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On Soft Lattice Theory

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Abstract

We revisit the term soft lattices and present some of its algebraic properties. Soft set lattice is redefined here in terms of supremum and infimum and some related results are established. The concept of upper bound and least upper bound, lower bound and greatest lower bound were defined in soft set context.

Keywords: Soft set, ordered soft lattice, partial ordered soft set, upper bound of a soft, least upper bound of a soft set, lower bound of a soft set, greatest lower bound of soft set, soft lattice

1.0 Introduction

Let U be a universal set and E be the set of all possible parameters under consideration with respect to U. Let the power set of U (i.e., the set of all subsets of U) be denoted by $P(U)$ and A is a subset of the parameters, E $(A \subseteq E)$. The parameters are attributes, characteristics or properties associated with the objects in U. Then we have the following definition [1-14]:

Definition 1.1

A pair (F, E) is called a soft set over U if and only if F is a mapping of E into the set of all subsets of the set U. That is, a soft set is a parametrized family of subsets of the set U. For all $e \in E$, $F(e)$ is considered as the set of e −approximate elements of the soft set (F, E).

Definition 1.2

A soft set (F, E) over a universe U is said to be *null* or *empty* soft set denoted by $\widetilde{\emptyset}$, if $\forall e \in E, F(e) = \emptyset$.

Definition 1.3

A soft set (F, A) over a universe U is called *absolute* or *universal soft set* denoted by $\widetilde{(F, A)}$ or $\widetilde{U}$, if $\forall e \in E, F(e) = U$.

Definition 1.4

Let $E = \{e_1, e_2, e_3, \ . \ . \ ., e_n\}$ be a set of parameters. The *not-set* of E denoted by $\neg E$ is defined as $\neg E = \{\neg e_1, \neg e_2, \neg e_3, \ . \ . \ ., \neg e_n)$.

Definition 1.5

The *complement* of a soft set (F, E), denoted by $(F, E)^c$, is defined as $(F, E)^c = (F^c, \neg E)$.

Where $F^c: \neg E \to P(U)$ is a mapping given by $F^c(\alpha) = U - F(\neg\alpha), \forall \alpha \in \neg E$. F^c is called the soft complement function of F. Consequently, $(F^c)^c = F$ and $((F, E)^c)^c = (F, E)$

Definition 1.6

Let (F, A) and (G, B) be any two soft sets over a common universe U, (F, A) is called a soft subset of (G, B), denoted by $(F, A) \widetilde{\subseteq} (G, B)$ if ;

(i) $A \subset B$, and

(ii) $\forall e \in A, F(e) = G(e)$

(F, A) is said to be a soft super set of (G, B), if (G, B) is a subset of (F, A) and it is denoted by $(F, A) \ \widetilde{\supseteq} \ (G, B)$.

*Definition*1.7

Two soft sets (F, A) and (G, B) over a common universe U are said to be soft equal, denoted by $(F, A) = (G, B)$, if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

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Definition 1.8
If (F,A) and (G,B) are two soft sets then "$(F,A)\ AND\ (G,B)$" denoted by $(F,A) \wedge (G,B)$ is defined as$(F,A) \wedge (G,B) = (H, A \times B)$, where $H(\alpha,\beta) = F(\alpha) \cap G(\beta)$, $\forall(\alpha,\beta) \in A \times B$.
Definition 1.9
If (F,A) and (G,B) are two soft sets then "$(F,\ A)\ OR\ (G,B)$" denoted by $(F,A) \vee (G,B)$ is defined as $(F,\ A) \vee (G,B) = (P,\ A \times B)$, where, $P(\alpha,\beta) = F(\alpha) \cup G(\beta), \forall\ (\alpha,\beta) \in A \times B$.
Definition 1.10
Let (F,A) and (G,B) be two soft sets over a common universe U. The union or extended union of $(F,A)$and (G,B), denoted by $(F,A) \cup (G,B)$ or $(F,A) \cup_E (G,B)$, is the soft set (H,C) satisfying the following conditions:
(i) $C = A \cup B$, (ii) $\forall e \in C$,

$$H(e) = \begin{cases} F(e) & \text{if } e \in A\backslash B \\ G(e) & \text{if } e \in B\backslash A \\ F(e) \cup G(e) & \text{if } e \in A \cap B \end{cases}$$

Definition 1.11
The intersection of two soft sets $(F,A)$and (G,B) over a common universe set U is the soft set (H,C), where $C = A \cap B$, and $\forall e \in C, H(e) = F(e)$or $G(e)$, we write$(F,A) \cap (G,B) = (H,C)$
Definition 1.12
The extended intersection of soft sets $(F,A)$and (G,B) over a common universe U, denoted by $(F,A) \cap_E (G,B)$, is the soft set (H,C), where $C = A \cup B\ \forall e \in C$ and

$$H(e) = \begin{cases} F(e) & \text{if } e \in A\backslash B \\ G(e) & \text{if } e \in B\backslash A \\ F(e) \cap G(e) & \text{if } e \in A \cap B \end{cases}$$

