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EOQ Model for Deteriorating Items that Exhibit Delay in Deterioration with Discrete Time

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Abstract

In this paper, we developed an Economic Order Quantity (EOQ) inventory model for delayed deteriorating items with discrete time and constant decay rate of on-hand inventory. The model was used to determine an optimum ordering quantity and replenishment cycle. The demand before deterioration is different from the demand after deterioration has set in which are both constant. Some numerical examples were given to illustrate the application of the model.

1.0 Introduction

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original [1]. Blood, fruits, gasoline, radioactive chemicals and grain products are examples of deteriorating commodities.

Ghare and Schrader [2] were the first to point out the effect of decay in inventory analysis. They developed an EOQ model for experimentally decay inventory. Covert and Philip [3] developed EOQ model for items with variable rate of deterioration by assuming Weibull density function for the rate of deterioration of the item. This work was generalized by Shah [4] to allow for Shortages and considered general deteriorating function. Misra [5] developed a deterministic model with a finite production rate. Shah and Jaiswal [6] presented an order-level inventory model for deteriorating items with a constant rate of deterioration. Their result is similar to those of Misra for a constant deterioration rate and to allow for shortages. Hwang and Sohn [7] developed a model for the management of deteriorating inventory under inflation with known price increases. Wee and Yu [8] developed an inventory model of deteriorating items with a temporary price discount by assuming deterioration to be a function of the on hand inventory. Hollier and Mark [9] developed a model for inventory replenishment policies of deteriorating items in a declining market. An EOQ model for deteriorating items with time varying demand and partial backlogging was also developed by Chang and Dye [10].

Patra and Ratha [11] developed an economic order quantity model for deteriorating items with increasing time varying demand cost. Also Patra and Ratha [12] developed a finite planning horizon inventory model for deteriorating items with stock-dependent where shortages are allowed and backlogged.

In all the literature above, the authors considered deterioration to begin as soon as the items are held in stock. In practical terms however, there are some items that do not start deteriorating immediately they are stocked. Some of these items include Potatoes, Yams, Tomatoes, Beans and so on. The depletion of these items will depend only on demand before deterioration sets in. When deterioration later sets in, depletion will depend on both demand and deterioration. This process is termed as non-instantaneous deterioration in Wu *et al*. [13] and delayed Deterioration in Musa and Sani [14], [15]. Wu *et al*.[13] developed an optimal replenishment model for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Ouyang et al [16] also constructed an inventory model for non-instantaneous deteriorating items when a supplier provides a permissible delay in payments. Ji [17] developed a deterministic inventory model for non instantaneous deteriorating items which starts with shortages and ends without shortages. Chung [18] provided a complete proof on the solution procedure for the model developed by Ouyang *et al*. [16] . Bindu and Garima [19] developed an inventory model with stock dependent and time varying decreasing demand. A temporary discount on selling price before the start of deterioration is given to enhance the demand in order to boost the inventory depletion rate. The work is an extension of Panda *et al*. [20] who developed an EOQ model for an infinite time horizon for perishable products with discounted selling price and stock dependent demand with non-instantaneous constant rate of deterioration.

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An EOQ model for some tropical items that exhibit delay in deterioration was developed by Musa and Sani [14]. Later Musa and Sani [15] developed inventory ordering policies of delayed deteriorating items under permissible delay in payments. Shalu [21] developed an inventory model for non-instantaneous deteriorating products with price and time dependent demand where shortages are allowed and is completely backlogged. Also Palanivel and Uthayakumar [22] developed an EOQ model for non-instantaneous deteriorating items with power demand, time dependent holding cost, partial backlogging and permissible delay in payments.

In all the above models, time was taken as a continuous variable, which is not always the case in practice. In real life problems, time is in many cases treated as a discrete variable. For instance, responses to a five-point rating scale can only take the values 1, 2, 3, 4 and 5. Time duration is normally counted in terms of complete units of days, weeks, months and so on.

