, reliability is a quantitative concept and this implies the need for a method of measuring reliability to eliminate some avoidable uncertainties. The objective of this study is to determine the reliability of Flash and Tiger Head dry cell batteries and to compare them. The result from the research indicates that the failure rates are λ_F = 0.269 for Flash batteries and λ_T = 0.497 for Tiger Head batteries. The reliability function are R(t) = $\ell^{-0.269t}$ for Flash batteries and R(t) = $\ell^{-0.497t}$ for Tiger Head batteries. Failure rate was established as a quality control parameter. Finally, failure-time distribution f(t) for both batteries are $f(t)_F = 0.269\ell^{-0.269t}$; t > 0 for Flash batteries and $f(t)_T = 0.269\ell^{-0.497t}$; t > 0 for Tiger Head batteries.

1.0 Introduction

Reliability is a word with many different meanings. When applied to human beings, it usually refers to that person's ability to perform certain tasks according to a specified standard. By extension the word is applied to a piece of equipment or a component of a large system to mean the ability of that equipment or component to fulfill what is required of it under a specified condition. The problem of assuring and maintaining reliability has many facets, including original equipment design, control of quality during production, life testing and design modifications. Reliability model is a dynamic mechanism, which could capture up the actual patterns of changes in any physical (machinery) system during its lifetime. There are two kinds of estimates; those based on the system operating and maintenance performance and those based on the model assumptions. The former reflect the engineering reality and the estimates are statistical (data-based) but the latter reflect only the mathematical modeling ideology [1]. We could also consider optimal replacement policy for stochastically failing equipment inaccessible to inspection. The policy was characterized by a single parameter, N. If equipment age is less than N, the appropriate action is to do nothing; if equal to N, the appropriate action is to replace the equipment. If it is repaired, the reliability of the system is restored to the state it was before the failure [2]. In this case, we talk of minimal repair and Non-homogeneous Poisson process (NHPP) is used to model the system if it shows a deteriorating failure rate; otherwise, a homogeneous Poisson process (HPP) is used if the system shows a constant failure rate. On the other hand, if that part of the affected system is entirely replaced, the reliability of the system will be as it were at the time of installation, in this case, we use Renewal process (RP) to model the system. The term minimal repair and perfect repair refer to NHPP or HPP and RP. We use the term as- bad- as old or same –as-old for minimal repairs and as – good as- new or same as new for perfect repairs. HPP has a constant failure rate λ, ie. HPP(λ) and NHPP has a failure rate that varies with time, ie. NHPP(λt).
Reliability is applied to model problems in the two broad classifications of systems, namely: **reparable and non-reparable systems**. A reparable system is a system which after failing to perform one or more of its functions satisfactorily can be restored to fully satisfactory performance by any method other than replacement of the entire system.

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Flash and Tiger Head dry cell batteries which are the main focus of this study belong to non-repairable system. A non-repairable system is a system that cannot be repaired if there is any failure to restore the system to its normal operational condition. That is, the system is discarded at first failure.

2.0 Theoretical Analysis

Several researches have been carried out to discover the reliability of components; systems; subsystems etc. Some research was done on the problems associated with measuring the reliabilities or survival of a system; components etc, and it was observed that most problems in reliability could be solved by adequate knowledge of mathematics and probability [3]. There is a need to quantify reliability. The most important reason for determining reliability should be on the economic basis [4]. It is cheaper to determine and maintain the reliability of a system than allowing it to fail before action is taken. In most cases, such a failure could cause irreparable lost. The best method of determining reliability is by life testing and it could be achieved by right or left censoring. They following statistical analysis such as Regression; Bayesian statistics; Maximum likelihood could be applied as modeling tools in determining reliability. Some researchers were of opinion that the reliabilities of components or systems depends on the usage and adaptability. In this case, reliability could be determined using Bathtub curve analysis and Weibull Modeling. Reliability can further be used to design and control the qualities of products [5]. Reliability is quantitative and can be used as a benchmark for quality control. Any product whose failure or control limit goes beyond a determined failure rate or fall below a determined reliability fall short of standard and therefore should be rejected. Empirical survival functions, $S^{\wedge}(t)$, could be estimated using Delta method. This approach established an easy way of calculating both survival and reliabilities of components or systems. Once the reliability of a system or its survival rate and its failure or intensity is determined, the distribution of that system can be obtained [6]. The reliabilities of components or systems could also be determined using a non-parametric approach. This can be done by constructed a table of data showing how the reliability, the probability density function and failure rate could be determined. The non-parametric method of finding reliability of a component or system does not depend on a known distribution; rather, the experimental units are distribution free [7]. Reliability; failure rate and failure-time distribution of a component or a system could again be estimated using the component count method. Through this approach, the individual component's failure rate is known with time and hence, the reliabilities [8]. The problem with this approach is that it is time consuming and may not be workable in situations where components are complex and difficult to observe by mere inspection. Reliability guarantees safety and confidence and must be quantified [9]. Direct mathematical calculations and the use of graphical methods have been found to be of immense help in estimating reliability. The graphical method is based on Logarithmic transformation, determination of the slope and intercept of the line from where the parameters of the model can be estimated. Some researchers used graphical method in estimating reliability. They showed how graphical method could be used to test for the goodness-of-fit for distributions using sample data [10]. This approach is more convenient as the parameters of the model that fit the data will be estimated from the graph without further calculations.