Definition 1.13
The restricted intersection of soft sets $(F,A)$and (G,B) over a common universe U, denoted by $(F,A) \cap_R (G,B)$, is the soft set (H,C), where $C = A \cap B \neq \emptyset$ such that $H(e) = F(e) \cap G(e), \forall e \in C$.
Definition 1.14
Let (F,A) and (G,B) be two soft sets over a common universe U such that $\cap B \neq \emptyset$. The restricted union of (F,A) and (G,B) , denoted by $(F,A) \cup_R (G,B)$, is defined as $(F,A) \cup_R (G,B) = (H,C)$, where $C = A \cup B$, and $\forall e \in C, H(e) = F(e) \cup G(e)$.
Definition 1.16
Let $(F,A)$and(G,B) be two soft sets over a common universe U such that $\cap B \neq \emptyset$. The restricted difference of $(F,A)$and (G,B) denoted by $(F,A) \sim_R (G,B)$, is defined as $(F,A) \sim_R (G,B) = (H,C)$, where $C = A \cap B$, and $\forall e \in C, H(e) = F(e) \setminus G(e)$.
Definition 1.17
The restricted symmetric difference of two soft sets (F,A) and (G,B) over a common universe U is defined as $(F,A)\,\Delta(G,B) = (F,A) \cup_R (G,B) \sim_R \left((F,A) \cup_R (G,B)\right)$
Various properties of these operations and algebraic structures defined on soft sets could be found in [10], and [11].

2.0 Soft lattice Theory

2.1 Redefined Concept of Conjunction (AND) and Disjunction (OR)

In this section, we defined some basic terms necessary to conceptualized soft lattice theory. We redefined the *Definition 1.8* and *1.9* as follows:
Definition 2.1
Given any two soft sets (F,A) and (G,B) over a common universe U, the disjunction of (F,A) and (G,B)denoted by $(F,A) \wedge (G,B)$ is defined as$(F,A) \wedge (G,B) = (H,\ A \cap B)$, where $H(\alpha) = F(\alpha) \cap G(\alpha), \forall \alpha \in A \cap B$.
Definition 2.2
Given any two soft sets (F,A) and (G,B) over a common universe U, the conjunction of (F,A) and (G,B)denoted by $(F,A) \vee (G,B)$ is defined as $(F,\ A) \vee (G,B) = (P,\ A \cup B)$, where $P(\alpha) = F(\alpha) \cup G(\alpha), \forall\ \alpha \in A \cup B$.
There are two standard ways of defining lattice in classical setting: Viz., based on algebraic structure or based on notion of order. So, in non-standard or nonclassical setting we define the soft lattice as follows:
Definition 2.3
Let (Γ,E) be a soft set. Let $A,B,C \subseteq E$ such that $(F,A),(G,B)$ and (H,C) are all defined. Then (Γ,E) together with the binary operations $\vee$ and $\wedge$ is called soft lattice if the following axioms are satisfied:
L1: (a) $(F,A) \vee (G,B) = (G,B) \vee (F,A)$
(b) $(F,A) \wedge (G,B) = (G,B) \wedge (F,A)$(Commutative laws)
L2: (a) $(F,A) \vee \left((G,B) \vee (H,C)\right) = \left((F,A) \vee (G,B)\right) \vee (H,C)$

(b) $(F,A) \vee ((G,B) \vee (H,C)) = ((F,A) \vee (G,B)) \vee (H,C)$ (Associative laws)

L3: (a) $(F,A) \vee (F,A) = (F,A)$

(b) $(F,A) \wedge (F,A) = (F,A)$ (Idempotent laws)

L4: (a) $(F,A) = (F,A) \vee ((F,A) \wedge (G,B))$

(b) $(F,A) = (F,A) \wedge ((F,A) \vee (G,B))$, (Absorption laws)

We denote the soft lattice $(\Gamma, E)$by $L(\Gamma, E)$. For convenience we simply write L. Where $\vee$ and $\wedge$ are as defined in *Definition 2.1* and *Definition 2.2.*

Example 2.1

Let $U = \{h_1, h_2, h_3, \dots, h_8\}$ be the sets of houses under consideration. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the sets of parameters, where e_1 stands for expensive, e_2= beautiful, e_3= wooden, e_4= cheap, e_5= in the green surrounding. Let $\Gamma: \mathrm{E} \to \mathrm{P(U)}$, then (Γ, E) is a soft set. Let $A = \{e_1, e_2, e_3\}$and $B = \{e_1, e_2, e_4, \}$, $C = \{e_1, e_3, e_4, e_5\}$ where A, B and C are subset of E such that (F, A), (G, B), (H, C) are all define. Then we show that the soft set (Γ, E) is a soft lattice, since if we

let$(\mathrm{F}, A) = \{ F(e_1) = \{h_1, h_2, h_4\}, F(e_2) = \{h_1, h_2, h_4, h_5\}, F(e_3) = \{h_1, h_2\}\}$

$(\mathrm{G}, B) = \{ G(e_1) = \{h_1, h_3, h_4\}, G(e_2) = \{h_1, h_4\}, G(e_4) = \{h_2, h_3, h_4\}\}$

$(\mathrm{H}, C) = \{ H(e_1) = \{h_2, h_4\}, H(e_3) = \{h_1, h_2, h_5\}, H(e_4) = \{h_3, h_4, h_5\}, H(e_5) = \{h_1, h_2, h_4, h_5\}\}$.