There are authors who developed inventory models for deteriorating items considering the time to be discrete but they are few. It is worth mentioning the pioneering work of Nahmias [23] in the study of discrete time inventory systems for perishable items. In the study, demand was assumed to be random in each period and products were assumed to have a certain life time which may be random. Dave [24] developed an order level inventory model for deteriorating items in which time variable was assumed to be discrete. In the work, a deterministic model with instantaneous delivery was considered, where it was assumed that a constant fraction of the on hand inventory deteriorated over time. Gupta and Jauhari [25] determined optimum ordering interval for constant decay rate inventory, under the condition of instantaneous replenishment with time assumed to be discrete. Aliyu and Boukas [26] developed discrete-time inventory models with deterministic or stochastic demand. Zhaotong and Liming [27] developed a discrete-time model for perishable inventory systems with geometric inter-demand times and batch demands. Castro and Alfa [28] proposed a life time replacement policy in discrete time for a single unit system. Ferhan *et al.*[29] developed an inventory model of deteriorating items on non-periodic discrete-time domains where time points may not be necessarily evenly spaced over a time interval.

In this present paper, a model for items that exhibit delay in deterioration is presented in which the time variable, t , is assumed to be discrete. Our aim is to remodel the work of Musa and Sani [14] to a situation where time is considered to be discrete as it is in many cases in reality.

2.0 The Mathematical Model

The following notation and assumptions are made in developing the model.

Notation

- $\mathbf{D}_1$ The demand rate (units per unit time) during the period before deterioration sets in.
- $\mathbf{D}_2$ The demand rate (units per unit time) after deteriorating sets in.
- $\mathbf{T}_1$ The time when deterioration sets in.
- **Q** The order quantity per order.
- **T** The inventory cycle length (time units).
- **C** The unit cost of the item.
- **K** The ordering cost per order.
- i The inventory carrying charge.
- λ The rate of deterioration in a unit time. ($0<\lambda<1$)
- **I (t)** The inventory level at any time t, before deterioration begins.
- $\mathbf{I}_d$ **(t)** The inventory level at any time t, after deterioration sets in.

Assumptions

- (i) Replenishment is instantaneous.
- (ii) The lead time is zero.
- (iii) Shortages are not allowed.
- (iv) There is no replacement or repair of the deteriorated items during the period under consideration.
- (v) Unconstrained suppliers capital (payment is made immediately the item is supplied)
- (vi) $T_1 \le T$

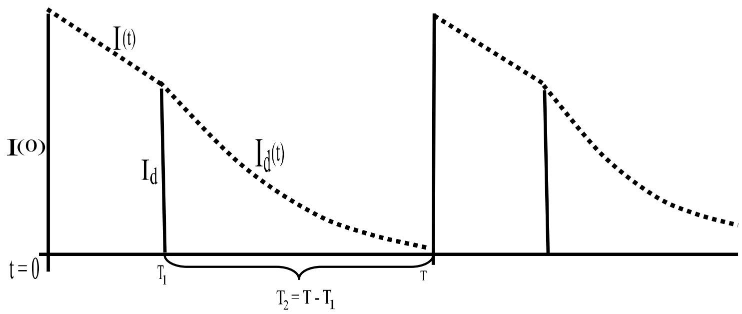

Fig 1: The position of inventory in every cycle

As it is under notation, T_1 is the time deterioration sets in, I_d is the inventory level at the time deterioration begins, T_2 is the difference between the cycle length T and the time when deterioration sets in. Also, I(t) is the inventory level at anytime t, in the region $(0 \le t \le T_1)$ .Where $I(0)$ is the initial inventory.