3.0 Experimental Work

The data used in this research work were entirely primary data collected from an experiment conducted in the department of Electrical Engineering Laboratory, University of Nigeria Nsukka at temperature $27^{o}C \pm 2^{o}C$. Two brands of batteries; Flash and Tiger Head (type R20 UM-1 D size) were used for the experiment. The instruments used include; (a). Stop watch for measuring the time. (b). Sunwa YX-360 TRES Multitester for measuring the potential difference (V); the current (I) and the resistance (R) of each batteries. (c). 1.5mm cable and tape. The batteries were put into life test for five hours and the reading at the beginning were taken and recorded. After three and five hours, other readings were taken and recorded. The outcomes of the experiment were presented in Tables 1, 2 and 3 respectively. In Table 1 prior to the experiment, Flash batteries have average power of 0.4 Watts while Tiger Head have average power of 0.3 Watts. After three hours into the experiment, the powers dropped to 0.1 Watts respectively. Finally, after five hours into the experiment, the power dropped to zero. The data in Table 2 shows the outcome of the repeat of the experiment a day after. The reading of the batteries before the experiment shows that both Flash and Tiger Head batteries have an average current of 0.150 (A) each. After one hour, the current dropped to 0.100 (A) and 0.300 (A) for Flash and Tiger Head batteries respectively. Also, the readings for the potential difference of both batteries were 0.900 (V) before the experiment as shown in Table 2. After one hour into the experiment, the potential difference dropped to 0.060 (V) and 0.100 (V) respectively. The power before the experiment were 0.135 (W) for both batteries. But an hour into the experiment shows a power drop of 0.006 and 0.030 respectively. In Table 3, between the interval of 296 – 297 (min), 4 Flash batteries and 7 Tiger Head batteries failed. Between the interval of 297 – 298 (min), 6 Flash batteries and 7 Tiger Head batteries failed. Between the interval of 298 – 299 (min), 3 Flash batteries and 7 Tiger Head batteries failed. Between the interval of 299 – 300 (min), 7 Flash batteries and 5 Tiger Head batteries failed. Between the interval of 300 – 301 (min), 10 Flash batteries and 4 Tiger Head batteries failed.

4.0 Sampling Technique Used

The researcher used a simple random sampling method to draw a sample of thirty (30) pairs each of Flash and Tiger Head batteries (type R20 UM-1 D Size) for the experiment.

5.0 Data Presentation

Table 1: Average Power (Watts) at time, t = 0, 3, 5 for Flash / Tiger batteries

Time (hrs)	Flash: Power(watts)	Tiger Head: Power(watts)
0	0.4	0.3
3	0.1	0.1
5	0.0	0.0

Table 2: Average Outcome of the Experiment a Day Latter, Time (hr)

	Flash (at T=0)	Flash (at T=1)	Tiger (at T=0)	Tiger (at T=1)
I (A)	0.150	0.100	0.150	0.300
V (V)	0.900	0.060	0.900	0.100
P (W)	0.135	0.006	0.135	0.030

Table 3: Time to Failure of Flash / Tiger Head Batteries (mins).

