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A Study of Pressure Derivatives Distribution of a Horizontal Well Subject To Simultaneous Two-Edge And Bottom Water Drive Mechanisms.

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Abstract

Pressure derivatives of the type $t_D \frac{\partial P_D}{\partial t_D}$ were investigated for a horizontal well subject to edge and bottom water drive mechanisms. The influence of dimensionless well length and wellbore radius was investigated. Characteristic curve shows that there are three distinct flow regimes (early flow regime, intermediate flow regime and the late time flow regime) with all the derivatives identical at early time and merges on a straight asymptote of slope equal to unity. The period of infinite activity is extended if the reservoir is much larger than the length of the well and the wellbore radius is small. The intermediate flow period is longer for a well with vertical flow period than a well with horizontal flow period.

1.0 Introduction

Dimensionless pressure derivative was first introduced into the petroleum literature be Bourdet et al. [1]. Pressure derivatives exposes both the wellbore and reservoir character more explicitly than the conventional test analysis techniques that are hitherto used in transient well test analysis.Adewole [2] studied the possible trends and characteristics most likely to be observed on derivative plots for a horizontal well subject to bottom water drive mechanism. He investigated factors affecting clean oil production in both rectangular and square oil field patterns. He concluded that large reservoirs have the tendency to produce clean oil longer than small reservoirs with small well completion and reservoir properties.

Ozkan etal. [3] studied a similar reservoir model but did not include the influence of field patterns on clean oil production and only laterally infinite reservoir pattern was discussed.

All pressure derivatives behaviors are identical at early times, and the curve merges on a straight asymptote of slope equal to unity. When the infinite acting radial flow regime has been reached, the second asymptotes, the one-half straight line is attained. Between the two asymptotes, each curve shows a specific shape much more pronounced than that of the usual pressure curves.

The pressure derivative method is a powerful diagnostic tool and it combines on the same log-log plot with dimensionless pressure [4]

Nomenclature

h = reservoir thickness, ft
h_D = dimensionless reservoir thichkness
l_D = dimensionless well length
s = source function
t_D = dimensionless time
x, y, z, = distance in the x, y, z, direction, ft, respectively
x_D, y_D, z_D = dimensionless distances in the x, y, and z direction respectively
k_x, k_y, k_z = permeability in the x, y and z direction respectively, md
p_D = dimensionless pressure
r_{wD} = dimensionless wellbore radius

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Reservoir and Mathematical Description of Model

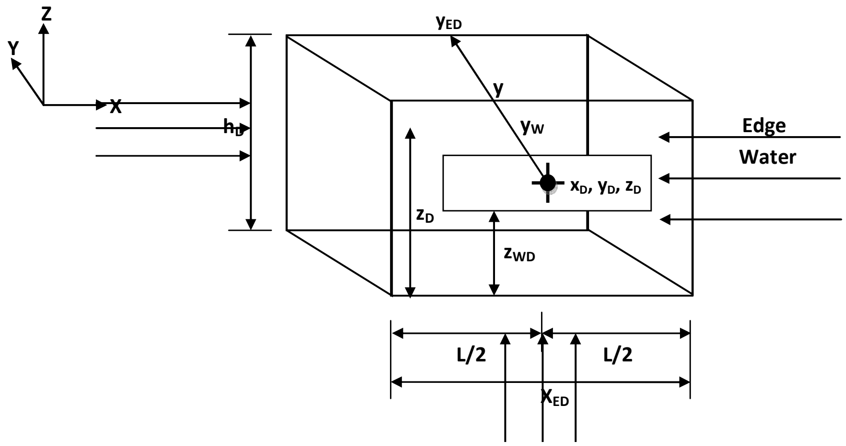

Fig. 1: Diagram of reservoir model:

Figure 1 above shows the bounded reservoir under study. It has a horizontal well producing oil of small and constant compressibility under a constant draw down rate pressure*Pi*. If reservoir well experiences a drawdown pressure of $\Delta P=Pi-P(x,y,z,t)$ for a certain time, *t*. it is intended to derive a generalized dimensionless pressure derivative distribution, as a function of reservoir, well and fluid properties. The horizontal well experiences water energy from the heel owing to edge water drive from the reservoir. The reservoir is anisotropic, with k_x, k_y and k_z as its permeabilities along the principal axes. The reservoir experiences a constant-pressure support (along the axis of the well) and a constant pressure (water) at the bottom and gas cap at the top (along the *z*-axis). Furthermore, it is seated at the toe (top) of the well and infinite laterally along the well width(*y*-direction). Hence, the horizontal well is a line source. Following the above description, the well is therefore located in an infinite slab reservoir in an infinite slab source with its bottom experiencing a constant-pressure and an infinitely far away top.

