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Application of Geometric Programming in Modelling of Solid Waste Products (Refuse): A Contribution In Combating Pollution, Uncontrolled Spending And Climate Change.

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Abstract

Climate change is the common word these days as it portrays massive alterations in weather patterns around the world and disruptions to economic growth in many regions. Poor waste management not only degrade the environment but cause climate change; Global warning; depletion of Ozone layer; rapid accumulation of greenhouse gases (GHGs) in the atmosphere; erosion; desertification; acid rain; draught, earthquake, tsunamis, tornadoes, increased intensity of sunlight, global food crisis; variation in seasons; flooding etc. In this study, the researchers were able to model the solid waste products (refuse) and the cost of waste management in Enugu State per month was determined. This will help in making informed decision; create employment opportunities, healthy environment, economic growth and stability, etc. Combating climate change is in line with achieving sustainable development.

1.0 Introduction

Climate change emanates from harmful practices in the environment and it is imperative to find a lasting solution to this rising trend. Solid waste products; domestic and industrial wastes are classified into biodegradable and non-biodegradable substances [1]. Nitrogen sulphide, Nitrous oxide, Nitrogen oxide, Methane, Carbon dioxide, etc. that ooze out of the decayed refuse dumps pollute the environment and these belong to greenhouse gases which cause global warming and climate change [2]. Refuse dumps cause spread of diseases and death to living organisms and if these refuse are burnt, it pollute the air and constitute aerosols which causes acid rain, health hazards and climate change [3,4]. Land degradation reduces the quality and productivity of land. Causes of land degradation include poor waste management, water and wind erosion, drought and desertification, acidification and salt accumulation, mineral extraction and heavy metal contamination [5]. A good waste management system should be profit oriented. The waste management authority is expected to make reasonable income from the waste products they gather by converting them into other processed forms that will attract revenue, create market and employment opportunities. The non-biodegradables can be recycled, which is cost effective than embarking on a fresh production process of such products. The biodegradables cannot be preserved for a very long time and cannot attract appreciable revenue; hence, it is pertinent to convert them to fertilizer, which can be commercialized. In Enugu State, the Waste Management Authority (ESWAMA) is charged with the responsibility of managing solid waste products. This body has different categories of trucks and other movable machines that collect and dispose solid waste products collected from homes and industries through dumpsters where waste products are dropped for easy collection and evacuation.

2.0 Theoretical Analysis

Geometric Programming is a class of non-linear optimization which reduces complicated non-linear program to a set of non-homogenous system of linear equations. It is used to minimize functions that are in the form of posynomials subject to constraints of the same type. The function in equation (1) is called the unconstrained posynomial.

$$f(x) = \sum_{j=1}^{N} C_j \prod_{i=1}^{m} x_i^{a_{ij}} \tag{1}$$

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Where
$C_j > 0$; $x_i > 0$; $a_{ij} \in R$
N = total number of terms in the posynomial
m = total number of variables in the posynomial
C_j = cost coefficient of the posynomial
x_i = primal decision variables in the posynomial
$\{a_{ij}\}$= exponent matrix which must be a real number.

It differs from other optimization techniques in the emphasis it places on the relative magnitudes of the terms of the objective function rather than the variables. Instead of finding optimal values of the decision variables first, geometric programming first finds the optimal value of the objective function. This feature has advantage in situations where the optimal value of the objective function may be all that is of interest [6, 7]. Geometric Programming derives its name from the fact that it is based on certain geometric concepts, such as orthogonality and arithmetic-geometric inequality. Geometric programming broadens the scope of linear programming applications and it is suited to model several types of important non-linear systems in physical sciences and engineering [8]. Geometric programming is very flexible and has been extended to different fields different from its original conceptions. In this study, Geometric programming has been extended to environmental studies to model solid waste products.
In geometric programming, the quantity N = m+1 is termed the degree of difficulty, where N denotes the total number of terms in all the posynomials and m denotes the number of decision variables. If N = m+1, the problem is said to have a zero degree of difficulty. In this case, the unknown y_j (j=1, 2,…,N) can be determined uniquely from the orthogonality and normality condition and the problem is said to have a unique solution. Geometric programming is not applicable to problems with negative degree of difficulty. The dual problem is solved when it is less complicated than the primal problem [6, 9]. In other words, we maximize the dual program whenever the degree of difficulty if greater than zero, that is, when the solution is not unique. The primal program is solved when the degree of difficulty is zero, that is, when the solution is unique. The degree of difficulty is the measure of computational difficulty of geometric program. But one can either maximize the dual or minimize the primal function since at optimality both yield the same optimal solution [6, 8, 9, 10]. From the equation (2), the minimum cost of waste management can be found without the knowledge of the optimal values of the decision variables.