Time (mins)	296– 297	297 – 298	298 – 299	299 – 300	300 - 301	301 – 302
No. failure(F)	4	6	3	7	10	0
No. failure(T)	7	7	7	5	4	0

6.0 Results and Discussion

Fitting Distribution to Sample Data

The empirical survival function, $S^{\wedge}(t)$, is defined by

$$S^{\wedge}(t) = \frac{1}{n} \text{ (Number of observation} \geq \text{t).} \quad \ldots\ldots\ldots\ldots \quad (1)$$

The survival function is a non-increasing step function with steps at the observed lifetimes. It is a non-parametric estimator of S(t) in the sense that it does not depend on assumptions relating to any specific probability model.

Goodness–of-fit Test (Graphical Method) One of the most useful methods of fitting distribution to both censored and uncensored data in reliability is by the use of graphical method. Perhaps the simplest methods are those based on fitting survival functions by eye to the sample data. One cannot, of course, easily draw an exponential, Weibull or lognormal survival function freehand. So, instead, we transform the problem so that the survival function is a straight line. However, for some simple models a transformation of the empirical survival function plot can be obtained easily so that the transformed plot should be roughly linear if the model is appropriate.

Exponential Goodness-of-fit Test: The approach here is to transform the survival function

$$S^{\wedge}(t) = \exp\{-\lambda t\} \Rightarrow -Ln\,(S^{\wedge}(t)) = \lambda t \quad \ldots\ldots\ldots\ldots \quad (2)$$

Where λ is the slope of the line and the plotting points are t_i and – Ln($S^{\wedge}(t)$). A rough linear plot through the origin indicates a constant hazard, λ, and hence an exponential model. The time / transformed survival function were presented in table 4 and 5 for Flash and Tiger Head batteries respectively. The survival function was estimated from equation (1) and its transform was based on equation (2).

Table 4: Time / Survival Function Table for Flash Batteries

Time(min)	**296 - 297**	**297 – 298**	**298 – 299**	**299 - 300**	**300 – 301**	**301 – 302**
$S^{\wedge}(t)$	1.000	0.8666	0.6666	0.5666	0.3333	0.000
– ln $S^{\wedge}(t)$	0.000	0.1432	0.4065	0.5681	1.0987	-

Table 5: Time / Survival Function Table for Tiger Head Batteries

Time(min)	**296 - 297**	**297 – 298**	**298 – 299**	**299 - 300**	**300 – 301**	**301 – 302**
$S^{\wedge}(t)$	1.000	0.7666	0.5333	0.3000	0.1333	0.0000
– ln $S^{\wedge}(t)$	0.000	0.2658	0.6287	0.2040	2.0152	-

A plot of – Ln($\hat{S}(t)$) against time (min) from Table 4 and 5 gives a linear graph that passed through the origin, from where we can determine the slope of the line λ . Again, the best line of fit was obtained using the method of least squares.

$$y_i = \alpha + \beta t_i \text{ ; i = 1, …, 5.} \qquad (3)$$

$$\hat{\alpha} = \bar{y} - \hat{\beta}\,\bar{t} \text{ and } \hat{\beta} = \frac{n\sum_{i=1}^{n} yt - \sum_{i=1}^{n} y \sum_{i=1}^{n} t}{n\sum_{i=1}^{n} t^2 - \left(\sum_{i=1}^{n} t\right)^2} \qquad (4)$$

Where α and β are the parameters to be determined by substituting the values in Table 6 and 7 into equation (4) for both Flash and Tiger Head batteries respectively. Equation (4) is a sample parameter used in estimating the population parameter in equation (3).

Table 6: The Least Square Estimation Table for Flash Battery.

Y	T	Yt	t^2	y*
0.0000	296.0000	-	87616.0000	
0.1432	297.0000	42.5300	88209.0000	
0.4070	298.0000	121.2900	88804.0000	0.4400
0.5700	299.0000	170.4300	89401.0000	
1.1000	300.0000	330.0000	90000.0000	0.9800
2.2200	1490.0000	664.2500	444030.0000	

$\hat{\alpha}$ = - 79.718 and $\hat{\beta}$ = 0.269

$$\therefore y = -79.718 + 0.269 t_i \text{ ; i= 1, . . . , 5.} \qquad (5)$$

Equation (5) is the fitted line and a plot of y* against t_i gives the best line of fit and its slope, λ = 0.2690. This also agrees with the result of plotting – Ln($\hat{S}(t)$) against t_i from Table 4.

Table 7. The Least Square Estimation Table for Tiger Head Battery.