2.0 Pressure Derivaties

Infinite Acting Period

During the infinite acting period, the pressure transit has not felt any of the external boundaries and according to equation (10) the pressure derivative is [5].

$$t_D\frac{\partial p_{Di}}{\partial t_D}=-t_D\frac{\beta h_D}{4}\sqrt{\frac{kk}{k_yk_z}}\left\{exp\left[-\frac{(y_D-y_{wD})^2\left(\frac{k}{k_y}\right)+(Z_D-Z_{wD})^2\frac{k}{k_z}}{4t_D}\right]\right\}\ldots\ldots \quad (1)$$

Where β is a function representing the short time approximation prior to the time the influence of the source boundary is felt.

$$\beta=2\ for x_D<\sqrt{\frac{k_x}{k}}\ldots\ldots \quad (2)$$

$$=0\text{ for }x_D>\sqrt{\frac{k_x}{k}}\ldots\ldots \quad (3)$$

$$=1\text{ for }x_D=\sqrt{\frac{k_x}{k}}\ldots\ldots \quad (4)$$

Early linear flow

If the well length is long in relation to the reservoir thickness and if there is a better vertical permeability than the horizontal permeability, then the flow feels the Z- boundaries first while the y- and x-boundaries are still infinite acting. According to equation (17) of [5], the pressure derivative of this flow period is

$$\boldsymbol{t_D\frac{\partial P_{D1}}{\partial t_D}}=\frac{t_D}{\sqrt{\pi}}\frac{1}{\sqrt{t_D}}\left[erf\frac{\sqrt{\frac{k}{k_x}}+x_D}{2\sqrt{t_D}}+erf\frac{\sqrt{\frac{k}{k_x}}-x_D}{2\sqrt{t_D}}\right].\ exp\frac{-(y_D-y_{wD})^2}{4t_D}\frac{k}{k_y}.\ \sum_{m=1}^{\infty}exp\left(-\frac{(2m-1)^2\pi^2t_D}{h_D^2}\frac{k}{k_h}\right)sin(2m-1)\pi\frac{z_{wD}}{h_D}sin(2m-1)\pi\frac{z_D}{h_D}\ldots\ldots(5)$$

First steady-state flow period

This period may occur if the wellbore length is short compared with the reservoir thickness and the horizontal permeability is larger than the vertical permeability. If this occurs then flow feels the X-boundaries while the y- and z- boundaries are still infinite acting. According to equation (24) of [1], the pressure derivative of this flow period is

$$t_D\frac{\partial p_{D2}}{\partial t_D}=2h_D\frac{1}{\pi}\sqrt{\frac{k^2}{k_yk_z}}\sum_{n=1}^{\infty}\frac{1}{n}exp\left(-\frac{4n^2\pi^2t_D}{X_{eD}^2}\right)sin\frac{n\pi}{XeD}\sqrt{\frac{k}{k_x}}\,sin\frac{n\pi X_{wD}}{X_{eD}}.sin\frac{n\pi X_D}{X_{eD}}.exp\left(-\frac{r_{wD}^2}{4t_D}\right)\ldots\ldots \quad (6)$$

Second steady-state flow period

This period may occur if the wellbore length is long compared to the reservoir thickness. During this flow period, the flow feels the z-boundaries while the x- and y- boundaries are still infinite acting. The required sources functions are an infinite reservoir plane source for the y axis, an infinite slab source for the x-axis and mixed boundaries for an infinite slab source in an infinite slab reservoir for the z-axis. According to equation (31) of [5], the pressure derivative of this flow period is

$$t_D \frac{\partial p_{D3}}{\partial t_D} = \sqrt{\pi t_D}\left[erf\frac{\sqrt{\frac{k}{k_x}}+x_D}{2\sqrt{t_D}}+erf\frac{\sqrt{\frac{k}{k_x}}-x_D}{2\sqrt{t_D}}\right]\left[exp\frac{-(y_D-y_{wD})^2}{4t_D}\frac{k}{k_y}\right].$$
$$\left[\sum_{m=1}^{\infty} exp\left(-\frac{(2m-1)^2\pi^2 t_D}{4h_D^2}\right)\sin(2m-1)\pi\frac{z_{wD}}{h_D}\sin(2m-1)\pi\frac{z_D}{h_D}\right]\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Late steady-state flow period