Depletion of inventory from the beginning of the cycle up to the time deterioration sets in will occur only due to demand. Since time is taken to be discrete, the difference equation describing the inventory level of the system at any time $t, (0 \le t \le T_1)$ is given by

$$\Delta I(t) = -D_1 \tag{1}$$

This can be solved as follows:

Since $\Delta f(x) = f(x+h) - f(x)$, where h is the step length,

then $\Delta I(t) = I(t+1) - I(t)$

for $t = 0,1,2,...,(\mathrm{m}-1), \mathrm{m} = T$,

$$\therefore \Delta I(0) = I(1) - I(0) = -D_1$$

$$\Rightarrow I(1) = I(0) - D_1$$

$$\Delta I(1) = I(0) - 2D_1$$

$$\Delta I(2) = I(0) - 3D_1$$

So that continuing up to t , we get

$$I(t) = I(0) - tD_1 \tag{2}$$

Using equation (2) and applying the boundary conditions at $t = T_1,\ I(t) = I_d$ and we have

$$I_d = I(0) - T_1 D_1$$

this implies

$$I(0) = I_d + D_1 T_1 \tag{3}$$

Substituting Equation (3) into (2) we get

$$I(t) = I_d + D_1 T_1 - tD_1$$

Or

$$I(t) = I_d + (T_1 - t)D_1 \tag{4}$$

Since $I_d(t)$ is the Inventory level at any time t after deterioration sets in and D_2 is the demand rate at the time deterioration begins, the decrease in Inventory level will hence forth depend on both deterioration and demand.

The difference equation which describes the instantaneous state of the Inventory over (T_1, T) is given by

$$\Delta I_d(t) = -\lambda I_d(t) - D_2 \tag{5}$$

$T_1 \le t \le T$,

However, $\Delta I_d(T_1) = I_d(T_1+1) - I_d(T_1) = -\lambda I_d(T_1) - D_2$ from equation (5)

$$\Rightarrow I_d(T_1+1) = (1-\lambda) I_d(T_1) - D_2$$

In a similar way,

$$I_d(T_1+2)=(1-\lambda)^2 I_d(T_1)+(\lambda-2)D_2$$

Similarly,

$$I_d(\mathrm{T}_1+3)=(1-\lambda)^3 I_d(T_1)+\left(\frac{(1-\lambda)^3-1}{\lambda}\right)\mathrm{D}_2$$

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$$\Rightarrow I_d(\mathrm{T}_1+s)=(1-\lambda)^s I_d(T_1)+\left(\frac{(1-\lambda)^s-1}{\lambda}\right)\mathrm{D}_2 \qquad (6)$$

for S = 0,1,........(T-T_1)

If $T_1+s=t \Rightarrow s=t-T_1$

$$\therefore I_d(t)=I_d(T_1)(1-\lambda)^{t-T_1}-\frac{D_2}{\lambda}(1-(1-\lambda)^{t-T_1}) \qquad (7)$$

At $t=T,\ \ I_d(T)=0$ equation (7) becomes

$$\Rightarrow 0=I_d(T_1)(1-\lambda)^{T-T_1}-\frac{D_2}{\lambda}(1-(1-\lambda)^{T-T_1}) \qquad (8)$$

However, $I_d(T_1)=I_d$

i.e. $$I_d=-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T}) \qquad (9)$$

Hence, substituting equation (9) into (4), we get:

$$I(t)=-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T})+(T_1-t)D_1 \qquad (10)$$

The total demand between T_1 and T = the demand rate at the onset of deterioration multiplied by the time period during which the item deteriorates.

$\therefore$ Total demand between T_1 and T = D_2T_2. Let $d(T_2)$ be the number of items that deteriorate during the time interval $[T_1,T]$, then

$$d(T_2)=I_d-D_2T_2 \qquad (11)$$

Substituting I_d from equation (9) into (11) we have

$$d(T_2)=-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T})-D_2T_2$$

i.e. $$d(T_2)=-\frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T}+\lambda T_2] \qquad (12)$$