The following axioms are satisfied:

(i) Commutativity

$(\mathrm{F}, A) \vee (\mathrm{G}, B) = (\mathrm{P}, A \cup B) = \{P(e_1) = \{h_1, h_2, h_3, h_4\}, P(e_2) = \{h_1, h_2, h_4, h_5\},\ P(e_3) = \{h_1, h_2\}\ ,\ P(e_4) = \{h_2, h_3, h_4\}\ \}$ and

$(\mathrm{G}, B) \vee (\mathrm{F}, A) = (\mathrm{T}, B \cup A) = \{T(e_1) = \{h_1, h_2, h_3, h_4\}, T(e_2) = \{h_1, h_2, h_4, h_5\}, T(e_3) = \{h_1, h_2\}, P(e_4) = \{h_2, h_3, h_4\}\}$

Hence,

$$(\mathrm{P}, A \cup B) = (\mathrm{T}, B \cup A)$$
$$\Rightarrow (\mathrm{F}, A) \vee (\mathrm{G}, B) = (\mathrm{G}, B) \vee (\mathrm{F}, A)$$

(ii) Associativity

$(\mathrm{F}, A) \vee ((\mathrm{G}, B) \vee (H, C)) = (\mathrm{F}, A) \vee (\mathrm{P}, B \cup C)$where $B \cup C = \{e_1, e_2, e_3, e_4, e_5\}$,

$(\mathrm{P}, B \cup C) = \{P(e_1) = \{h_1, h_2, h_3, h_4\}, P(e_2) = \{h_1, h_2, h_4, h_5\}, P(e_3) = \{h_1, h_2, h_5\}$, $P(e_4) = \{h_2, h_3, h_4, h_5\}, P(e_2) = \{h_1, h_2, h_4, h_5\}\ , P(e_5) = \{h_1, h_2, h_4, h_5\}\}$ then

$(\mathrm{F}, A) \vee (\mathrm{P}, B \cup C) = (\mathrm{T}, A \cup (B \cup C))$where$A \cup (B \cup C) = \{e_1, e_2, e_3, e_4, e_5\}$

$(\mathrm{T}, A \cup (B \cup C)) = \{T(e_1) = \{h_1, h_2, h_3, h_4\}, T(e_2) = \{h_1, h_2, h_4, h_5\}, T(e_3) = \{h_1, h_2, h_5\}$, $T(e_4) = \{h_2, h_3, h_4, h_5\}, T(e_5) = \{h_1, h_2, h_4, h_5\}\ \}$

Also we have,

$((F, A) \vee (G, B)) \vee (H, C) = (J, A \cup B) \vee (H, C)$where $A \cup B = \{e_1, e_2, e_3, e_4\}$,

$(J, A \cup B) = \{J(e_1) = \{h_1, h_2, h_3, h_4\}, J(e_2) = \{h_1, h_2, h_4, h_5\}, J(e_3) = \{h_1, h_2\}, J(e_4) = \{h_2, h_3, h_4\}\}$ then

$(J, A \cup B) \vee (H, C) = (K, (A \cup B) \cup C)$where $(A \cup B) \cup C = \{e_1, e_2, e_3, e_4, e_5\}$,

$(K, (A \cup B) \cup C) = \{K(e_1) = \{h_1, h_2, h_3, h_4\}, K(e_2) = \{h_1, h_2, h_4, h_5\}$, $K(e_3) = \{h_1, h_2, h_5\}, K(e_4) = \{h_2, h_3, h_4, h_5\}$, $K(e_5) = \{h_1, h_2, h_4, h_5\}\ \}$.

Hence, $(\mathrm{T}, A \cup (B \cup C)) = (K, (A \cup B) \cup C)$

$$\Rightarrow (F, A) \vee ((G, B) \vee (H, C)) = ((F, A) \vee\ (G, B)) \vee (H, C)$$

(iii) Idempotent

$(\mathrm{F}, A) = \{ F(e_1) = \{h_1, h_2, h_4\}, F(e_2) = \{h_1, h_2, h_4, h_5\}, F(e_3) = \{h_1, h_2\}\}$,

$$(F, A) \vee (F, A) = (H, A \cup A) = (H, A)$$

$(H, A) = \{ H(e_1) = \{h_1, h_2, h_4\}H(e_2) = \{h_1, h_2, h_4, h_5\}, H(e_3) = \{h_1, h_2\}\}$.