The flow feels the X- and Z- boundaries while the y- boundary is still infinite acting. According to equation (38) of [5], the pressure derivative for this period is

$$t_D\frac{\partial p_{D4}}{\partial t_D} = 8\sqrt{\frac{kt_D}{\pi k_y}}\sum_{n=1}^{\infty}\frac{1}{n}exp\left(-\frac{(n^2\pi^2)}{x_{eD}^2}\frac{k}{k_h}t_D\right)\sin n\frac{\pi}{x_{eD}}\sqrt{\frac{k}{k_x}}\sin n\pi\frac{x_{wD}}{x_{eD}}\cos n\pi\frac{x_D}{x_{eD}}\sum_{m=1}^{\infty}exp\left(-\frac{(2m-1)^2\pi^2 t_D}{h_D^2}\frac{k}{k_h}\right)sin(2m-1)\pi\frac{z_{wD}}{h_D}sin(2m-1)\pi\frac{z_D}{h_D}\ldots\ldots\ldots(8)$$

3.0 Result and Discussion

Dimensionless pressure derivatives were computed for equations (1), (5), (6), (7) and(8). Results of our numerical computation are shown in Tables1 to 6 and plotted in Figures 2 to 6. Results shows in both cases of vertical and horizontal permeability, that there are three flow regimes (infinite acting flow period, intermediate flow period and late time flow period). The period of clean oil production is longer in a reservoir with horizontal permeability than a reservoir with vertical permeability. The intermediate flow period is longer in a reservoir with vertical permeability than a reservoir with horizontal permeability.

TABLE 1: PRESSURE DERIVATIVES FOR R_{WD}=0.0001, L_D=0.5, 1.0, 2.5, and 10 FORINFINTE ACTING PEROID

T_D	$P'_D(L_D=0.5, R_{WD}=1E-4)$	$P'_D(L_D=1, R_{WD}=1E-4)$	$P'_D(L_D=2.5, R_{WD}=1E-4)$	$P'_D(L_D=10, R_{WD}=1E-4)$
1.0E-05	0.828787	0.414393	0.165757	4.143935E-02
1.0E-04	0.828974	0.414487	0.165795	4.144868E-02
1.0E-03	0.828992	0.414496	0.165798	4.144961E-02
1.0E-02	0.828994	0.414497	0.165799	4.144970E-02
0.1	0.828994	0.414497	0.165799	4.144971E-02
1.0	0.828994	0.414497	0.165799	4.144971E-02
10.0	0.828994	0.414497	0.165799	4.144971E-02
100.0	0.828994	0.414497	0.165799	4.144971E-02
1000.0	0.828994	0.414497	0.165799	4.144971E-02
10000.0	0.828994	0.414497	0.165799	4.144971E-02
100000.	0.828994	0.414497	0.165799	4.144971E-02

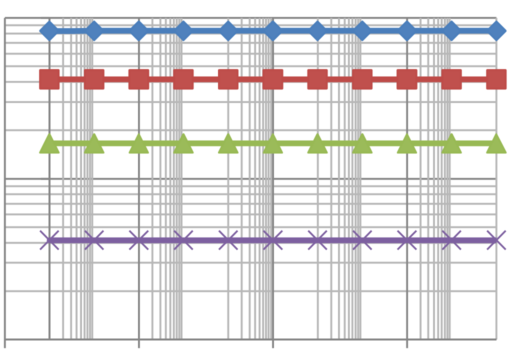

FIG. 2:DERIVATIVE PLOT FOR R_{WD}=0.0001, L_D=0.5, 1.0, 2.5, AND 10 FOR INFINTE ACTING PERIOD