The total number of units in inventory during a cycle is determined from the build up of inventory as follows:
From equation (4),

$$I(0)=I_d+(T_1-0)D_1$$
$$I(2)=I_d+(T_1-2)D_1$$
$$I(3)=I_d+(T_1-3)D_1$$

Since $$I_d=-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T})$$

Continuing up to T_1-1 we get

$$I(T_1-1)=I_d+D_1$$ So that at T_1 we get

$I(T_1) = I_d$

Thus the total number of units in the period $[0, T_1]$ is

$$\sum_{t=0}^{T_1} I(\text{t}) = (\text{T}_1+1)\,\text{I}_\text{d} + (0+1+2+3+...+(\text{T}_1-1)+\text{T}_1)\,\text{D}_1$$

$$= \frac{T_1+1}{2}\left(2I_d + T_1 D_1\right)$$

So that the average number of units per unit time in the period [0, T_1] is given as

$$I_A(t) = \frac{1}{T_1+1}\sum_{t=0}^{T_1} I(\text{t})$$

$$= \frac{\left[2I_d + T_1 D_1\right]}{2} \qquad (13)$$

In a Similar way, to obtain

$\sum_{t=T_1+1}^{T} I_d(\text{t})$ for $T_1+1 \le t \le T$ we proceed as follows:

for $t = T_1+1$ $\quad I_d(\text{t}) = (1-\lambda)\,\text{I}_\text{d} + \left(\frac{(1-\lambda)-1}{\lambda}\right)D_2$

for $t = T_1+2$ $\quad I_d(\text{t}) = (1-\lambda)^2\,\text{I}_\text{d} + \left(\frac{(1-\lambda)^2-1}{\lambda}\right)D_2$

for $t = T_1+3$ $\quad I_d(\text{t}) = (1-\lambda)^3\,\text{I}_\text{d} + \left(\frac{(1-\lambda)^3-1}{\lambda}\right)D_2$

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for $t = T-1$ $\quad I_d(\text{t}) = (1-\lambda)^{(\text{T}-\text{T}_1)-1}\,\text{I}_\text{d} + \left(\frac{(1-\lambda)^{(\text{T}-\text{T}_1)-1}-1}{\lambda}\right)D_2$

for $t = T$ $\quad I_d(\text{t}) = (1-\lambda)^{(\text{T}-\text{T}_1)}\,\text{I}_\text{d} + \left(\frac{(1-\lambda)^{(\text{T}-\text{T}_1)}-1}{\lambda}\right)D_2$

Let $1-\lambda = q$

$$\therefore \sum_{t=T_1+1}^{T} I_d(\text{t}) = I_d\left[q+q^2+q^3+...+q^{(\text{T}-\text{T}_1)}\right] + \frac{D_2}{\lambda}\left[q+q^2+q^3+...+q^{(\text{T}-\text{T}_1)} - (\text{T}-\text{T}_1)\right]$$

$$= \frac{I_d\left[\lambda q(1-\text{q}^{(\text{T}-\text{T}_1)})\right] + D_2\left[q(1-\text{q}^{(\text{T}-\text{T}_1)}) - (1-q)(\text{T}-\text{T}_1)\right]}{(1-\text{q})\lambda}$$

Since $1-\lambda = q \Rightarrow 1-q = \lambda$

This implies $\sum_{t=T_1+1}^{T} I_d(\text{t}) = \dfrac{I_d\left[\lambda(1-\lambda)(1-(1-\lambda)^{(\text{T}-\text{T}_1)})\right] + D_2\left[(1-\lambda)(1-(1-\lambda)^{(\text{T}-\text{T}_1)}) - \lambda(\text{T}-\text{T}_1)\right]}{\lambda^2}$

$$= \frac{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2}\left[\lambda I_d + D_2 - \frac{\lambda(T-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right] \qquad (14)$$

Hence $$\sum_{t=0}^{T} I(t) = \frac{T_1+1}{2}\left[2I_d + T_1 D_1\right] + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2}\left[\lambda I_d + D_2 - \frac{\lambda(T-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right]$$

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Therefore, average inventory per unit time will be $\frac{1}{T_1+1}\sum_{t=0}^{T_1} I(t) + \frac{1}{T-T_1}\sum_{t=T_1+1}^{T} I_d(t)$

$$= \frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2 (T-T_1)}\left[\lambda I_d + D_2 - \frac{\lambda(T-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right]$$

(16)

**Inventory Carrying Cost (or Holding Cost)**

Let the inventory carrying cost or holding cost be $C_H$, which is the cost associated with the storage of the inventory until it is sold or used.