Hence,$(H, A) = (F, A)$

$$\Rightarrow (F, A) \vee (F, A) = (F, A)$$

(iv) Absorption

$$(F, A) = (F, A) \vee ((F, A) \wedge (G, B))$$

$(F, A) = \{ F(e_1) = \{h_1, h_2, h_4\}, F(e_2) = \{h_1, h_2, h_4, h_5\}, F(e_3) = \{h_1, h_2\}\}$and

$(F, A) \vee ((F, A) \wedge (G, B)) = (F, A) \vee (P, A \cap B)$, where $A \cap B = \{e_1, e_2\}$,

$(P, A \cap B) = \{ P(e_1) = \{h_1, h_4\}, P(e_2) = \{h_1, h_4\},$

$(F, A) \vee (P, A \cap B) = \big(K, A \cup (A \cap B)\big)$

$= \{ K(e_1) = \{h_1, h_2, h_4\}, K(e_2) = \{h_1, h_2, h_4, h_5\}, K(e_3) = \{h_1, h_2\}\}$

i.e. $(F, A) = \big(K, A \cup (A \cap B)\big)$

$\Rightarrow (F, A) = (F, A) \vee \big((F, A) \wedge (G, B)\big)$

Hence, the soft set (Γ, E) defined on U is a soft lattice.

Definition 2.4

2.2 Soft Lattice in Terms of Supremum and Infimum

A soft set (Γ, E) is called an ordered soft set if the parameter E is ordered.

Example 2.2

Let U be a universal set, let $P(U)$ be the power set of U. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ where e_1 stands for fair, e_2= good, e_3=better, e_4= best, e_5= excellent, and $e_1 \subseteq e_2 \subseteq e_3 \subseteq e_4 \subseteq e_5$. Let $\Gamma: E \rightarrow P(U)$, then (Γ, E) is an ordered soft set.

Remark 2.1

If (Γ, E) is order soft set then for $A, B, C \subseteq E$, $(F, A), (G, B)$, (H, C) are all order soft sets.

Definition 2.5

A binary relation $\subseteq$ defined on the set of parameters E is a partial order on E if for every $, B, C \subseteq E, (F, A), (G, B)$, (H, C) are defined such that the following axioms are satisfied

(i)$(F, A) \cong (F, A)$ (Reflexivity)

(ii)$(F, A) \cong (G, B)$ and $(G, B) \cong (F, A) \Rightarrow (F, A) = (G, B)$ (Antisymmetry)

(iii)$(F, A) \cong (G, B)$ and $(G, B) \cong (H, C) \Rightarrow (F, A) \cong (H, C)$ (Transitivity)

If, in addition, for every $A, B \subseteq E$

(iv))$(F, A) \cong (G, B)$ or $(G, B) \cong (F, A)$, then we say $\cong$ is a total order on E.

A non- empty soft set (Γ, E) with a partial order on it is called *partially orderedsoft set* denoted as $\big((\Gamma, E), \cong\big)$. If the relation is total order then we say that $\big((\Gamma, E), \cong\big)$ is called *totally orderedsoft set.*

Example 2.3

Let U be a universal set, let $P(U)$ be the power set of U. Let $E = \{e_1 = fair, e_2 = good, e_3 = better, e_4 = best, e_5 = excellent\}$ be the set of ordered parameters defined on U. Let $\Gamma: E \rightarrow P(U)$, then (Γ, E) is an ordered soft set

Let $A, B, C \subseteq E$ such that $A = \{e_1. e_2, e_3\}$, $B = \{e_1. e_2, e_3\}$, $C = \{e_1. e_2, e_3, e_4\}$,and $(F, A), (G, B), (H, C)$are all defined. Then we show that (Γ, E) is a partial ordered soft set.

Let $(F, A) = \{ F(e_1) = \{h_1, h_2, h_4\}, F(e_2) = \{h_1, h_4\}, F(e_3) = \{h_2, h_3, h_4\}$

$(G, B) = \{ G(e_1) = \{h_1, h_2, h_4\}, G(e_2) = \{h_1, h_4\}, G(e_3) = \{h_2, h_3, h_4\}$

$(H, C) = \{ H(e_1) = \{h_1, h_2, h_3, h_4\}, H(e_2) = \{h_1, h_4, h_5\}, H(e_3) = \{h_2, h_3, h_4\},$

$H(e_4) = \{h_2, h_3, h_4, h_5\}\}$

Since, $F(e_1) \subseteq F(e_1),\ F(e_2) \subseteq F(e_2), F(e_3) \subseteq F(e_3)$,

$\Rightarrow (F, A) \cong (F, A)$ (Reflexivity)

(i) Since, $F(e_1) \subseteq G(e_1),\ F(e_2) \subseteq G(e_2), F(e_3) \subseteq G(e_3)$,

$\Rightarrow (F, A) \cong (G, B)$

Also, $G(e_1) \subseteq F(e_1),\ G(e_2) \subseteq F(e_2), G(e_3) \subseteq F(e_3)$,

$\Rightarrow (G, B) \cong (F, A)$

$i.e\ (G, B) = (F, A)$ (Antisymmetry)

(iii) $F(e_1) \subseteq G(e_1),\ F(e_2) \subseteq G(e_2), F(e_3) \subseteq G(e_3)$,

$\Rightarrow (F, A) \cong (G, B)$and

$G(e_1) \subseteq H(e_1),\ G(e_2) \subseteq H(e_2), G(e_3) \subseteq H(e_3)$,

$\Rightarrow (G, B) \cong (H, C)$

$i.e\ (F, A) \cong (H, C)$ (Transitivity)

Hence, we say (Γ, E) is a partial ordered soft set defined on U.

