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The Exponential Paraletrix

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Abstract

This paper established the theory of exponential paraletrix using the hearty multiplication of rhotrix and presents results and characteristics of this concept. Unlike exponential matrix, where the matrix is required to be square, exponential paraletrix exist for any type of order, so long that the heart of the paraletrix exists. We extend to show that the set of this exponential paraletrix forms an Abelian group under the hearty multiplication.

Keywords: Paraletrix; exponential paraletrix; convergence; group, hearty multiplication

1.0 Introduction

The concept of Paraletrix was introduced by Aminu and Michael [1] as an extension of Rhotrix [2] when the number of rows is not equal to the number of columns. It worth mentioning to say that not all paraletrix has heart, Aminu and Michael [1]. Let R and Q be two paraletrices such that the heart of the paraletrices exist;

$$R=\left\langle \begin{array}{ccccc} & a_1 & & & \\ a_2 & a_3 & a_4 & & \\ & a_5 & a_6 & a_7 & \\ & & a_8 & a_9 & a_{10} \\ & & & a_{11} & \end{array} \right\rangle,\quad Q=\left\langle \begin{array}{ccccc} & b_1 & & & \\ b_2 & b_3 & b_4 & & \\ & b_5 & b_6 & b_7 & \\ & & b_8 & b_9 & b_{10} \\ & & & b_{11} & \end{array} \right\rangle, \qquad (1)$$

The hearty multiplication of paraletrix R and Q using Ajibade's multiplication of rhotrix [2] is extended to paraletrices R and Q [1]

$$R\circ Q=\left\langle \begin{array}{ccccc} & c_1 & & & \\ c_2 & c_3 & c_4 & & \\ & c_5 & c_6 & c_7 & \\ & & c_8 & c_9 & c_{10} \\ & & & c_{11} & \end{array} \right\rangle \text{ where each } c_i = a_i h(Q) + b_i h(R) \ni \ i = 1,2,3,\cdots,11 \ni \ i \neq 6$$

and $c_6 = h(Q)h(R)$ for

the heart of the paraletrix $R\circ Q$, and $h(Q) = b_6$ and $h(R) = a_6$

where $h(R)$ and $h(Q)$ are the hearts of palatrices R and Q respectively

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Also, addition of paraletrix R and Q is

$$R+Q=\left\langle \begin{array}{ccccc} & d_1 & & & \\ d_2 & d_3 & d_4 & & \\ & d_5 & d_6 & d_7 & \\ & & d_8 & d_9 & d_{10} \\ & & & d_{11} & \end{array} \right\rangle \text{ where } d_i = a_i + b_i \ \ni \ i = 1,2,3,\cdots,11$$

In addition, the multiplication of paraletrix R and Q using Sani [3] rhotrix approach is only possible whenever the number of columns of R is equal to the number of rows of Q for any arbitrary Paraletrix.

scalar multiplication, let $\lambda \in \mathbb{R}$ then

$$\lambda R = \lambda \left\langle \begin{array}{ccccc} & a_1 & & & \\ a_2 & a_3 & a_4 & & \\ & a_5 & a_6 & a_7 & \\ & & a_8 & a_9 & a_{10} \\ & & & a_{11} & \end{array} \right\rangle = \left\langle \begin{array}{ccccc} & \lambda a_1 & & & \\ \lambda a_2 & \lambda a_3 & \lambda a_4 & & \\ & \lambda a_5 & \lambda a_6 & \lambda a_7 & \\ & & \lambda a_8 & \lambda a_9 & \lambda a_{10} \\ & & & \lambda a_{11} & \end{array} \right\rangle$$

Throughout this work, we considered the hearty multiplication of paraletrix [1].

2.0 Definitions of Useful Terms

The definition of concepts below will help in our discussion of useful results in this paper[6]:

The Exponential series: the expansion for exponential function is obtained from Taylor series as follows;

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{(x - x_0)^2}{2!} f''(x_0) + \frac{(x - x_0)^3}{3!} f'''(x_0) + \cdots$$

setting $x_0 = 0$,

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\Rightarrow e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$ is called the exponential series of the exponential function e^x

Identity Paraletrix:is a Paraletrix whose other entries are all zeros except the entry for the heart of the Paraletrix which is one.