Y	T	Yt	t^2	y*
0.0000	296.0000	-	87616.000	
0.2700	297.0000	80.1900	88209.0000	
0.6300	298.0000	187.7400	88804.0000	0.8200
1.2000	299.0000	358.8000	89401.0000	
2.0200	300.0000	607.0000	90000.0000	1.8200
4.1200	1490.0000	1232.7300	444030.0000	

$\hat{\alpha}$ = - 147.282 and $\hat{\beta}$ = 0.497 and

$$\therefore y^* = -147.282 + 0.497 t_i \qquad (6)$$

Equation (6) is the fitted line and a plot of y*against t_i gives the best line of fit and its slope, λ = 0.497. This also agrees with the result of plotting – Ln($\hat{S}(t)$) against t_i from Table 5.

A line could be fitted by least squares but the usual requirement of independence and constant variance are lacking here. But a number of researches using such methods seemed to give similar estimates in spite of the lack of any adequate theoretical justification. For most practical purposes fitting by eye is perfectly satisfactory; if the fit is good there is little room for serious error and if the fit is poor no amount of sophistication in the fitting procedure will compensate for an inadequate model [10]. Hence, exponential distribution fit the sample data in this study.

Parameters of the Model:

λ_F = 0.269 and λ_T = 0.497;

Where λ_F and λ_T are the failure rate of Flash and Tiger Head batteries respectively.

MTTF $= \alpha_F = 3.72$ and $\alpha_T = 2.01$

where α_F and α_T are mean time to failure of Flash and Tiger Head batteries respectively.

Reliability $R_F(t) = \ell^{-\lambda t} = \ell^{-0.269\,t}$ ……………………………………….. (7)

Reliability $R_T(t) = \ell^{-\lambda t} \;\; \ell^{-0.497\,t}$ ……………………………………….. (8)

Equation (7) and (8) are the reliabilities of Flash and Tiger Head batteries respectively. Other distributions such as Weibull and Lognormal distributions were tested but they failed.

Failure -Time Distributions

$f(t) = \lambda.R(t)$; t > 0. ..……………………………………………………… (9)

$\therefore \;\; f(t)_F = 0.269\ell^{-0.269t}$; t > 0. …………………………………………... (10)

and

$f(t)_T = 0.497\ell^{-0.497t}$; t > 0. ……..……………………………………… (11)

Equation (9) is the failure – time distribution function or the probability density function (pdf). Equations (10) and (11) are the failure – time distribution function of both Flash and Tiger Head batteries respectively.

Comparison Based on Analysis

Flash batteries have a failure rate λ_F = 0.269 and Tiger Head batteries have a failure rate, λ_T = 0.497. Also, the mean time to failure of Flash battery is greater than that of Tiger Head battery, that is, $\alpha_F = 3.72$ and $\alpha_T = 2.01$

Comparison Based on Observed Time to Failure

From Table 3, ten Flash batteries survived up to five hours while only four Tiger Head batteries survived up to five hours. From Table 2, the result shows that Tiger Head batteries can be used further while Flash batteries cannot be used further because of the observed overflow of chemical used in producing it, while the reverse was the case for Tiger Head batteries.

Failure Rate as a Quality Control Parameter:

At the commencement of any manufacturing process, the specification (quality) to be met by the product to be manufactured is usually made since any manufacturing process is subject to variations; every effort is made during the production of the material to meet the required specification (quality). Also, at the end of production, the produced items are inspected to see whether they meet the desired quality before being passed on to the consumers. Hence, from our research, a Flash battery whose failure rate exceeds λ_F = 0.269 and its reliability fall below $\ell^{-0.269\,t}$ is out of control. Also, a Tiger Head battery whose failure rate exceeds λ_F = 0.497 and its reliability fall below $\ell^{-0.497\,t}$ is out of control

7.0 Conclusion

Both the Flash and the Tiger Head batteries have average failure rate, λ_F = 0.269 and λ_T 0.497 respectively. The reliability is improving if λ_F < 0.269 and λ_T < 0.497 but deteriorating if otherwise. The failure - time distribution for both batteries were obtained with the failure time distribution $f(t)_F = 0.269\ell^{-0.269t}$ and $f(t)_T = 0.497\ell^{-0.497t}$. From the failure - time distribution, the reliability and failure rate of Flash and Tiger Head batteries can be obtained at any given time.

8.0 References

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