Definition 2.6

Upper bound of a soft set

Let (Γ, E) be a partial ordered soft set. Let $A \subseteq E$ such that (F, A) is also a partial ordered soft set. Then a set $\Gamma(e)$ in (Γ, E) is called an *upper bound* for (F, A) if

$F(a) \subseteq \Gamma(e), e \in E\ \forall F(a) \in (F, A)$, and $a \in A$

Definition 2.7

Least upper bound of a soft set

Let (Γ, E) be a partial ordered soft set. Let $A \subseteq E$ such that(F, A) is also a partial ordered soft set. Then a set $\Gamma(e)$ in $(\Gamma, E), e \in E$ is called the *least upper bound* (lub), or *supremum* of $(\mathrm{F}, A)\big(sup(\mathrm{F}, A)\big)$ if $\Gamma(e)$ is an *upper bound*of (F, A), and if $\forall F(a) \in (F, A), \forall a \in A, \exists e_1 \in E$ such that $\mathrm{F}(a) \subseteq \Gamma(e_1), \forall \mathrm{F}(a) \in (\mathrm{F}, A) \Rightarrow \Gamma(e) \subseteq \Gamma(e_1)$ {i.e., $\Gamma(e)$ is the smallest among the *upper bound* of (F, A)}.

Definition 2.8

Lower bound of a soft set

Let (Γ, E) be a partial ordered soft set. Let $A \subseteq E$ such that(F, A) is also a partial ordered soft set. Then a set $\Gamma(e)$ in (Γ, E) is called a *lower bound* for (F, A)if $\Gamma(e) \subseteq \mathrm{F}(a), e \in E, \forall \mathrm{F}(a) \in (\mathrm{F}, A), \forall a \in A$

Definition 2.9

Greatest lower bound of a soft set

Let (Γ, E) be a partial ordered soft set. Let $A \subseteq E$ such that(F, A) is also a partial ordered soft set. Then a set $\Gamma(e)$ in $(\Gamma, E), e \in E$ is called the *greatest lower bound* (g. l. b), or *infimum* of $(\mathrm{F}, A)\big(imf(\mathrm{F}, A)\big)$ if $\Gamma(e)$ is a *lower bound*of (F, A), and if $\forall F(a) \in (F, A), \forall a \in A, \exists e_1 \in E$ such that $\Gamma(e_1) \subseteq \mathrm{F}(a), \forall \mathrm{F}(a) \in (\mathrm{F}, A), \forall a \in A \Rightarrow \Gamma(e_1) \subseteq \Gamma(e)$ {i.e., $\Gamma(e)$ is the greatest among the lower bound of (F, A)}.

Example 2.4

Let (Γ, E) be a partial ordered soft set defined on a universal set $U = \{h_1, h_2, \dots, h_6\}$. Let $E = \{e_1, e_2, e_3, e_4, e_5\}$ be the set of parameters. Let $A \subseteq E$where $A = \{e_1, e_2, e_3\}$. Then (F, A) is a partial ordered soft subset of (Γ, E).

Let $(\Gamma, E) = \{\Gamma(e_1) = \{h_1, h_2, h_3, h_4\}, \Gamma(e_2) = \{h_1, h_2, h_3, h_4\}, \Gamma(e_3) = \{h_1, h_2, h_4\},$
$\Gamma(e_4) = \{h_2, h_3, h_4, h_5\}, \Gamma(e_5) = \{h_1, h_2, h_3, h_4, h_5\}\}$ and
$(\mathrm{F}, A) = \{F(e_1) = \{h_1, h_2\}, F(e_2) = \{h_1, h_2, h_3\}, F(e_4) = \{h_4\}\}$

Hence, we say that $\Gamma(e_1) \in (\Gamma, E)$ is an upper bound of (F, A), since for every $F(a)$ in (F, A), $F(a) \subseteq \Gamma(e_1)$. Also $\Gamma(e_1)$ is the least upper bound or supremum of (F, A), since there exists $\Gamma(e_2)$ in (Γ, E) such that $\Gamma(e_1)$ is an upper bound of (F, A) and $F(a) \subseteq \Gamma(e_2)$ for every $F(a)$ in $(\mathrm{F}, A) \Rightarrow \Gamma(e_1) \subseteq \Gamma(e_2)$. Hence the supremum of $(\mathrm{F}, A)$is $\Gamma(e_1)$, written $sup\{(\mathrm{F}, A)\} = \Gamma(e_1)$.