Heart of a Paraletrix: just like a rhotrix, the heart of a Paraletrix is simply defined as the element in a Paraletrix which divides the Paraletrix into two equal parts. The heart of a Paraletrix if it exists is located at the $\frac{1}{2}\left[\frac{1}{2}((m \times n)+1)\right]+1$ entry of the Paraletrix, Where m and n are the number of rows and columns of the Paraletrix respectively. The heart of a paraletrix is said to exist if the heart is present.

Group: let V be a non- empty set and $\circ$ be a binary operation. Then the order pair (V, $\circ$) is called a group if for any $x, y \in V$, the following conditions are satisfied with respect to the binary operation; closure property, associativity, every non-zero element has inverse, identity element is contain in V. in addition, if commutativity is included, then we call it an abelian group.

Ring: let V be a non- empty set and$+, \circ$be two binary operations. Then the ordered pair $(V, \circ, +)$ is called a ring if for any $x, y \in V$, the following conditions are satisfied; (V, $\circ$)is a an abelian group and (V,+) satisfies closure,associativity,distributive properties. In addition, it is called a commutative ring if commutativity rule is satisfied.

3.0 The Exponential Paraletrix

From the definition of exponential series, consider a paraletrix $P_{m,n}$ with m rows and n columns to take the place of x in the definition of exponential series. Without loss of generality, denote $P_{m,n} = P$, then;

$$e^P = I + P + \frac{P^2}{2!} + \frac{P^3}{3!} + \frac{P^4}{4!} + \cdots + \frac{P^n}{n!} + \cdots \qquad (2)$$

$e^P = \sum_{k=0}^{\infty} \frac{P^k}{k!}$, where P is a paraletrix and I is the identity paraletrix,and e^P is also another paraletrix of the same dimension with P, and ! means factorial.

3.1 Convergence of Exponential Paraletrix

Let $P_1, P_2, P_3, P_4 \cdots$ be a sequence of mxn dimensional paraletrix. Then, the sequence is said to converge to an mxn dimensional paraletrix Q if for each $i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots n$, we have that the sequence of entries $(P_k)_{ij}$ converges to Q_{ij} in the usual sense. i.e for $\in > 0$ there exist some $N \in \mathbb{N} \ni k \geq N$ which implies $|(P_k)_{ij} - Q_{ij}| < \in$.

Now, the series $P_1 + P_2 + P_3 + \cdots$ converges means that the sequence of partial sum $P_1 + P_2 + P_3 + \cdots$ converges (in the case of our paraletrix, $I, I + P, I + P + \frac{P^2}{2!}, I + P + \frac{P^2}{2!}, \cdots$), hence, the exponential paraletrix e^P converges.

3.2 Derivation of the Exponential Paraletrix

consider the Paraletrix

$$P_{m,n} = \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m-1\,1} & & & h(P) & & & a_{2n-1} & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & a_{m\,n-2} & a_{m-1\,n-2} & a_{m-2\,n-1} & a_{m-2\,n-1} & a_{m-2\,n} & & \\ & & & a_{m\,n-1} & a_{m-1\,n-1} & a_{m-1\,n} & & & \\ & & & & a_{m\,n} & & & & \end{array} \right\rangle$$

where m, n is the number of rows and columns respectively of the Paraletrix.