Average holding Cost ($H_A$) = unit cost of item multiplied by inventory carrying charge ( i%) multiplied by average inventory

$$\therefore \quad H_A = i\% \text{ x unit cost x}\left[\frac{1}{T_1+1}\sum_{t=0}^{T_1} I(t) + \frac{1}{T-T_1}\sum_{t=T_1+1}^{T} I_d(t)\right] \qquad (17)$$

$$= ic\left(\frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2 (T-T_1)}\left[\lambda I_d + D_2 - \frac{\lambda(T-T_1)D_2}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right]\right)$$

$$= ic\left(\frac{\left[2I_d + T_1 D_1\right]}{2} + \frac{D_2(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2 (T-T_1)}\left[(1-\lambda)^{(T_1-T)} - \frac{\lambda(T-T_1)}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right]\right) \qquad (18)$$

**Total Variable Cost**

The total variable cost per unit time in a cycle is given by

$$C(T) = \frac{K}{T} + C\frac{d(T_2)}{T} + H_A$$

$$= \frac{K}{T} + \frac{C}{T}\left(\frac{-D_2}{\lambda}\left[1-(1-\lambda)^{(T_1-T)} + (T-T_1)\lambda\right]\right)$$

$$+ic\left[\frac{-D_2}{\lambda} + \frac{D_2}{\lambda}(1-\lambda)^{T_1-T} + \frac{T_1 D_1}{2} + \frac{D_2(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}{\lambda^2 (T-T_1)}\left[(1-\lambda)^{(T_1-T)} - \frac{\lambda(T-T_1)}{(1-\lambda)\left[1-(1-\lambda)^{(T-T_1)}\right]}\right]\right]$$

(19) Since $T$ must be a non-negative integer the conditions for $C(T)$ to have a minimum at $T = T^*$ are

$C(T^*) \le C(T^*-1)$ and $C(T^*) \le C(T^*+1)$

$\Rightarrow C(T^*) - C(T^*-1) \le 0$ and $C(T^*+1) - C(T^*) \ge 0$

$\Rightarrow \Delta C(T^*-1) \le 0$ and $\Delta C(T^*) \ge 0$

$\Rightarrow \Delta C(T^*-1) \le 0 \le \Delta C(T^*)$

$$\Delta C(T^*-1) \le 0 \le \Delta C(T^*)$$

$$C(T^*) - C(T^*-1) \le 0 \le C(T^*+1) - C(T^*) \qquad (20)$$
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i.e

$$\frac{-K}{D_2}+\frac{c}{\lambda}-cT_1+cT(1-\lambda)^{T_1-T}-\frac{c(1-\lambda)^{T_1-T}}{\lambda}+icT(T-1)(1-\lambda)^{T_1-T}$$
$$+\frac{icT(T-1)(1-\lambda)^{T_1-T+1}\left[\lambda(T-T_1)-1\right]+icT(T-1)(1-\lambda)}{\lambda^2(T-T_1)(T-1-T_1)}\leq 0\leq$$
$$\frac{-K}{D_2}+\frac{c}{\lambda}-cT_1+\frac{c(1-\lambda)^{T_1-T-1}\left[\lambda T+\lambda-1\right]}{\lambda}+icT(T+1)(1-\lambda)^{T_1-T-1}$$
$$+\frac{icT(T+1)(1-\lambda)^{T_1-T}\left[\lambda(T+1-T_1)-1\right]+icT(T+1)(1-\lambda)}{\lambda^2(T-T_1)(T+1-T_1)}$$

This implies

$$cT(1-\lambda)^{T_1-T}-\frac{c(1-\lambda)^{T_1-T}}{\lambda}+icT(T-1)(1-\lambda)^{T_1-T}+\frac{icT(T-1)(1-\lambda)^{T_1-T+1}\left[\lambda(T-T_1)-1\right]+icT(T-1)(1-\lambda)}{\lambda^2(T-T_1)(T-1-T_1)}\leq\frac{K}{D_2}-\frac{c}{\lambda}+cT_1\leq$$
$$\frac{c(1-\lambda)^{T_1-T-1}\left[\lambda T+\lambda-1\right]}{\lambda}+icT(T+1)(1-\lambda)^{T_1-T-1}+\frac{icT(T+1)(1-\lambda)^{T_1-T}\left[\lambda(T+1-T_1)-1\right]+icT(T+1)(1-\lambda)}{\lambda^2(T-T_1)(T+1-T_1)} \quad (21)$$