$$\text{Min } f(x) = f^*(x) = \prod_{j=1}^{n}\left(\frac{C_j}{y_j}\right)^{y_j} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

The sufficient conditions for optimality are known as orthogonality and normality conditions which form a system of non-homogeneous linear equations. This can be expressed in matrix notation as follows:

$$AY = B \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Where A is the coefficient matrix, Y and B are the column vectors consisting of unknown dual variables and constants, respectively. A unique value of Y can be obtained by solving the system of linear equations $Y = A^{-1}B$ if the degree of difficulty is zero. The values of Y_j must satisfy the orthogonality and normality conditions. For the given value of f*(x) and unique value of Y_j, the solutions to the set of primal decision variables x_i can be obtained from equation (4)

$$C_j \prod_{i=1}^{m}(x_i)^{a_{ij}} = y^*_j \; f^*(x) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Where $\{a_{ij}\}$ = exponent matrix which must have a real number, $x_i > 0$, $C_j > 0$ and f*(x) = the optimal objective function and y^*_j is the optimal dual decision variables.

The Geometric Programming Model of the Study

Often times, a real life problem has some associated constraints because of limited resources. Hence, we maximize the dual function given by

$$Max \;\; f(y) = \prod_{k=0}^{m}\prod_{j=1}^{N_k}\left(\frac{C_{kj}}{\lambda_{kj}}\sum_{i=1}^{N_k}\lambda_{kl}\right)^{\lambda_{kj}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

Subject to AY = B
Where m = the total number of variables in the geometric program; N_k = the total number of terms in the geometric program; λ_{kj} = the dual decision variables and C_{kj} = the cost coefficient in the geometric program. The function in equation (5) is the constrained dual geometric program.

3.0 Data Presentation

Table 1: Average Maintenance Cost per Month / Trucks and their capacities (in tons)

S/No	**Truck Types**	**Ave. km Covered**	**Others**	**Fuel/Oil**	**Total Cost(₦)**	**Capacity (tons)**
1	Bedford Diesel trip.	5000	7500	5536	13,036	9
2	Mercedes 1513 trip.	5000	7000	5871	12,871	9
3	Roll-off truck	5000	8500	6020	14,520	12
4	Compactor truck	5000	8000	5978	13,978	10
5	Swaraj Mazda	5000	7000	6408	13,408	5
6	Service truck	5000	7000	6042	13,042	5
	Total Cost		**45,000**	**35,855**	**80,855**	**50**

Source: ESWAMA (2015)

Table2: Cost of labour per truck and Average Trips per Month.

S/No	**Truck Types**	**Driver**	**Loaders**	**Cost per month**	**Average trips**
1	Bedford Diesel trip.	24,000	36,000	60,000	4
2	Mercedes 1513 trip.	28,000	36,000	64,000	4
3	Roll-off truck	26,000	36,000	62,000	4
4	Compactor truck	23,000	36,000	59,000	4
5	Swaraj Mazda T3500	28,000	36,000	64,000	4
6	Service truck	30,000	36,000	66,000	4
	Total	**159,000**	**216,000**	**375,000**	**24**

Source: ESWAMA (2015)

4.0 Formulation of the Problem

C_1 = Cost of Maintenance: Let X_1 = expenditure on other maintenance and X_3 = expenditure on fuel/oil.
C_2 = Cost of Labour: Let X_1 = expenditure on drivers and X_2 = expenditure on waste loaders. **C_3 = Cost of Collection and maintenance of dump sites:** Let X_2 = expenditure on collecting waste products from some special (remote) arrears and X_3 = maintenance of dump sites.
C_4 = Volume (tons) of these wastes: Each volume contains the three categories of waste product. Let X_1 take care of the biodegradable, X_2 take care of the non-recyclable non-biodegradable and let X_3 take care of the recyclable non-biodegradable content of the volume (ton) of the trucks.