$h(P) = a_{\text{h}}$; is the heart of the Paraletrix.located at the $\frac{1}{2}\left[\frac{1}{2}((m \times n)+1)\right]+1$ entry

Then the Exponential Paraletrix becomes;

$$e^P = \left\langle \begin{array}{ccccccccc} & & & & 0 & & & & \\ & & & 0 & 0 & 0 & & & \\ & & 0 & 0 & 0 & 0 & 0 & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & & & 1 & & & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & 0 & 0 & 0 & 0 & 0 & & \\ & & & 0 & 0 & 0 & & & \\ & & & & 0 & & & & \end{array} \right\rangle + \left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m-1\,1} & & & h(P) & & & a_{2n-1} & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & a_{m\,n-2} & a_{m-1\,n-2} & a_{m-2\,n-1} & a_{m-2\,n-1} & a_{m-2\,n} & & \\ & & & a_{m\,n-1} & a_{m-1\,n-1} & a_{m-1\,n} & & & \\ & & & & a_{m\,n} & & & & \end{array} \right\rangle +$$

$$\frac{1}{2!}\left[\left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m-1\,1} & & & h(P) & & & a_{2n-1} & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & a_{m\,n-2} & a_{m-1\,n-2} & a_{m-2\,n-1} & a_{m-2\,n-1} & a_{m-2\,n} & & \\ & & & a_{m\,n-1} & a_{m-1\,n-1} & a_{m-1\,n} & & & \\ & & & & a_{m\,n} & & & & \end{array} \right\rangle\right]^2 + \frac{1}{3!}\left[\left\langle \begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m-1\,1} & & & h(P) & & & a_{2n-1} & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & a_{m\,n-2} & a_{m-1\,n-2} & a_{m-2\,n-1} & a_{m-2\,n-1} & a_{m-2\,n} & & \\ & & & a_{m\,n-1} & a_{m-1\,n-1} & a_{m-1\,n} & & & \\ & & & & a_{m\,n} & & & & \end{array} \right\rangle\right]^3 + \cdots\cdots$$

e^P is the exponential Paraletrix for the Paraletrix P, where h(P) denotes the heart of the Paraletrix P.
Using the Heart multiplication, Aminu [3] defines the following result on a rhotrix R.

$$\text{if } R = \left\langle \begin{array}{ccc} & a_1 & \\ a_2 & h(R) & a_3 \\ & a_4 & \end{array} \right\rangle, \text{ then } R^n = \left\langle \begin{array}{ccc} & na_1h(R) & \\ na_2h(R) & h(R)^n & na_3h(R) \\ & na_4h(R) & \end{array} \right\rangle$$

We extend this idea of Aminu [5] on paraletrix to compute the exponential paraletrix.

$$P^{k}=\left\langle\begin{array}{ccccccccc}
 & & & & ka_{11}h(P) & & & & \\
 & & & ka_{31}h(P) & ka_{21}h(P) & ka_{12}h(P) & & & \\
 & & ka_{51}h(P) & ka_{41}h(P) & ka_{32}h(P) & ka_{22}h(P) & ka_{13}h(P) & & \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
ka_{m1}h(P) & ka_{m-1\,1}h(P) & & & [h(P)]^{k} & & & ka_{2n-1}h(P) & ka_{1n}h(P) \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 & & ka_{m\,n-2}h(P) & ka_{m-1\,n-2}h(P) & ka_{m-2\,n-1}h(P) & ka_{m-2\,n-1}h(P) & ka_{m-2\,n}h(P) & & \\
 & & & ka_{m\,n-1}h(P) & ka_{m-1\,n-1}h(P) & ka_{m-1\,n}h(P) & & & \\
 & & & & ka_{m\,n}h(P) & & & &
\end{array}\right\rangle$$

for $k=2,3,4,\dots$

Now, suppose

$$e^{P}=\left\langle\begin{array}{ccccccccc}
 & & & & b_{11} & & & & \\
 & & & b_{31} & b_{21} & b_{12} & & & \\
 & & b_{51} & b_{41} & b_{32} & b_{22} & b_{13} & & \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
b_{m1} & b_{m-1\,1} & & & h(e^{P}) & & & b_{2\,n-1} & b_{1n} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 & & b_{m\,n-2} & b_{m-1\,n-2} & b_{m-2\,n-1} & b_{m-2\,n-1} & b_{m-2\,n} & & \\
 & & & b_{m\,n-1} & b_{m-1\,n-1} & b_{m-1\,n} & & & \\
 & & & & b_{m\,n} & & & &
\end{array}\right\rangle$$

where $h(e^{P})$ is the heart of the Paraletrix.