The EOQ Corresponding to T is determined as follows:

$$EOQ = D_1T_1+D_2T_2+d(T_2)$$
$$=D_1T_1+D_2(T-T_1)+d(T_2)$$

and from Equation(12)

$$EOQ=D_1T_1+D_2(T-T_1)-\frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T}+\lambda T_2]$$
$$=D_1T_1+D_2(T-T_1)-\frac{D_2}{\lambda}[1-(1-\lambda)^{T_1-T}+\lambda(T-T_1)]$$
$$=D_1T_1+D_2(T-T_1)-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T})-D_2(T-T_1)$$
$$=D_1T_1-\frac{D_2}{\lambda}(1-(1-\lambda)^{T_1-T}) \quad (22)$$

Table 1: Tabulation of the Solutions of ten different numerical examples for different values of the parameters as indicated

S/N	D_1 (units per day)	D_2 (units per day)	T_1	C	K	i	λ	T	EOQ
1	800/365	300/365	7days	60	2300	0.13/365	0.4	10days	22.80
2	500/365	250/365	14days	50	2000	0.15/365	0.5	16days	23.29
3	1000/365	500/365	21days	70	2500	0.14/365	0.6	22days	60.96
4	2000/365	500/365	28 days	80	3000	0.12/365	0.3	29days	155.40
5	900/365	400/365	35 days	35	3500	0.11/365	0.35	37days	90.58
6	1200/365	1000/365	6 days	40	2200	0.10/365	0.2	11days	47.83
7	1500/365	900/365	13 days	65	2400	0.15/365	0.25	15days	61.10
8	2000/365	1500/365	20 days	75	2800	0.16/365	0.45	21days	117.06
9	2500/365	2000/365	27 days	85	3200	0.17/365	0.32	28days	192.99
10	2300/365	1700/365	36 days	55	3700	0.18/365	0.37	37days	234.24

In Table 1 the values of the parameters used are the same as those of Musa and Sani [14].The division by 365 days is to take care of the discrete nature of our work which takes the unit of time to be days. The following are our observations: In many cases our T is less than their T. This will reduce the holding cost of the item, since the item will stay for less number of days in the stock. Our T will also reduce deterioration thereby reducing the deterioration cost.

Table 2: Effect of changing the demand rate D_1 on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	1200/365	300/365	7 days	60	2300	0.13/365	0.4	10days	30.47
2	1000/365	300/365	7 days	60	2300	0.13/365	0.4	10days	26.64
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	600/365	300/365	7 days	60	2300	0.13/365	0.4	10days	18.96
5	400/365	300/365	7 days	60	2300	0.13/365	0.4	10days	15.13

Table 3: Effect of changing the demand rate D_2 on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	800/365	400/365	7 days	60	2300	0.13/365	0.4	10days	25.29
2	800/365	350/365	7 days	60	2300	0.13/365	0.4	10days	24.04
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	250/365	7 days	60	2300	0.13/365	0.4	10days	21.55
5	800/365	200/365	7 days	60	2300	0.13/365	0.4	11days	24.54

Table 4: Effect of changing the cost of item, C, on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	800/365	300/365	7 days	80	2300	0.13/365	0.4	10days	22.80
2	800/365	300/365	7 days	70	2300	0.13/365	0.4	10days	22.80
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	300/365	7 days	50	2300	0.13/365	0.4	10days	22.80
5	800/365	300/365	7 days	40	2300	0.13/365	0.4	11days	29.14

Table 5: Effect of changing the ordering cost, K, on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	800/365	300/365	7 days	60	2700	0.13/365	0.4	10days	22.80
2	800/365	300/365	7 days	60	2500	0.13/365	0.4	10days	22.80
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	300/365	7 days	60	2100	0.13/365	0.4	10days	22.80
5	800/365	300/365	7 days	60	1900	0.13/365	0.4	10days	22.80