Definition 2.10

Redefined soft lattice in terms of supremum and infimum.

A partial ordered soft set (Γ, E) is a soft lattice denoted by $L(\Gamma, E)$ if and only if for every partial ordered soft subset $(\mathrm{F}, A)$of (Γ, E), the supremum of (F, A) and the infimum of (F, A) exists in (Γ, E).

Definition 2.11

Let (Γ, E) be a soft set. Let $A, B, C \subseteq E$ such that $(F, A), (G, B)$ and (H, C) are all defined. Then (Γ, E) together with the binary operations $\vee$ and $\wedge$ is called soft semilattice if the following axioms are satisfied:

L1: (a) $(F, A) \vee (G, B) = (G, B) \vee (F, A)$
(b) $(F, A) \wedge (G, B) = (G, B) \wedge (F, A)$ (Commutative laws)

L2: (a) $(F, A) \vee \big((G, B) \vee (H, C)\big) = \big((F, A) \vee (G, B)\big) \vee (H, C)$
(b) $(F, A) \vee \big((G, B) \vee (H, C)\big) = \big((F, A) \vee (G, B)\big) \vee (H, C)$ (Associative laws)

L3: (a) $(F, A) \vee (F, A) = (F, A)$
(b) $(F, A) \wedge (F, A) = (F, A)$ (Idempotent laws)

Theorem 2.1

A finite partial ordered soft set with a finite ordered parameter sets defined on it is a soft lattice.

Proof

Suppose that (Γ, E) is a partial ordered soft, and E is a set of ordered parameters defined on the soft set(F, E). Since the soft set (Γ, E) is finite then, it implies that supremum and infimum of (Γ, E) exist. Then the soft set (Γ, E) in which the supremum and infimum exist is called a soft lattice.

Theorem 2.2

Definition 2.3 and Definition 2.10 are equivalent.

Proof

To proof that the two definitions are equivalent, we proceed as follows:

(i) If L is a lattice by Definition 2.3, then define $\subseteq$ on L by $\Gamma(a) \subseteq F(b)$ if and only if $\Gamma(a) = \Gamma(a) \wedge \Gamma(b)$, for $\Gamma(a), \Gamma(b) \in L$

(ii) If $L$is a lattice by the Definition 2.10, then define the operation $\wedge$ and $\vee$ by $\Gamma(a) \vee \Gamma(b) = sup\{\Gamma(a), \Gamma(b)\}$, and $\Gamma(a) \wedge \Gamma(b) = inf\{\Gamma(a), \Gamma(b)\}$.

Suppose that L is a lattice by Definition 2.3, and $\subseteq$ is define as in (i).

From $\Gamma(a) \wedge \Gamma(a) = \Gamma(a)$, it follows that

$\Gamma(a) \subseteq \Gamma(a)$(Reflexivity)

If $\Gamma(a) \subseteq \Gamma(b)$and $\Gamma(b) \subseteq \Gamma(a)$ then,

$\Gamma(a) = \Gamma(a) \wedge \Gamma(b)$ and $\Gamma(b) = \Gamma(b) \wedge \Gamma(a)$;

Hence, $\Gamma(a) = \Gamma(b)$. (Antisymmetry)

If $\Gamma(a) \subseteq \Gamma(b)$ and $\Gamma(b) \subseteq \Gamma(c)$ then,

$\Gamma(a) = \Gamma(a) \wedge \Gamma(b)$ and $\Gamma(b) = \Gamma(b) \wedge \Gamma(c)$,

so, $\Gamma(a) = \Gamma(a) \wedge \big(\Gamma(b) \wedge \Gamma(c)\big)$;

$= \big(\Gamma(a) \wedge \Gamma(b)\big) \wedge \Gamma(c)$

$= \Gamma(a) \wedge \Gamma(c)$.

Hence, $\Gamma(a) \subseteq \Gamma(c)$. (Transitivity)

This shows that $\subseteq$ is a partial ordered on L.

Also, from $\Gamma(a) = \Gamma(a) \wedge \big(\Gamma(a) \vee \Gamma(b)\big)$ and $\Gamma(b) = \Gamma(b) \wedge \big(\Gamma(a) \vee \Gamma(b)\big)$

We have, $\Gamma(a) \subseteq \Gamma(a) \vee \Gamma(b)$and $\Gamma(b\,) \subseteq \Gamma(a) \vee \Gamma(b)$,

$\Rightarrow \Gamma(a) \vee \Gamma(b)$is an upper bound of both $\Gamma(a)$ and $\Gamma(b)$.