Then by applying equation (1), we have;

$$b_{11}=0+a_{11}+a_{11}h(P)+\frac{2}{2!}a_{11}[h(P)]+\frac{3}{3!}a_{11}[h(P)]+\frac{4}{4!}a_{11}[h(P)]+\frac{5}{5!}a_{11}[h(P)]+\cdots$$

$$b_{11}=\left[0+a_{11}+a_{11}[h(P)]+\frac{3}{3!}a_{11}[h(P)]\ +\frac{4}{4!}a_{11}[h(P)]+\frac{5}{5!}a_{11}[h(P)]+\cdots\right]$$

$$b_{11}=a_{11}\{1+h(P)[e-1]\}$$

similarly,

$$b_{12}=a_{12}\{1+h(P)[e-1]\}$$

$$b_{13}=a_{13}\{1+h(P)[e-1]\}$$

$\vdots$

$\vdots$

$$b_{mn}=a_{mn}\{1+h(P)[e-1]\}$$

and

$$h(e^{R})=\left[1+h(P)+\frac{1}{2!}[h(P)]^{2}+\frac{1}{3!}[h(P)]^{3}+\frac{1}{4!}[h(P)]^{4}+\frac{1}{5!}[h(P)]^{5}+\cdots\right]$$

$$h(e^{P})=e^{h(P)}.$$

$$e^{P}=\left\langle\begin{array}{ccccccccc}
 & & & & a_{11}\{1+h(P)[e-1]\} & & & & \\
 & & & a_{31}\{1+h(P)[e-1]\} & a_{21}\{1+h(P)[e-1]\} & a_{12}\{1+h(P)[e-1]\} & & & \\
 & & a_{51}\{1+h(P)[e-1]\} & a_{41}\{1+h(P)[e-1]\} & a_{32}\{1+h(P)[e-1]\} & a_{22}\{1+h(P)[e-1]\} & a_{13}\{1+h(P)[e-1]\} & & \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
a_{m1}\{1+h(P)[e-1]\} & & & & e^{h(P)} & & & & a_{1n}\{1+h(P)[e-1]\} \\
\dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\
 & & a_{m\,n-2}\{1+h(R)[e-1]\} & a_{m-1\,n-2}\{1+h(P)[e-1]\} & a_{m-2\,n-1}\{1+h(P)[e-1]\} & a_{m-2\,n-1}\{1+h(P)[e-1]\} & a_{m-2\,n}\{1+h(P)[e-1]\} & & \\
 & & & a_{m\,n-1}\{1+h(P)[e-1]\} & a_{m-1\,n-1}\{1+h(P)[e-1]\} & a_{m-1\,n}\{1+h(P)[e-1]\} & & & \\
 & & & & a_{mn}\{1+h(P)[e-1]\} & & & &
\end{array}\right\rangle$$

$$e^{P}=\{1+h(P)[e-1]\}\circ\left\langle\begin{array}{ccccccccc} & & & & a_{11} & & & & \\ & & & a_{31} & a_{21} & a_{12} & & & \\ & & a_{51} & a_{41} & a_{32} & a_{22} & a_{13} & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m-1\,1} & & & \dfrac{e^{h(P)}}{\{1+h(P)[e-1]\}} & & & a_{2n-1} & a_{1n} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & a_{m\;n-2} & a_{m-1\;n-2} & a_{m-2\;n-1} & a_{m-2\;n-1} & a_{m-2\;n} & & \\ & & & a_{m\;n-1} & a_{m-1\;n-1} & a_{m-1\;n} & & & \\ & & & & a_{m\;n} & & & & \end{array}\right\rangle, \qquad (3)$$

where $\circ$ is a dot multiplication. and (3) is the exponential Paraletrix.