Objective Function

The objective is to minimize the cost of waste disposal subject to the volume available for disposal for the month, refer to the last two columns of table 1 and 2 for the total cost, volume of waste available for the month and average trips.

$$\text{Minimize} \quad f(x) = C_1X_1X_3 + C_2X_1X_2 + C_3X_2X_3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{6}$$

Subject to

$$\frac{C_4}{X_1X_2X_3} \le 24 \text{ trips per month.} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{7}$$

That is, a maximum of 24 trips of waste products are allowed for transporting 50 tons of solid waste products per month. At optimality in geometric programming, minimization of f(x) = maximization of f(y). Maximization is easier because it is a concave function constrained by linear equations, Ax = b (i.e. orthogonality and normality conditions). Equation (6) and (7) are the objective function and constraint equations respectively. Hence, we have

$$\text{Maximize} \quad f(y) = 80{,}855x_1x_3 + 375{,}000x_1x_2 + 50{,}000x_2x_3 + \frac{2.0833}{x_1x_2x_3} \quad \ldots\ldots\ldots \tag{8}$$

$$\text{Subject to} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & -1 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Test for the Degree of Difficulty:

N = 4; m = 3

N = m+1 $\Rightarrow$ N - m - 1 $\Rightarrow$ 4 - 3 - 1 = 0

The degree of difficulty is zero and the dual decision variables have a unique solution from equation (3) and thereafter, the optimal primal decision variables can be determined using equation (4).

Now Writing each component in terms of U_i's ; i = 1, …, 4, we have

$$U_1 = 80{,}855 x_1 x_3 \; ; \; U_2 = 375{,}000 x_1 x_2 \; ; \; U_3 = 50{,}000 x_2 x_3 ; \; U_4 = 2.0833 x_1^{-1} x_2^{-1} x_3^{-1}$$

Hence $f(y) = \left(\frac{C_1}{y_1}\right)^{y_1}\left(\frac{C_2}{y_2}\right)^{y_2}\left(\frac{C_3}{y_3}\right)^{y_3}\left(\frac{C_4 y_4}{y_4}\right)^{y_4}$ ………………….. (9)

$f(y) = \left(\frac{80{,}855}{y_1}\right)^{y_1}\left(\frac{375000}{y_2}\right)^{y_2}\left(\frac{50{,}000}{y_3}\right)^{y_3}(2.0833)^{y_4}$ We now solve for y to obtain $y = A^{-1}b$

$$\therefore \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1/3 & 1/3 & -2/3 & 1/3 \\ 1/3 & 1/3 & 1/3 & -2/3 \\ 1/3 & -2/3 & 1/3 & 1/3 \\ 2/3 & -1/3 & -1/3 & -1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} \Rightarrow y_1 = y_2 = y_3 = \frac{1}{3}; y_4 = \frac{2}{3}$$

$$f(y) = \left(\frac{80{,}855}{1/3}\right)^{1/3}\left(\frac{375000}{1/3}\right)^{1/3}\left(\frac{50{,}000}{1/3}\right)^{1/3}(2.0833)^{2/3}$$

$f(y) = (62.3395)(104.0042)(53.1118)(1.6310) =$ ₦ 561,641.50

$f(y) = f^*(x) =$ ₦ 561,641.50

This is the minimum expenditure on waste (refuse) management per month.

5.0 Results and Discussion

Enugu State Waste Management Authority (ESWAMA) spends about ₦561, 641.50 on waste management per month. A waste management model was developed in this study and applied to model solid waste product. Our interest was on the overall expenditure on refuse disposal, hence; the various contributions of the primal decision variables were not considered since they were not required. The determination of the cost of waste management per month will help ESWAMA and Enugu State government in planning.

6.0 Conclusion

Application of this model will create a disciplined waste management. It will help the government to determine the approximate amount of money she is spending on waste disposal at any given period of time (control spending on waste management). The study indicates the need for the establishment of fertilizer and other recycling plants. A disciplined waste management will save our environment from degradation and effectively combat climate change. All these will lead to sustainable development, economic growth and development in Enugu state in particular and Nigeria at large. It will reduce carbon dioxide and other greenhouse gases emission; this is in line with clean development initiative and combating climate change is in line with achieving sustainable development.

7.0 References

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