If $\Gamma(a) \subseteq \Gamma(u)$and $\Gamma(b) \subseteq \Gamma(u)$, where $\Gamma(u) = \big(\Gamma(a) \vee \Gamma(b)\big)$then,

$\Gamma(a) \vee \Gamma(u) = \big(\Gamma(a) \vee \Gamma(u)\big) \vee \Gamma(u) = \Gamma(u)$, and

$\Gamma(b) \vee \Gamma(u) = \big(\Gamma(b) \vee \Gamma(u)\big) \vee \Gamma(u) = \Gamma(u)$.

So, $\big(\Gamma(a) \vee \Gamma(u)\big) \vee \big(\Gamma(b) \vee \Gamma(u)\big) = \Gamma(u) \vee \Gamma(u) = \Gamma(u)$;

Hence, $\big(\Gamma(a) \vee \Gamma(b)\big) \vee \Gamma(u) = \Gamma(u)$,

Now, $\big(\Gamma(a) \vee \Gamma(b)\big) \wedge \Gamma(u) = \big(\Gamma(a) \vee \Gamma(b)\big) \wedge \big[\big(\Gamma(a) \vee \Gamma(b)\big) \vee \Gamma(u)\big]$

$= \Gamma(a) \vee \Gamma(b)$(by the absorption law),

$\Rightarrow \Gamma(a) \vee \Gamma(b) \subseteq \Gamma(u)$.

Thus, $\Gamma(a) \vee \Gamma(b) = sup\{\Gamma(a), \Gamma(b)\}$.

In the same way we see that$\Gamma(a) \wedge \Gamma(b) = inf\{\Gamma(a), \Gamma(b)\}$..

Similarly, from definition 2.10, the $sup\{\Gamma(a), \Gamma(b)\} = \Gamma(a) \vee \Gamma(b)$ and the $inf\{\Gamma(a), \Gamma(b)\} = \Gamma(a) \wedge \Gamma(b)$,the result follows.

Theorem 2.4

Given a soft semilattice (Γ, E), define $\Gamma(\mathrm{e}_1) \subseteq \Gamma(\mathrm{e}_2)$ if and only if $\Gamma(\mathrm{e}_1) \wedge \Gamma(\mathrm{e}_2) = \Gamma(\mathrm{e}_1)$. Then $\big((\Gamma, E), \subseteq\big)$ is an ordered soft set in which every pair of elements has greatest lower bound.

Proof

Let $\big((\Gamma, E), \subseteq\big)$ be a soft semilattice, and define $\subseteq$ as above. First we check that $\subseteq$ is partial ordered.

(i) Reflexive

$$\Gamma(\mathrm{e}_1) \wedge \Gamma(\mathrm{e}_1) = \Gamma(\mathrm{e}_1)$$
$$\Rightarrow \Gamma(\mathrm{e}_1) \subseteq \Gamma(e_1)$$

(ii) Antisymmetry

If $\Gamma(e_1) \subseteq \Gamma(e_2)$ and $\Gamma(e_2) \subseteq \Gamma(e_1)$, then

$$\Gamma(e_1) = \Gamma(\mathrm{e}_1) \wedge \Gamma(e_2)$$
$$= \Gamma(e_2) \wedge \Gamma(e_2)$$
$$= \Gamma(e_2)$$

(iii) Transitivity

If $\Gamma(e_1) \subseteq \Gamma(e_2) \subseteq \Gamma(e_3)$, then

$$\Gamma(e_1) \wedge \Gamma(\mathrm{e}_3) = \big(\Gamma(\mathrm{e}_1) \wedge \Gamma(e_2)\big) \wedge \Gamma(e_3)$$
$$= \Gamma(e_1) \wedge \big((e_2) \wedge \Gamma(\mathrm{e}_3)\big)$$
$$= \Gamma(e_1) \wedge \Gamma(e_2) = \Gamma(e_1), \text{ so } \Gamma(e_1) \subseteq \Gamma(e_3).$$

Since, $\big(\Gamma(e_1) \wedge \Gamma(e_2)\big) \wedge F(e_1) = \Gamma(e_1) \wedge \big(\Gamma(\mathrm{e}_1) \wedge \Gamma(e_2)\big)$

$= \big(\Gamma(e_1) \wedge \Gamma(\mathrm{e}_1)\big) \wedge \Gamma(e_2)$

$= \Gamma(e_1) \wedge \Gamma(e_2)$

We have, $\Gamma(e_1) \wedge \Gamma(e_2) \subseteq F(e_1)$

Similarly, $\Gamma(e_1) \wedge \Gamma(e_2) \subseteq \Gamma(e_2)$.

Thus, $\Gamma(e_1) \wedge \Gamma(\mathrm{e}_2)$ is a lower bound for $\{\Gamma(e_1), \Gamma(\mathrm{e}_2)\}$. To see that it is the greatest lower bound, suppose$\Gamma(e_3) \subseteq \Gamma(e_1)$ and $\Gamma(\mathrm{e}_3) \subseteq \Gamma(\mathrm{e}_2)$. Then

$$\Gamma(e_3) \wedge \big(\Gamma(e_1) \wedge \Gamma(\mathrm{e}_2)\big) = \big(\Gamma(e_3) \wedge \Gamma(e_1)\big) \wedge \Gamma(e_2)$$
$$= \Gamma(e_3) \wedge \Gamma(e_2) = \Gamma(e_3)$$

So, $\Gamma(e_3) \subseteq \Gamma(e_1) \wedge \Gamma(e_2)$, as desired.