Theorem 3.1; Let P be a Paraletrix with heart h(P), then a necessary and sufficient condition for the exponential Paraletrix to exist is that h(P) must exist.

Proof: the result follows directly from equation (3), since the exponential Paraletrix depend on the heart of the Paraletrix h(P), hence the proof.

Theorem 3.2;let P be a Paraletrix. Then $e^{P}=P+I$ *Wheneverh(P) =0.*

Proof: let P be a *Paraletrix*, since h(P) =0, implies that the exponential *Paraletrix*existfrom (theroem3.1). Now, suppose P is such that there exist at least one entry in the *Paraletrix* P such that $a_{ij}\neq 0$ then, e^{P} cannot be I. also h(P) =0 implies $e^{h(P)}=e^{0}=1$ and $\{1+h(P)(e-1)$ becomes 1. then, from (3), $e^{P}=P+I$. hence the proof.

Corollary 3.1; if P= 0 *Paraletrix, then* $e^{P}=e^{0}=I$

Proof: The result follows directly from (theorem 3.2.) Since each $a_{ij}=0$ for all i, j therefore $e^{P}=P+I=0+I=I$. Hence the proof.

Theorem 3.3; *let P be a Paraletrix. if* $h(P)=1$, *Then* $e^{P}=e\circ P$, where $\circ$ denotes scalar multiplication of paraletrix

Proof: the result follows directly from equation (3).

Corollary 3.2; *let P be a Paraletrix. If* $P=I$, *Then* $e^{P}=e^{I}=e\circ P$

Proof ; from (theorem 3.3), replacing P with identity paraletrix. Hence the result follows since the heart of identity Paraletrix is 1.

Theorem 3.4; let PandQ be Paraletrix, $\forall\lambda,\beta\in\mathbb{R}$ *and* $\bullet$ denotes the usual hearty multiplication. *Then the following statements are true;*

(i) $e^{P(\lambda+\beta)}=e^{P\lambda}e^{P\beta}$.

(ii) if h(e^{P}) $\neq 0$, then e^{P} is invertible, $\left(e^{P}\right)^{-1}$ exist and is equal to e^{-P} and $e^{P}\circ(e^{P})^{-1}=e^{P}\circ e^{-P}=e^{P-P}=e^{0}=I$

(iii) $e^{P^{T}}=(e^{P})^{T}$

Proof:

i. since $\lambda,\beta\in\mathbb{R}$, then $\lambda+\beta\in\mathbb{R}$, hence from exponential property of $e^{x+y}=e^{x}e^{y}$,and our exponential paraletrix (3) the result follows directly

ii. h(e^{P}) $\neq 0$, implies that e^{P} is invertible, then its inverse exist say $[e^{P}]^{-1}$,by property of exponential, $[e^{P}]^{-1}=1/e^{P}=e^{-P}$ and therefore $e^{P}e^{-P}$, from i, gives $e^{P-P}=e^{0}=I$, from corollary 3.1

iii. since e^{P} is also a Paraletrix, therefore $e^{P^{T}}$ and $(e^{P})^{T}$ are also Paraletrices, and from equation (3), the result follows directly

Theorem 3.5; Let V={ e^{p_i} for i=1,2,3,4,5,6,..... such that e^{p_i} is an exponential Paraletrix} then the ordered pair

$(V, \circ)$ form an abelian group.

Proof: It is easy to show that $e^{P_1} \circ e^{P_2} \in V$ from the definition of Paraletrix addition and for the fact that they are the same order. This implies that closure property is satisfied.