Table 6: Effect of changing the inventory carrying charge rate i on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	800/365	300/365	7 days	60	2300	0.09/365	0.4	10days	22.80
2	800/365	300/365	7 days	60	2300	0.11/365	0.4	10days	22.80
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	300/365	7 days	60	2300	0.15/365	0.4	10days	22.80
5	800/365	300/365	7 days	60	2300	0.17/365	0.4	10days	22.80

Table 7: Effect of changing the deterioration rate λ on the decision variables

S/N	D_1	D_2	T_1	C	K	I	λ	T	EOQ
1	800/365	300/365	7 days	60	2300	0.13/365	0.3	11days	24.00
2	800/365	300/365	7 days	60	2300	0.13/365	0.35	11days	26.10
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	300/365	7 days	60	2300	0.13/365	0.45	10days	24.50
5	800/365	300/365	7 days	60	2300	0.13/365	0.50	9days	20.27

Table 8: Effect of changing the time T_1on the decision variables

S/N	D_1	D_2	T_1	C	K	i	λ	T	EOQ
1	800/365	300/365	11days	60	2300	0.13/365	0.4	14days	31.57
2	800/365	300/365	9 days	60	2300	0.13/365	0.4	12days	27.18
3	800/365	300/365	7 days	60	2300	0.13/365	0.4	10days	22.80
4	800/365	300/365	5 days	60	2300	0.13/365	0.4	8days	18.42
5	800/365	300/365	3 days	60	2300	0.13/365	0.4	7days	20.37

3.0 Discussion on Results

Changes in the values of parameters may occur due to uncertainties in any decision-making situation. In order to examine the effects of these changes, a sensitivity analysis will be of great help. Using the computational results as shown in Tables 2 – 7 with respect to the Parameters D_1, D_2, c, K, i, λ and T_1, we get the following observations:

(1) From Table 2, a higher value of D_1 results in a higher value of EOQ but the value of remains almost unchanged. This implies that increase in the demand rate before deterioration will result in the increase of the Economic Order Quantity (EOQ) and T remains unchanged. This is expected since at that time there is no deterioration so more stock can be kept to cater for the increase in demand.

(2) From Table 3, a higher value of D_2 results in a higher value of EOQ but lower value of T. This implies that increase in the demand rate when deterioration has set in will result in the increase of the Economic Order Quantity, but decrease in the cycle length. This is also to be expected since there is more demand and decrease in T is to reduce the deterioration.

(3) From Table 4, a higher value of C, the item's cost, does not have effect on the EOQ. Also in table 5 a higher value of ordering cost K does not affect the EOQ. Likewise in table 6 a higher value of carrying charge i does not change the value of EOQ and cycle length.

(4) From Table 7, a higher value of the rate of deterioration results in lower values of EOQ and T. This is expected because if deterioration rate is high, EOQ should be low to avoid much deterioration of the item.

(5) From Table 8, a higher value of T_1 results in higher value of EOQ. This is expected since if T_1 is high it will take more time before deterioration sets in and hence the EOQ will be high.

4.0 Conclusion

In this paper, an economic order quantity (EOQ) model on inventory of deteriorating items which exhibit delay in deterioration taking time to be discrete is presented. Demand before deterioration sets in is different from demand after deterioration has set in. The depletion of the items before deterioration sets in is dependent solely on demand but when deterioration sets in, depletion now depends on both demand and deterioration. Items that mostly exhibit this property are farm produce like tomatoes, potatoes, yams, beans and so on. Our objective is to minimize cost and based on the sensitivity analysis carried out, changes in the parameters D_1, D_2, T_1 and λ result in some significant changes of the EOQ. It is therefore very important to estimate them very appropriately in using the model. Another important issue is that the T value from our model is usually smaller than that from Musa and Sani [14].This gives a smaller annual inventory cost due to smaller holding cost and smaller deterioration cost.

5.0 References

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