3.0 Hasse Diagram for soft lattice

Soft set can be represented on Hasse Diagram. Let (Γ, E) be a partial ordered soft set, let E be the set of parameters such that$\Gamma: E \rightarrow P(U)$. Let $\Gamma(a), \Gamma(b), \Gamma(c) \in (\Gamma, E)$, we say that $\Gamma(b)$ covers $\Gamma(a)$ or $\Gamma(a)$ is covered by $\Gamma(b)$, if $\Gamma(a) \subseteq \Gamma(b)$, and whenever $\Gamma(a) \subseteq \Gamma(c) \subseteq \Gamma(b) \Rightarrow \Gamma(a) = \Gamma(c)$ or $\Gamma(c) = \Gamma(b)$. We use the notation $\Gamma(a) \prec \Gamma(b)$ to denote $\Gamma(a)$ is covered by $\Gamma(b)$.

The closed interval $[\Gamma(a), \Gamma(b)]$ is defined to be the set of $\Gamma(c)$ in (Γ, E) such that$\Gamma(a) \subseteq \Gamma(c) \subseteq \Gamma(b)$, and the open interval $(\Gamma(a), \Gamma(b))$ is the set of $\Gamma(c)$ in (Γ, E) such that $\Gamma(a) \subset \Gamma(c) \subset \Gamma(b)$. Now, we represent each element of (Γ, E) by a small circle "o". If $\Gamma(a) \prec \Gamma(b)$ then we draw the circle for $\Gamma(b)$ above the circle for $\Gamma(a)$, joining the two circles with a line segment. From this diagram we can recaptured the relation $\subseteq$ by noting that $\Gamma(a) \subseteq \Gamma(c)$ holds if and only if for some finite $\Gamma(c_1), \Gamma(c_1), \ldots, \Gamma(c_n)$ from (Γ, E) we have $\Gamma(a) = \Gamma(c_1) \prec \Gamma(c_1) \ldots \Gamma(c_{n-1}) \prec \Gamma(c_n) = \Gamma(b)$.

Some examples of Hasse Diagram of soft lattice.

Let (Γ, E) be a partial ordered soft set, let E be the set of parameters such that $\Gamma: E \rightarrow P(U)$. Let $\Gamma(a), \Gamma(b), \Gamma(c), \Gamma(d), \Gamma(e), \Gamma(f), \Gamma(g), \Gamma(h) \in (\Gamma, E)$, we use the Hasse Diagram to represent the following:

(i) $\Gamma(a) \prec \Gamma(b)$, $\Gamma(a) \prec \Gamma(c)$ and $\Gamma(d) \prec \Gamma(c)$

(ii) $\Gamma(d) \prec \Gamma(a) \prec \Gamma(h)$,$\Gamma(f) \prec \Gamma(b)$, $\Gamma(g) \prec \Gamma(e) \prec \Gamma(a)$ and $\Gamma(e) \prec \Gamma(c) \prec \Gamma(h)$

(iii) $\Gamma(c) \prec \Gamma(a)$, $\Gamma(d) \prec \Gamma(a)$,$\Gamma(c) \prec \Gamma(b)$ and $\Gamma(d) \prec \Gamma(b)$

(iv) $\Gamma(b) \prec \Gamma(a)or\Gamma(a) \prec \Gamma(b)$

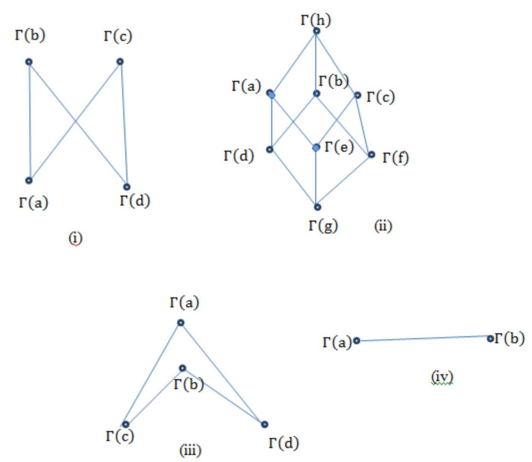

Figure 1: Representation of elements of (Γ, E) in Hasse Diagram.

4.0 Conclusion

This paper briefly presents some fundamentals of soft set theory[2, 4, 7, 10, 11],to show that soft set has enough developed basic supporting tools through which various algebraic structures in theoretical point of view could be developed. We redefined the concepts of lattices using soft sets theory and present their properties. In particular, a perception named soft lattice is introduced where some related results were established. We represent the elements of soft lattice on Hasse Diagram [15], in soft set context.

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