1. Let $e^{P_1} \circ (e^{P_2} \circ e^{P_3}) = (e^{P_1} \circ e^{P_2}) \circ e^{P_3}$ this result is true because heart multiplication of Paraletrix is commutative.
2. There exist an element $e^0 \in V \ni e^P \circ e^0 = e^0 \circ e^P = e^P$. the zero exponential Paraletrix say e^0, satisfies this condition.
3. $\exists\, [e^P]^{-1} \in V$ whenever $h(e^P) \neq 0 \ni [e^P]^{-1} \circ e^P = e^P \circ [e^P]^{-1} = e^P \circ e^{-P} = e^{P-P} = e^0$. This also comes directly from negation of (3) which in turns satisfies the condition.
4. Since hearty multiplication of *Paraletrix* is commutative and by (3), $e^{P_1} \circ e^{P_2} = e^{P_2} \circ e^{P_1}$

Since closure, commutativity, associativity, identity element and inverse element conditions are satisfied, hence $(V, \circ)$ forms an abelian group.

Theorem 3.6; there does not exist any paraletrix $\in \mathbb{R} \ni e^P =$ null paraletrix

Proof: the proof follows from the definition of exponential paraletrix, the value $\frac{e^{h(P)}}{\{1+h(P)[e-1]\}} \neq 0$ for any value of $h(P) \in \mathbb{R}$, $\{1+h(P)[e-1]\}$ can never be zero, hence their product can never be zero, which implies that the heart of an exponential paraletrix cannot be zero, hence the result.

4.0 Examples on Exponential Paraletrix

4.1 Consider;

$$R = \left\langle \begin{array}{ccccc} & & 1 & & \\ & 2 & 3 & 4 & \\ 5 & 6 & 7 & 8 & 9 \\ & 10 & 11 & 12 & \\ & & 13 & & \end{array} \right\rangle$$

, the heart of the Rhotrix is 7

then, e^R using (3), becomes;

$$e^R = \left\langle \begin{array}{ccccc} & & [1+7(e-1)] & & \\ & 2[1+7(e-1)] & 3[1+7(e-1)] & 4[1+7(e-1)] & \\ 5[1+7(e-1)] & 6[1+7(e-1)] & e^7 & 8[1+7(e-1)] & 9[1+7(e-1)] \\ & 10[1+7(e-1)] & 11[1+7(e-1)] & 12[1+7(e-1)] & \\ & & 13[1+7(e-1)] & & \end{array} \right\rangle$$

$$e^R = [1+7(e-1)] \left\langle \begin{array}{ccccc} & & 1 & & \\ & 2 & 3 & 4 & \\ 5 & 6 & \frac{e^7}{[1+7(e-1)]} & 8 & 9 \\ & 10 & 11 & 12 & \\ & & 13 & & \end{array} \right\rangle$$

4.2. Consider;

$$P_{3,7} = \left\langle \begin{array}{ccccc} & 2 & & & \\ -3 & 6 & 7 & & \\ & 2 & 3 & 6 & \\ & & 5 & -4 & 1 \\ & & & 8 & \end{array} \right\rangle$$, here, the heart h(R)=3, therefore applying (3),

we have;

$$e^{P} = \left\langle \begin{array}{ccccc} & 2[1+3(e-1)] & & & \\ -3[1+3(e-1)] & 6[1+3(e-1)] & 7[1+3(e-1)] & & \\ & 2[1+3(e-1)] & e^{3} & 6[1+3(e-1)] & \\ & & 5[1+3(e-1)] & -4[1+3(e-1)] & [1+3(e-1)] \\ & & & 8[1+3(e-1)] & \end{array} \right\rangle$$

$$e^{P} = [3e-2] \left\langle \begin{array}{ccccc} & 2 & & & \\ -3 & 6 & 7 & & \\ & 2 & \dfrac{e^{3}}{[3e-2]} & 6 & \\ & & 5 & -4 & 1 \\ & & & 8 & \end{array} \right\rangle$$

5.0 Conclusion

In this paper, we have derived the concept of exponential paraletrix; an extension of rhotrix using the hearty multiplication of Ajibade [2].Unlike in matrix where the Matrix need to be squared, wehave shown that a necessary and sufficient condition for the exponential paraletrix to exist is that its heart h(P) must exist. We further show that the set of exponential paraletrix with a binary operation (hearty multiplication of paraletrix), forms a Group with respect to hearty multiplication of paraletrix.

6.0 References

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